ANALYSIS AND DESIGN OF HYBRID STRUCTURES USING OPTIMIZATION STRATEGIES*

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Summary

The paper gives a general overview and concerns with a specified set of computer-aided analysis modules for hybrid structures loaded by extreme excitations. All problems are solved by methods of linear, quadratic or nonlinear mathematical optimization, that leads to very effective and economic design solutions. All approaches are derived from general optimization problem that can be easily altered to conform to specific design tasks. Some advantages and possibilities of hybrid structural modeling (single or mixed model-supported) are discussed. The methods will be illustrated by an example structure and optimization schemes.

1 Introduction

The development of numerical methods in the last decades leads to the availability of a huge amount of tools for the analysis of non-linear structures. Besides dominating technologies performing incremental iterative solving some strategies methods based on mathematical optimization are increasingly applied. These optimization algorithms are principally qualified for solving several classes of initial and boundary value problems. The application of these algorithms is beneficial if a certain amount of subsidiary conditions have to be fulfilled while considering design objectives [1].

Especially the design tasks in civil engineering correspond to this type of numerical interface. That’s why mechanical problems can be descriptively formulated as optimization problems. The advantages over strategies basing on the solution of several linear equilibrium systems result from using inequality condition i.e. for the formulation of the limit state conditions (plasticity-, contact conditions etc.) and result from the presence of an objective function for the specification of a design intentions. The non-linear behavior of structures can be either implemented as non-linear equations or as a combination of equality and inequality conditions. The application of those methods offers multifaceted possibilities supporting the solution of analysis and design problems in engineering.

Hereby special problems for revitalization of existing buildings have to be considered, that include problems caused by the change of the static system combined with the weakening of the stability system of the structure, the loss of symmetry and the loss of stabilizing vertical loads resulting from the break down of parts of the structure. The research is mainly connected to the extensive efforts done for the revitalization of panel structures built up in a huge amount in the eastern part of Germany (Fig. 1).

The proof of structural capacities is often problematical according to classical analysis. Using non-linear reserves by introducing and utilizing extended but moderate plasticity in predefined zones, a design strategy for this kind of structures can be derived.

The engineering and mechanical basics for the treatment of such kind of tasks in engineering practice are only insufficiently provided. If using concepts for newly built houses they may

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affect unrealistic estimations of the structural behavior, non-economic solutions or even a limitation of given planning choices. Despite the availability of sophisticated analysis tools the coverage of initial damage states and the quantification of given reserves i.e. from the nonlinearity of materials may cause significant problems in engineering practice if standards and codes have to be fulfilled. A development of a set of commonly usable methods for the evaluation of the structural behavior of hybrid systems is necessary.

Figure 1 Panel structures before and after revitalization

In the following chapters selected concepts and methods for modeling and calculation of hybrid structures will be provided that beyond code regulations give the possibility of a methodical consistent estimation of the live cycle of a structure starting from the origin up to the revitalized structure. Using models on the basis of mathematical optimization the coactions of existing structural members with new coupling and supplemental elements can be assessed. Moreover synergy effects can be utilized to ensure an optimal design of hybrid structures. The approaches rely on practical demands and are characterized by a close relationship to codes and by a moderate extension of existing concepts. This paper will provide a general overview for solving hybrid model problems with help of linear and nonlinear optimization algorithms. The specific content of vectors and matrices can be found in the reference literature [1-4].

2 Models and Methods

2.1 General considerations

Hybrid structures can be characterized by the connection of at least two structural elements. Hereby the properties of the structural parts, of the coupling method and of the behavior of the hybrid system are of major interest. Several criteria can be stated, Fig. 2 will give an overview of problems that will be considered in this paper. Most commonly used is the connection of different structural components with different structural system or materials. Moreover the different components can be distinguished by its age and therefore by it’s different pre-damage state. The other way arises from using hybrid models whereas the most interesting for this publication are the use of coupled models for the structure and the appropriate cross-sections and the use of hybrid models consisting of finite element (FEM) and element free (i.e. Element Free Galerkin methods EFG) parts.

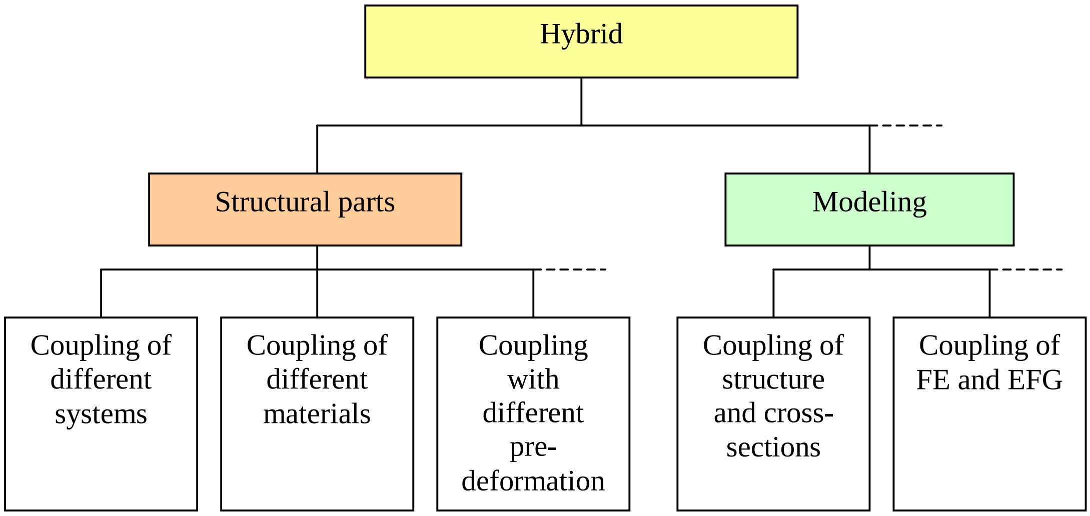


Figure 2 Different kinds of considerations for hybrid structures

2.2 General optimization model

For the analysis of hybrid structures a general optimization problem can be stated in Tab. 1, which can be later on easily modified for various purposes. The problem is given in a matrix scheme form so that changes in the matrix composition will be obvious.

Table 1 General optimization scheme for elasto-plastic structures

conditions		u	λ	p	1		
objective OF	(	$f(u,\lambda,p)$				$) \rightarrow$	Min
equilibrium EC	(	A^TQA	$-A^TQ\,A_p$	$-f_0$	$-A^TQ\varepsilon_0$	$) =$	0
plasticity PC	(	A_p^TQA	$-A_p^TQA_p$		$-A_p^TQ\varepsilon_0 - s_u$	$) \leq$	0
complementary CC	λ^T (	A_p^TQA	$-A_p^TQA_p$		$-A_p^TQ\varepsilon_0 - s_u$	$) =$	0
non-negativity NC	(		-1			$) \leq$	0
additional AC	(	$F(u,\lambda,p)$				$) \leq$	0

Herein u is the vector of deflections, λ is the vector of Lagrange multipliers, A, Q and A_p are the matrices of equilibrium, inverse flexibility and yield surface. The vectors f_0, s_u and ε_0 represent the external forces, the constant part of the yield function and possible pre-strains in the structure. However other material properties can be used as well in this case the used material law can be either linear elastic or linear elastic- ideal plastic. Intended changes between both behaviors can be organized by avoiding or implementing the plasticity conditions. Beyond classical conditions several additional conditions like contact or deformation limits can be applied as well.

Because all necessary conditions for the description of the structural behavior are already given in the subsidiary conditions the objective function can be freely used to organize several kinds of design optimizations. This complies to the most common tasks in engineering practice concerning limit state analysis, i.e. for the calculation of limit forces or the limit resistances considering several design restrictions.

This scheme applies also for the analysis of structures as well as for the consideration of cross-sections. Only the content of the vectors and matrices have to altered appropriately. Moreover the general structure of the scheme will remain the same even the method of discretisation can be changed. So this model applies for Difference Methods, for Finite Elements as well as for so called Element Free discretisations. It is obvious that with this kind of mechanical description of the problems a more or less general description is given.

2.3 Specialized optimization models

2.3.1 Coupling of different systems

The connection of different systems can be done in most cases by direct equating of appropriate degrees of freedom at nodes. In Fig.2 an example structure consisting of two types of systems is given. The first one is a panel type structure whereas the second can be considered as a frame structure. Enforcing the stiffness of the both structures can be reached by coupling of both systems as typical means of revitalization.

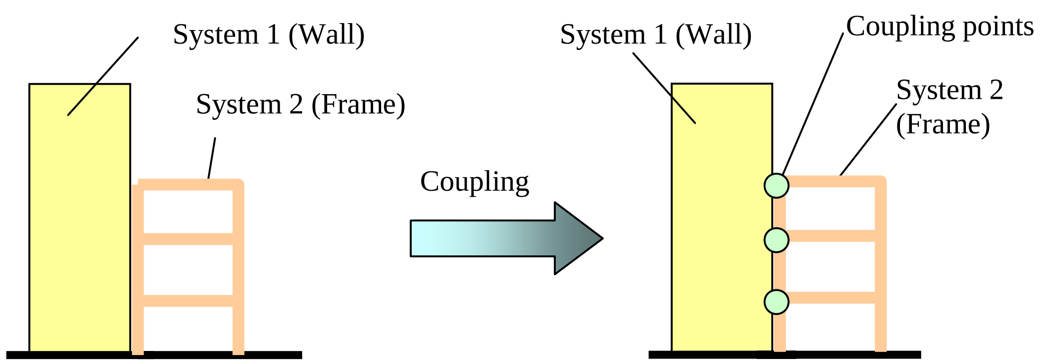


Figure 3 Example for the coupling of different systems

The second component is modeled by using beam elements. At the interface points (green) the systems will be coupled directly. In the optimization scheme different colors indicate different parts of the model. The Introduction of additional coupling conditions as later described in Chapter 2.3.5 is another option.

Table 3 Optimization scheme for the coupling of different systems

		u_1	u_i	u_2	λ_1	λ_2	p	1		
OF	(				$f(u,\lambda,p)$				) →	Min
EC	(	A^TQA_1	A^TQA_{1i}		$- A^TQ A_{p1}$		$- f_{01}$		) =	0
PC	(	$A_p{}^TQA_1$	$A_p{}^TQA_{1i}$		$- A_p{}^TQA_{p1}$			$- s_{u1}$	) ≤	0
CC	λ^T (	$A_p{}^TQA_1$	$A_p{}^TQA_{1i}$		$- A_p{}^TQA_{p1}$			$- s_{u1}$	) =	0
NC	(				-1				) ≤	0
EC	(		A^TQA_{21}	A^TQA_2		$- A^TQ A_{p2}$	$- f_{02}$		) =	0
PC	(		$A_p{}^TQA_{2i}$	$A_p{}^TQA_2$		$- A_p{}^TQA_{p2}$		$- s_{u2}$	) ≤	0
CC	λ^T (		$A_p{}^TQA_{2i}$	$A_p{}^TQA_2$		$- A_p{}^TQA_{p2}$		$- s_{u2}$	) =	0
NC	(					-1			) ≤	0
AC	(				$F(u,\lambda,p)$				) ≤	0

2.3.2 Coupling of different materials

The connection of different materials can be done almost the same way as in Chapter 2.3.2. Only the content of the matrices and vectors describing the material behavior (Q, s_u and A_p) have to be changed [2]. An Example is shown in Fig. 4.

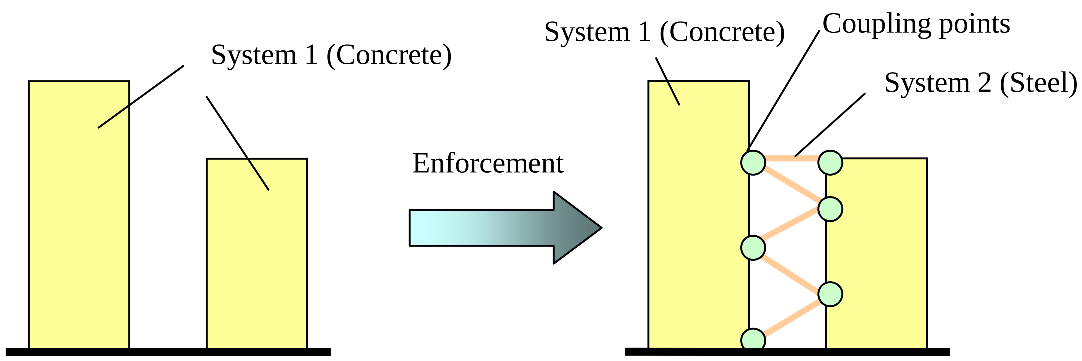


Figure 4 Example for the coupling of different materials

2.3.3 Coupling of members with different age and pre-deformation

Damage or pre-deformations in structures can be consistently described by introducing residual or pre-strains ε_0 for the affected members of the structure. So only little changes to the optimization scheme given in Tab. 3 have to be applied. Arising problems are shown for example in Fig. 5 where the old structural component with pre-strains will be extended by a new layer only facing loads when external loading conditions will change. This is a common task because in revitalization processes normally the loads will vary. The respective optimization scheme is given in Tab. 4.

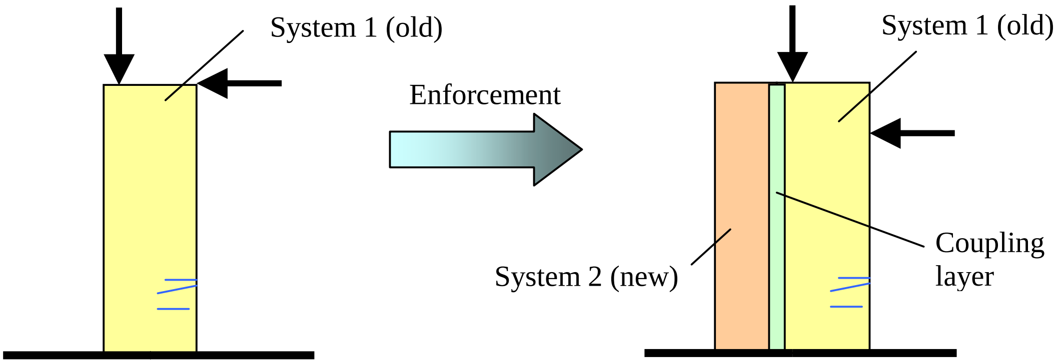


Figure 5 Example for the coupling of members with different pre-deformation

Table 4 Optimization scheme for the coupling of different pre-deformations

		u_1	u_i	u_2	λ_1	λ_2	p	1		
OF	(	$f(u,\lambda,p)$							) →	Min
EC	(	A^TQA_1	A^TQA_{1i}		$- A^TQ\,A_{p1}$		$- f_{01}$	$- A^TQ\varepsilon_0$	) =	0
PC	(	$A_p{}^TQA_1$	$A_p{}^TQA_{1i}$		$- A_p{}^TQA_{p1}$			$- A_p{}^TQ\varepsilon_0 - s_{u1}$	) ≤	0
CC	λ^T (	$A_p{}^TQA_1$	$A_p{}^TQA_{1i}$		$- A_p{}^TQA_{p1}$			$- A_p{}^TQ\varepsilon_0 - s_{u1}$	) =	0
NC	(				-1				) ≤	0
EC	(		A^TQA_{21}	A^TQA_2		$- A^TQ\,A_{p2}$	$- f_{02}$		) =	0
PC	(		$A_p{}^TQA_{2i}$	$A_p{}^TQA_2$		$- A_p{}^TQA_{p2}$		$- s_{u2}$	) ≤	0
CC	λ^T (		$A_p{}^TQA_{2i}$	$A_p{}^TQA_2$		$- A_p{}^TQA_{p2}$		$- s_{u2}$	) =	0
NC	(					-1			) ≤	0
AC	(	$F(u,\lambda,p)$							) ≤	0

2.3.4 Selective coupling of structures and cross-section models

In a lot of cases a simple linear elastic- ideal plastic consideration of the material law for the calculation of reinforced concrete structures is appropriate. But good approximations of the structural behavior in local points for hybrid systems need special considerations and a higher detailing degree. In those cases the model will normally switched to a layer or fiber model type with the effect that the model unknowns and the amount of necessary pre-information increases considerably. Using selective coupling only at exposed points of the structure can mitigate the efforts. For that reason a coupling of several model layers can be organized by connecting parameters of the structure with those of the cross-section at that specific location. Fig. 6 gives an impression of that procedure. This strategy confirms with the known multi scale analysis and uses directly advantages given by the parallel calculation and direct feedback of both models. Tab. 5 gives the optimization scheme, wherein only new interface conditions (IC) have to be introduced that implements the transformation function H between parameters [3].

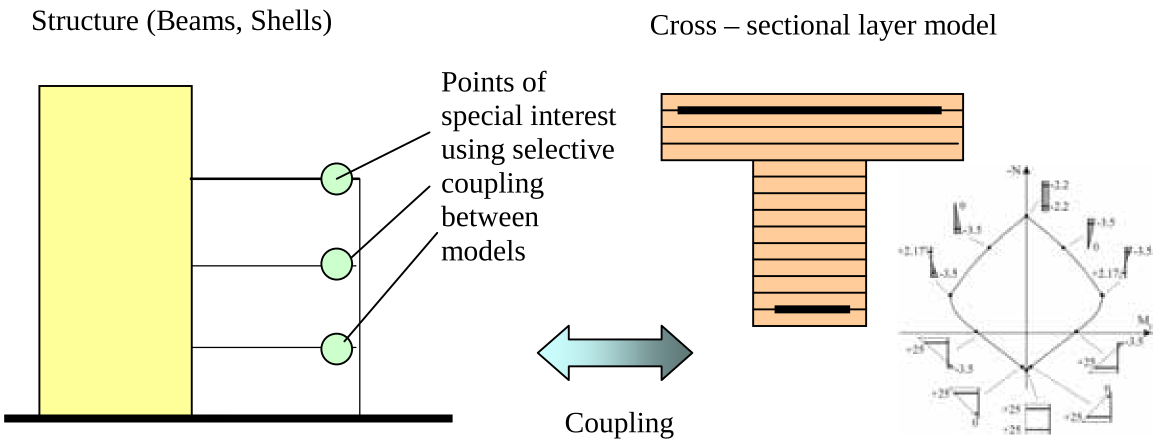


Figure 6 Example for the coupling of selected model layers

Table 5 Optimization scheme for coupling of different models

		u_1	u_{i1}	u_{i2}	u_2	λ_1	λ_2	p	1		
OF	(	$f(u,\lambda,p)$								$) \rightarrow$	Min
EC	(	A^TQA_1	A^TQA_{1i}			$- A^TQ\,A_{p1}$		$- f_{01}$		$) =$	0
PC	(	$A_p^TQA_1$	$A_p^TQA_{1i}$			$- A_p^TQA_{p1}$			$- s_{u1}$	$) \leq$	0
CC	λ^T (	$A_p^TQA_1$	$A_p^TQA_{1i}$			$- A_p^TQA_{p1}$			$- s_{u1}$	$) =$	0
NC	(					-1				$) \leq$	0
IC			H_1	$- H_2$						=	0
EC	(			A^TQA_{21}	A^TQA_2		$- A^TQ\,A_{p2}$	$- f_{02}$		$) =$	0
PC	(			$A_p^TQA_{2i}$	$A_p^TQA_2$		$- A_p^TQA_{p2}$		$- s_{u2}$	$) \leq$	0
CC	λ^T (			$A_p^TQA_{2i}$	$A_p^TQA_2$		$- A_p^TQA_{p2}$		$- s_{u2}$	$) =$	0
NC	(						-1			$) \leq$	0
AC	(	$F(u,\lambda,p)$								$) \leq$	0

2.3.5 Coupling of models with different discretization strategies

Besides the development of alternative solving strategies several research activities have their focus on the improvement of the quality of the results. A lot of proposals deal with the alternation of the basis polynoms or the amount of nodes belonging to an element. Alternatively methods for the adaptive mesh improvement have been developed. The application of these methods typically leads to an increase in the amount of unknowns. On the other hand most optimization algorithms perform better with less design parameters and subsidiary conditions.

These problems can be avoided while having still a high quality approximation. It can be done by breaking down the traditional finite element boundaries. This strategy has been applied by so-called meshless or element free technologies. These methods were successfully applied in structural engineering solving non-linear analysis problems [4]. The application leads to structural matrices with fewer unknowns but with more density (bandwidth).

Therefore in revitalization processes the advantages of several models can be utilized as necessary. Using mixed models the implementation in optimization problems is rather simple. Because of the incompatibility of the unknowns of the EFG model with FE parameters a direct coupling is impossible. That's why the transformation functions must be provided in that case. These can be implemented as further conditions the way as seen in Tab. 5. An example gives Fig. 7.

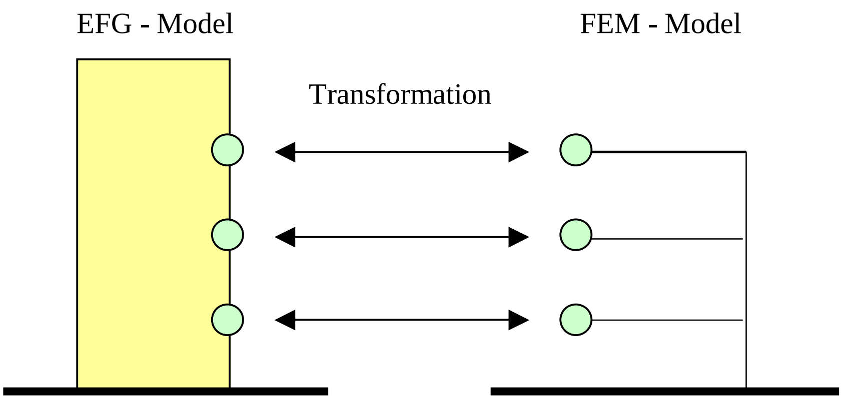


Figure 7 Example for coupling of models using different discretisation techniques

The paper showed different types of modeling of hybrid structures with means of mathematical optimization. Even if the considered non-linear problems have a certain complexity the formulation within an optimization scheme is relatively simple. This method approves its capabilities for the application in engineering design.

3 Example

A typical revitalization problem for concrete panel structures will be examined. The gabled of that 6-storey structure (18m height, 12m width) is supposed to be altered so that one third of the area can be used as windows. Therefore main parts of the stiffening system will be lost. The integrity and performance of the revitalized structure have to be proven by using the optimization strategies presented in Chapter 2. As a performance factor the maximum deformation and the ultimate load in plastic limit state will be evaluated. Only standard vertical and left side wind load cases will be considered. The panel structure parts will be EFG discretized. All steel frames are modeled using FE method. Inelastic pre-deformations of the origin structures will be considered during calculations of the revitalized systems. For discussion four cases will be considered:

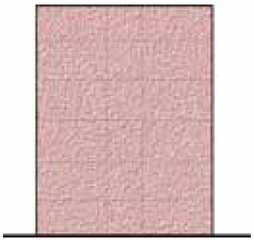
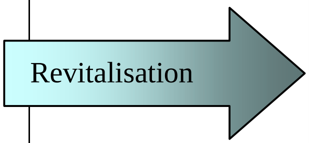
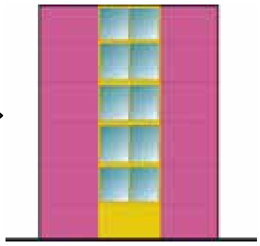
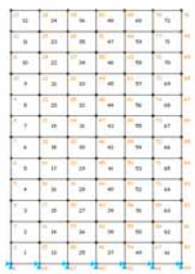
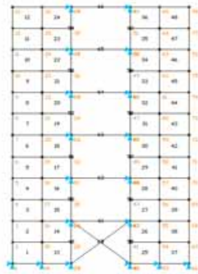
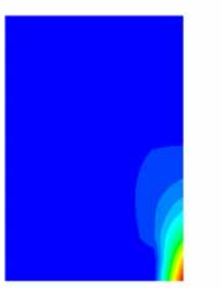
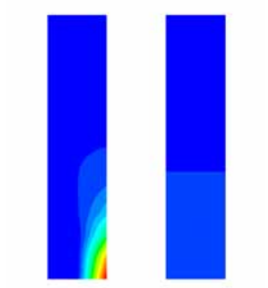
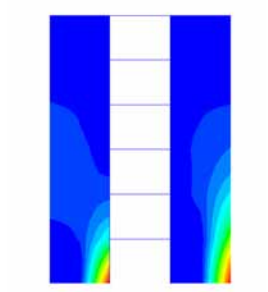
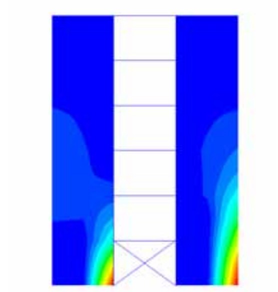
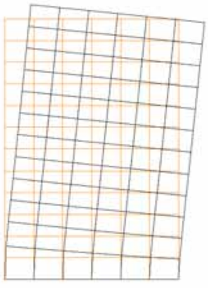
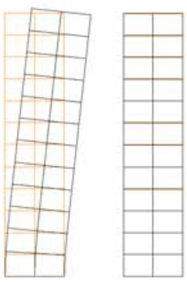
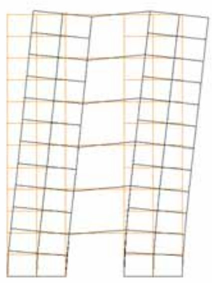
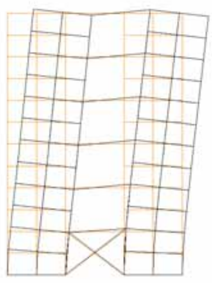
Case a) Original structure (upper bound)

Case b) Revitalized structure without coupling of stiffening walls (fictitious for comparison; lower bound)

Case c) Revitalized structure with additional coupling through moment resisting steel frame

Case d) Revitalized structure with coupling through moment resisting steel frame with brace in the first floor

Table 6 Example

			
Case a) Origin Structure	Case b) Fictitious; without coupling	Case c) Coupling with moment resisting steel frame	Case d) Coupling with moment resisting steel frame and brace
			
Plastic limit load factor p=2.2	p=0.4	p=0.9	p=1.1
Stresses in limit state 			
Ultimate Deflection u = 6.43e-3 [m]	u = 11.06e-3	u = 5.28e-3	u = 6.75e-3
Deform. limit state 			

The statical systems and results for the calculations are given in Tab. 6. It is obvious that the desired performance of p≥1.0 for the revitalized structure can be achieved by application of variant d) just by implementing moment resting frames and one brace in the first floor. The ultimate limit force will be increased by simultaneously maintaining an acceptable deformation compared with the origin structure. Intensive discussions and examples for coupling structures and cross-sections selectively can be found in [3].

4 Conclusions

The investigations show a good adaptability of the optimization methods to the design of hybrid structures consisting of several types, materials and pre-configurations. As well as single domain models, mixed structure-cross-sectional models can be used. With this method the advantages of both finite element and meshless methods can be utilized most suitable. This approach gives best support to practical design. Therefore these methods can be considered to be a promising alternative to traditional methods in structural analysis.

5 References

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