



Decay of polarized muon at rest as a source of polarized antineutrino beam

W. Sobków^{a,*}, S. Ciechanowicz^a, M. Misiaszek^b

^a Institute of Theoretical Physics, University of Wrocław, Pl. M. Born 9, PL-50-204 Wrocław, Poland

^b M. Smoluchowski Institute of Physics, Jagiellonian University, ul. Reymonta 4, PL-30-059 Kraków, Poland

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ABSTRACT

In this Letter, we indicate the possibility of using the decay of polarized muons at rest (DPMaR) as a source of the transversely polarized electron antineutrino beam. Such a beam could be used to probe new effects beyond standard model such as: time reversal violation, existence of right-chirality (anti)neutrinos. The (anti)neutrinos are assumed to be Dirac fermions with non-zero mass. We analyze a scenario with the participation of the complex exotic vector, scalar and tensor couplings of the right-chirality electron antineutrinos in addition to the standard vector coupling of the left-chirality ones. We show that the energy–angle distribution of the electron antineutrinos from the DPMaR depends on the interference terms between standard and exotic couplings, which are proportional to the transverse components of the antineutrino spin polarization and independent of an antineutrino mass. It allows to calculate the flux of electron antineutrinos and the expected number of recoil electrons in the elastic antineutrino–electron scattering ($\bar{\nu}_e e^-$), where the incoming antineutrino beam comes from the DPMaR and is transversely polarized. Our analysis is model-independent and consistent with the current upper limits on the non-standard couplings. The results are presented in a limit of infinitesimally small mass for all particles produced in the muon decay.

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1. Introduction

Decay of polarized muon at rest is the appropriate process to test the time reversal violation (TRV) and space–time structure of the leptonic charged weak interactions. It may also be an important source of information on the right-chirality neutrinos. According to the Standard Model (SM) [1–3], the dominant weak interaction responsible for the DPMaR has a vector-axial (V–A) structure [4], which has been confirmed by precise measurements of the electron observables and of the neutrino energy spectrum. It is worthwhile remarking the high-precision measurement of the angle–energy spectrum of positrons made recently by TWIST Collaboration [5] and the KARMEN experiment [6], in which the energy distribution of electron neutrinos emitted in positive muon decay at rest has been measured. Although the SM agrees with the experimental results, there are numerous theoretical reasons for which SM cannot be viewed as a ultimate theory. The standard theory does not clarify why parity is violated in the weak interaction and what is the mechanism behind this violation. The maximal parity violation is empirically based. The another funda-

mental problem is impossibility of explaining the observed baryon asymmetry of Universe [7] through a single CP-violating phase of the Cabibbo–Kobayashi–Maskawa quark-mixing matrix (CKM) [8]. Presently the CP violation is observed only in the decays of neutral K- and B-mesons [9]. This situation has led to the appearance of various non-standard schemes, in which the exotic V + A, scalar (S), tensor (T), pseudoscalar (P) couplings of the interacting right-chirality neutrinos and new CP-breaking phases can appear. We mean left–right symmetric models (LRSM) [10,11], composite models (CM) [12], leptoquarks models (LQ) and the models with extra dimensions (MED) [13]. In the MED all the particles of the SM are trapped on the three-brane, while the right-chirality neutrinos can move in the extra dimensions. This mechanism explains why the interactions of right-chirality neutrinos with the SM particles are extremely small and have never been observed so far.

The problem of the nature of leptonic weak interactions plays a key role in the context of non-vanishing neutrino masses predicted by the neutrino oscillation experiments and of possible lepton-number violation. Admittance of the S, T, P interactions also allows to test the possibility of lepton-number violation in the muon decay. However, if the final neutrinos are massless and unobserved, the lepton-number-violating effects cannot be observed in the electron observables [14].

One should clearly stress that both electron observables and energy spectrum of (anti)neutrinos include mainly the contributions from the squares of coupling constants of the right-chirality

* Corresponding author.

E-mail addresses: sobkow@sunflower.ift.uni.wroc.pl (W. Sobków), ciechano@sunflower.ift.uni.wroc.pl (S. Ciechanowicz), misiaszek@zefir.if.uj.edu.pl (M. Misiaszek).

neutrinos and at most from the interferences within exotic couplings, that are both very tiny. Although the transverse electron (positron) polarization contains the interference terms between the standard vector and non-standard scalar couplings, but unfortunately both antineutrino and neutrino are left-chirality. In the mentioned above observables, all the eventual interference terms between the standard and exotic couplings are strongly suppressed by a tiny $\bar{\nu}_e$ mass. In such a situation, it seems meaningful to search for new quantities with the linear terms from the exotic couplings obtained in model-independent way. It would allow to compare various non-standard gauge-model predictions with experiments. Moreover, interference effects would be larger than the quadratic contributions of exotic couplings at the same experimental precision. We mean neutrino observables consisting only of the interference terms between the standard coupling of left-chirality neutrinos and exotic couplings of the right-chirality ones and independent of the neutrino mass. These quantities would also make it possible to look for the non-standard T-violating phases. In this Letter, an analysis of electroweak interactions Standard Model structure is carried out for the possibility of testing its limit effects. One of the main goals is to show how the presence of exotic vector, scalar, tensor couplings of the right-chirality electron antineutrinos ($\bar{\nu}_e$) in addition to the standard vector coupling of the left-chirality ones affects the energy–angle distribution of the $\bar{\nu}_e$ from the DP-MaR. Having the spectral function, we calculate the flux of $\bar{\nu}_e$, both for the SM prediction and for the case of mixture of the left- and right-chirality $\bar{\nu}_e$. The other purpose is to calculate the azimuthal distribution of the recoil electrons from the $\bar{\nu}_e e^-$ scattering, when the incoming transversely polarized $\bar{\nu}_e$ beam comes from the DP-MaR. It allows to find the expected event number for assumed detector configuration. Our analysis is model-independent and the calculations are made in the limit of infinitesimally small mass for all particles produced in the DP-MaR. The density operators [15] for the polarized initial muon and for the polarized outgoing $\bar{\nu}_e$ are used, see Appendix A. We use the system of natural units with $\hbar = c = 1$, Dirac–Pauli representation of the γ -matrices and the $(+, -, -, -)$ metric [16].

The Letter is organized as follows: Section 2 contains the basic assumptions, notation used for description of the polarized muon decay. In Section 3 we present the formula for the angle–energy distribution of $\bar{\nu}_e$ from the DP-MaR. In Section 4 the scattering of the transversely polarized $\bar{\nu}_e$ beam from the DP-MaR on the unpolarized electron target is analyzed. Last section gives conclusions. Appendix A includes the formulas for the four-vector $\bar{\nu}_e$ polarization and density operator used in calculations.

2. Basic assumptions – polarized muon decay at rest

We assume that the DP-MaR ($\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$) is a source of the $\bar{\nu}_e$ beam. One ought notice that if one takes into account the positive muon decay ($\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$), the conclusions will be the same as for the μ^- decay.

We admit a presence of the exotic scalar g_{LR}^S, g_{LL}^S , tensor g_{LR}^T and vector g_{RL}^V, g_{RR}^V couplings in addition to the standard vector g_{LL}^V coupling. It means that the outgoing $\bar{\nu}_e$ flux is a mixture of the left-chirality $\bar{\nu}_e$ produced in the g_{LL}^V weak interaction and the right-chirality ones produced in the $g_{LR,LL}^S, g_{LR}^T, g_{RL,RR}^V$ weak interactions. As our analysis is carried out in the limit of vanishing $\bar{\nu}_e$ mass, the left-chirality $\bar{\nu}_e$ has positive helicity, while the right-chirality one has negative helicity, see [17]. The muon neutrino is left-chirality for the $g_{LL}^V, g_{LR}^S, g_{LR}^T$ couplings (negative helicity, when $m_{\nu_\mu} \rightarrow 0$), and right-chirality for the g_{RR}^V, g_{LL}^S couplings (positive helicity, when $m_{\nu_\mu} \rightarrow 0$). In the SM, only g_{LL}^V is non-zero value. Because we allow for the non-conservation of the combined

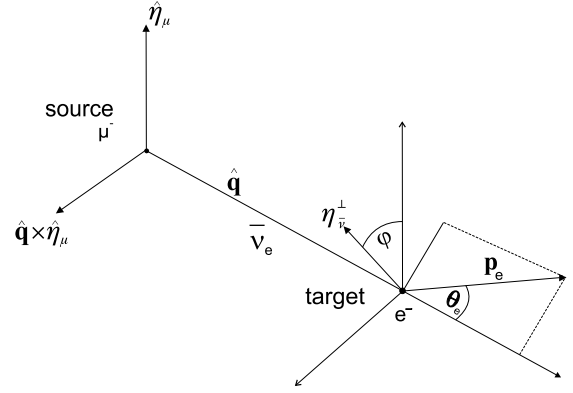


Fig. 1. Figure shows the production plane of the $\bar{\nu}_e$ for the process $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$. $\eta_{\bar{\nu}_e}^\perp$ – the transverse polarization of the outgoing antineutrino.

symmetry CP, all the coupling constants are complex. The amplitude is of the form:

$$M_{\mu^-} = \frac{G_F}{\sqrt{2}} \left\{ g_{LL}^V (\bar{u}_e \gamma_\alpha (1 - \gamma_5) v_{\nu_e}) (\bar{u}_{\nu_\mu} \gamma^\alpha (1 - \gamma_5) u_\mu) \right. \\ + g_{RL}^V (\bar{u}_e \gamma_\alpha (1 + \gamma_5) v_{\nu_e}) (\bar{u}_{\nu_\mu} \gamma^\alpha (1 - \gamma_5) u_\mu) \\ + g_{RR}^V (\bar{u}_e \gamma_\alpha (1 + \gamma_5) v_{\nu_e}) (\bar{u}_{\nu_\mu} \gamma^\alpha (1 + \gamma_5) u_\mu) \\ + g_{LR}^S (\bar{u}_e (1 + \gamma_5) v_{\nu_e}) (\bar{u}_{\nu_\mu} (1 + \gamma_5) u_\mu) \\ + g_{LL}^S (\bar{u}_e (1 + \gamma_5) v_{\nu_e}) (\bar{u}_{\nu_\mu} (1 - \gamma_5) u_\mu) \\ \left. + \frac{g_{LR}^T}{2} (\bar{u}_e \sigma_{\alpha\beta} (1 + \gamma_5) v_{\nu_e}) (\bar{u}_{\nu_\mu} \sigma^{\alpha\beta} (1 + \gamma_5) u_\mu) \right\}, \quad (1)$$

where v_{ν_e} and \bar{u}_e (u_μ and \bar{u}_{ν_μ}) are the Dirac bispinors of the outgoing electron antineutrino and electron (initial muon and final muon neutrino), respectively. $G_F = 1.1663788(7) \times 10^{-5} \text{ GeV}^{-2}$ (0.6 ppm) (MuLan Collaboration) [18] is the Fermi constant. The coupling constants are denoted as $g_{LL,RL,RR}^V$ and $g_{LR,LL}^S, g_{LR}^T$ respectively to the chirality of the final electron and initial stopped muon. The initial muon is at rest and polarized. The unit vector in the LAB system $\hat{\eta}_\mu$ denotes the muon polarization for a single muon decay. The production plane is spanned by the direction of the muon polarization $\hat{\eta}_\mu$ and of the outgoing electron antineutrino LAB momentum unit vector \hat{q} , Fig. 1. As is known, in this plane, the polarization vector $\hat{\eta}_\mu$ can be expressed, with respect to the \hat{q} , as a sum of the longitudinal component of the muon polarization ($(\hat{\eta}_\mu \cdot \hat{q})\hat{q}$) and transverse component of the muon polarization η_{μ}^\perp , that is defined as $\eta_{\mu}^\perp = \hat{\eta}_\mu - (\hat{\eta}_\mu \cdot \hat{q})\hat{q}$.

By $\hat{\eta}_{\bar{\nu}_e}$, $(\hat{\eta}_{\bar{\nu}_e} \cdot \hat{q})\hat{q}$, and $\hat{\eta}_{\bar{\nu}_e}^\perp$ we denote the unit polarization vector, its longitudinal component, and transverse component of the outgoing $\bar{\nu}_e$ in its rest system, respectively, Fig. 1. Then, $\hat{\eta}_{\bar{\nu}_e} \cdot \hat{q} = +1$ is the polarization longitudinal component of the left-chirality $\bar{\nu}_e$ for the standard g_{LL}^V coupling, while $\hat{\eta}_{\bar{\nu}_e} \cdot \hat{q} = -1$ is the polarization longitudinal component of the right-chirality $\bar{\nu}_e$ for the exotic $g_{LR}^S, g_{LL}^S, g_{RL,RR}^V$ couplings.

3. Energy–angle distribution of electron antineutrinos

The formula for the energy and angular distribution of the $\bar{\nu}_e$ coming from the DP-MaR is of the form:

$$\frac{d^2 \Gamma}{dy d\Omega_\nu} = \left(\frac{d^2 \Gamma}{dy d\Omega_\nu} \right)_{(V)} + \left(\frac{d^2 \Gamma}{dy d\Omega_\nu} \right)_{(S+T)} \\ + \left(\frac{d^2 \Gamma}{dy d\Omega_\nu} \right)_{(V_{RL}+V_{RR})} + \left(\frac{d^2 \Gamma}{dy d\Omega_\nu} \right)_{(V_{S+T})}, \quad (2)$$

$$\left(\frac{d^2\Gamma}{dy d\Omega_\nu}\right)_{(V)} = \frac{G_F^2 m_\mu^5}{128\pi^4} \{ |g_{LL}^V|^2 y^2 (1-y)(1 + \hat{\eta}_{\bar{\nu}} \cdot \hat{\mathbf{q}}) \times (1 + \hat{\eta}_\mu \cdot \hat{\mathbf{q}}) \}, \quad (3)$$

$$\begin{aligned} \left(\frac{d^2\Gamma}{dy d\Omega_\nu}\right)_{(S+T)} &= \frac{G_F^2 m_\mu^5}{3072\pi^4} (1 - \hat{\eta}_{\bar{\nu}} \cdot \hat{\mathbf{q}}) |g_{LR}^S|^2 y^2 \\ &\times \left\{ [(3-2y) - (1-2y)\hat{\eta}_\mu \cdot \hat{\mathbf{q}}] \right. \\ &+ \left| \frac{g_{LL}^S}{g_{LR}^S} \right|^2 [(3-2y) + (1-2y)\hat{\eta}_\mu \cdot \hat{\mathbf{q}}] \\ &+ 4 \left| \frac{g_{LR}^T}{g_{LR}^S} \right|^2 [(15-14y) - (13-14y)\hat{\eta}_\mu \cdot \hat{\mathbf{q}}] \\ &\left. + 4 \operatorname{Re} \left(\frac{g_{LR}^{T*}}{g_{LR}^S} \right) [(4y-3) - (4y-5)\hat{\eta}_\mu \cdot \hat{\mathbf{q}}] \right\}, \quad (4) \end{aligned}$$

$$\begin{aligned} \left(\frac{d^2\Gamma}{dy d\Omega_\nu}\right)_{(V_{RL}+V_{RR})} &= \frac{G_F^2 m_\mu^5}{768\pi^4} (1 - \hat{\eta}_{\bar{\nu}} \cdot \hat{\mathbf{q}}) y^2 \{ |g_{RL}^V|^2 [(3-2y) \\ &+ (1-2y)\hat{\eta}_\mu \cdot \hat{\mathbf{q}}] \\ &+ 6 |g_{RR}^V|^2 (1-y)(1 - \hat{\eta}_\mu \cdot \hat{\mathbf{q}}) \}, \quad (5) \end{aligned}$$

$$\begin{aligned} \left(\frac{d^2\Gamma}{dy d\Omega_\nu}\right)_{(V_{S+VT})} &= \frac{G_F^2 m_\mu^5}{256\pi^4} y^2 (1-y) \\ &\times \{ [\operatorname{Re}(g_{LL}^V g_{LR}^{S*}) - 6 \operatorname{Re}(g_{LL}^V g_{LR}^{T*})] (\eta_{\bar{\nu}}^\perp \cdot \hat{\eta}_\mu) \\ &+ [\operatorname{Im}(g_{LL}^V g_{LR}^{S*}) - 6 \operatorname{Im}(g_{LL}^V g_{LR}^{T*})] \\ &\times \eta_{\bar{\nu}}^\perp \cdot (\hat{\mathbf{q}} \times \hat{\eta}_\mu) \}. \quad (6) \end{aligned}$$

Here, $y = \frac{2E_\nu}{m_\mu}$ is the reduced $\bar{\nu}_e$ energy for the muon mass m_μ , it varies from 0 to 1, and $d\Omega_\nu$ is the solid angle differential for $\bar{\nu}_e$ momentum $\hat{\mathbf{q}}$.

Eq. (6) includes two interference terms between the standard g_{LL}^V and exotic g_{LR}^S, g_{LR}^T couplings, so it is linear in the exotic couplings contrary to Eqs. (4) and (5). It is necessary to point out that the above formula is presented after the integration over all the momentum directions of the outgoing electron and muon neutrino.

If the $\hat{\eta}_\mu \cdot \hat{\mathbf{q}} = 0$ the interference part can be rewritten in the following way:

$$\begin{aligned} \left(\frac{d^2\Gamma}{dy d\Omega_\nu}\right)_{(V_{S+VT})} &= \frac{G_F^2 m_\mu^5}{256\pi^4} |\eta_{\bar{\nu}}^\perp| |\eta_\mu^\perp| |g_{LL}^V| |g_{LR}^S| \\ &\times \left\{ \cos(\phi - \alpha_{VS}) \right. \\ &\left. - 6 \left| \frac{g_{LR}^T}{g_{LR}^S} \right| \cos(\phi - \alpha_{VT}) \right\} y^2 (1-y), \quad (7) \end{aligned}$$

where ϕ is the angle between the direction of $\eta_{\bar{\nu}}^\perp$ and the direction of η_μ^\perp only, cf. Fig. 1; $\alpha_{VS} \equiv \alpha_V^{LL} - \alpha_S^{LR}$, $\alpha_{VT} \equiv \alpha_V^{LL} - \alpha_T^{LR}$ are the relative phases between the g_{LL}^V and g_{LR}^S, g_{LR}^T couplings.

It can be noticed that the relative phases α_{VS}, α_{VT} different from 0, π would indicate the CP violation in the CC weak interaction. We see that in the case of the transversely polarized antineutrino beam coming from the polarized muon decay, the interference terms between the standard coupling g_{LL}^V and exotic $g_{LR}^{S,T}$ couplings do not vanish in the limit of vanishing electron-antineutrino and muon-neutrino masses. This independence of the neutrino mass makes the measurement of the relative phases

Table 1
Current limits on the non-standard couplings.

| Coupling constants | SM | Current limits |
|--------------------|----|----------------|
| $ g_{LL}^V $ | 1 | > 0.960 |
| $ g_{LR}^V $ | 0 | < 0.025 |
| $ g_{RL}^V $ | 0 | < 0.104 |
| $ g_{RR}^V $ | 0 | < 0.031 |
| $ g_{LL}^S $ | 0 | < 0.550 |
| $ g_{LR}^S $ | 0 | < 0.074 |
| $ g_{RL}^S $ | 0 | < 0.412 |
| $ g_{RR}^S $ | 0 | < 0.062 |
| $ g_{LL}^T $ | 0 | 0 |
| $ g_{LR}^T $ | 0 | < 0.021 |
| $ g_{RL}^T $ | 0 | < 0.103 |
| $ g_{RR}^T $ | 0 | 0 |

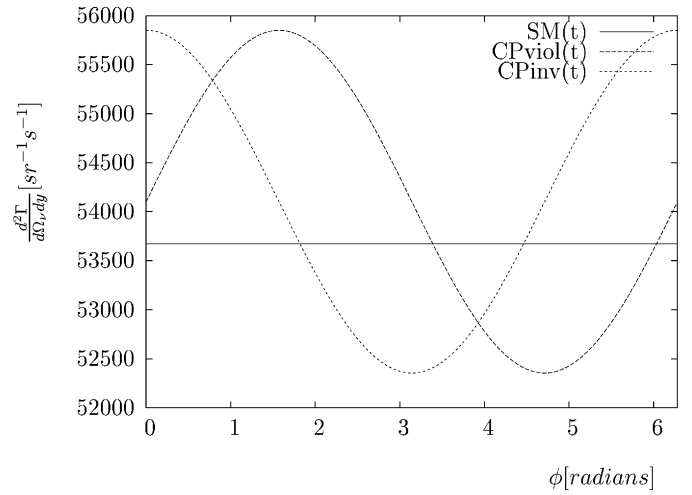


Fig. 2. Plot of the $\frac{d^2\Gamma}{dy d\Omega_\nu}$ as a function of ϕ for assigned $y = 2/3$, when $\hat{\eta}_{\bar{\nu}} \cdot \hat{\mathbf{q}} = 0$, $\hat{\eta}_{\bar{\nu}} \cdot \hat{\mathbf{q}} = 0.822$, $|\eta_{\bar{\nu}}^\perp| = 0.570$, $|\eta_\mu^\perp| = 1$: (a) solid line is for the V-A interaction; (b) time reversal violation, $\alpha_{VS} = \pi/2$, $\alpha_{VT} = 3\pi/2$ (long-dashed line); (c) time reversal conservation, $\alpha_{VS} = 0$, $\alpha_{VT} = \pi$ (short-dashed line).

α_{VS}, α_{VT} between these couplings possible. The interference part, Eq. (7), includes only the contributions from the transverse component of the initial muon polarization η_μ^\perp and the transverse component of the outgoing antineutrino polarization $\eta_{\bar{\nu}}^\perp$. Both transverse components are perpendicular with respect to the $\hat{\mathbf{q}}$.

Using the current data [19], see Table 1, we calculate the upper limit on the magnitude of the transverse antineutrino polarization and lower bound for the longitudinal antineutrino polarization, see [17]:

$$\begin{aligned} |\eta_{\bar{\nu}}^\perp| &= 2\sqrt{Q_L^{\bar{\nu}}(1 - Q_L^{\bar{\nu}})} \leq 0.570, \\ \hat{\eta}_{\bar{\nu}} \cdot \hat{\mathbf{q}} &= 2Q_L^{\bar{\nu}} - 1 \geq 0.822, \quad (8) \end{aligned}$$

$$\begin{aligned} Q_L^{\bar{\nu}} &= 1 - \frac{1}{4} (|g_{LR}^S|^2 + |g_{LL}^S|^2) - 3|g_{LR}^T|^2 - |g_{RL}^V|^2 - |g_{RR}^V|^2 \\ &\geq 0.911, \quad (9) \end{aligned}$$

where $Q_L^{\bar{\nu}}$ is the probability of the $\bar{\nu}_e$ to be left-chirality. The above limits are computed for the normalized values of coupling constants.

Fig. 2 illustrates the possible effect from the terms with interference between the standard and exotic couplings. We note that Eq. (3) after integration over all the $\bar{\nu}_e$ directions (with $|g_{LL}^V| = 1$, $\hat{\eta}_{\bar{\nu}} \cdot \hat{\mathbf{q}} = +1$) is the same as Eq. (7) in [17] (with $Q_L^{\bar{\nu}} = 1$, $\omega_L = 0$, $\eta_L = 0$, neglecting the masses of the neutrinos and of the electron

as well as radiative corrections). We see that for $\hat{\eta}_\mu \cdot \hat{\mathbf{q}} = -1$ only the exotic part with the squared coupling constants survives:

$$\begin{aligned} \frac{d^2\Gamma}{dy d\Omega_\nu} &= \frac{G_F^2 m_\mu^5}{768\pi^4} (1 - \hat{\eta}_{\bar{\nu}} \cdot \hat{\mathbf{q}}) |g_{LR}^S|^2 y^2 \\ &\times \left\{ (1-y) \left(1 + 28 \left| \frac{g_{LR}^T}{g_{LR}^S} \right|^2 + 12 \left| \frac{g_{RR}^V}{g_{LR}^S} \right|^2 \right. \right. \\ &\left. \left. - 8 \operatorname{Re} \left(\frac{g_{LR}^{T*}}{g_{LR}^S} \right) \right) + 2 \left| \frac{g_{RL}^V}{g_{LR}^S} \right|^2 + \frac{1}{2} \left| \frac{g_{LL}^S}{g_{LR}^S} \right|^2 \right\}. \end{aligned} \quad (10)$$

However in this case there is always an ambiguity in measurement of $\bar{\nu}_e$ direction (systematic error), so a contribution from the standard left-chirality $\bar{\nu}_e$ is always present. It means that Eq. (10) is strongly suppressed.

After the integration of Eqs. (3), (4), (5), (6), the muon lifetime is as follows:

$$\tau = \frac{192\pi^3}{m_\mu^5 G_F^2} \left(\frac{1}{A} \right), \quad (11)$$

where $A = |g_{LL}^V|^2 + \frac{1}{4}(|g_{LR}^S|^2 + |g_{LR}^T|^2) + |g_{RL}^V|^2 + |g_{RR}^V|^2 + 3|g_{LR}^T|^2$. Because the muon lifetime observable is measured, so the admittance of the exotic couplings means that the standard coupling g_{LL}^V should be decreased in order to get $A = 1$.

If the $\bar{\nu}_e$ beam comes from the unpolarized muon decay, the energy and angular distribution of the $\bar{\nu}_e$ consists only of two parts; standard V and exotic ($S + T + V_{RL} + V_{RR}$), i.e. Eqs. (3), (4), (5) for $\hat{\eta}_\mu \cdot \hat{\mathbf{q}} = 0$. If one puts $E_{\bar{\nu}} = m_\mu/2$ (i.e. $y = 1$) in both parts, the standard V contribution vanishes, while the exotic one survives.

It is necessary to stress that the effects coming from the $\bar{\nu}_e$ mass and mixing are very small and they may be neglected. In order to show this we use the final density matrix for the mass states m_1, m_2 of $\bar{\nu}_e$ to avoid breaking the fundamental principles of Quantum Field Theory. We assume that at the $\bar{\nu}_e$ detector (target) $\bar{\nu}_e = \cos\theta \bar{\nu}_1 + \sin\theta \bar{\nu}_2$. In this way the differential antineutrino spectrum is of the form:

$$\begin{aligned} \frac{d^2\Gamma}{dy d\Omega_\nu} &= \cos^2\theta \frac{d^2\Gamma}{dy_1 d\Omega_\nu} + \sin^2\theta \frac{d^2\Gamma}{dy_2 d\Omega_\nu} \\ &= \gamma_{(V,S,T)}^{(\phi)} \left[y_1^2 (1-y_1) + \sin^2\theta (4-6y_1) \frac{\delta m_\nu^2}{m_\mu^2} \right. \\ &\left. + O\left(\frac{\delta m_\nu^2}{m_\mu^2}\right) \right], \end{aligned} \quad (12)$$

where $\gamma_{(V,S,T)}^{(\phi)} = \frac{G_F^2 m_\mu^5}{256\pi^4} |\eta_{\bar{\nu}}^\perp| |\eta_\mu^\perp| |g_{LL}^V| |g_{LR}^S| \cdot [\cos(\phi - \alpha_{VS}) - 6 \frac{g_{LR}^T}{g_{LR}^S} |\cos(\phi - \alpha_{VT})|]$. We see that at $y_1 = \frac{2}{3}$ the linear contribution from the mass mixing is absent, besides $\frac{\delta m_\nu^2}{m_\mu^2}$ is of the order of 10^{-19} , so this effect is very negligible.

4. Azimuthal distribution of recoil electrons

As the direct measurement of the interfering left- and right-chirality $\bar{\nu}_e$ in the DPMAr is impossible, one proposes to use the elastic $\bar{\nu}_e e^-$ scattering as the detection process of new signals. The electron $\bar{\nu}_e$ beam from the DPMAr would be scattered off the unpolarized target-electrons and the azimuthal distribution of the recoil electrons would be measured. Our analysis is made for the detector in the shape of flat circular ring, while the $\bar{\nu}_e$ source is located in the center of the ring detector and polarized perpendicularly to the ring. To give the expected event number, we need

the following quantities indicated in Table 2. In addition, we must know the differential cross section for the $\bar{\nu}_e e^-$ scattering. The transition amplitude for this process is of the form:

$$\begin{aligned} M_{\bar{\nu}_e e} &= \frac{G_F}{\sqrt{2}} \left\{ (\bar{u}_{e'} \gamma^\alpha (c_V^L - c_A^L \gamma_5) u_e) (\bar{\nu}_{\bar{\nu}_e} \gamma_\alpha (1 - \gamma_5) \nu_{\bar{\nu}_e}) \right. \\ &+ (\bar{u}_{e'} \gamma^\alpha (c_V^R + c_A^R \gamma_5) u_e) (\bar{\nu}_{\bar{\nu}_e} \gamma_\alpha (1 + \gamma_5) \nu_{\bar{\nu}_e}) \\ &+ c_S^R (\bar{u}_{e'} u_e) (\bar{\nu}_{\bar{\nu}_e} (1 - \gamma_5) \nu_{\bar{\nu}_e}) \\ &\left. + \frac{c_T^R}{2} (\bar{u}_{e'} \sigma^{\alpha\beta} u_e) (\bar{\nu}_{\bar{\nu}_e} \sigma_{\alpha\beta} (1 - \gamma_5) \nu_{\bar{\nu}_e}) \right\}. \end{aligned} \quad (13)$$

The all coupling constants are complex and denoted with the superscripts L and R as $c_V^{L,R}, c_A^{L,R}, c_S^R, c_T^R$ respectively to the incoming $\bar{\nu}_e$ of left- and right-chirality. We assume that the incoming $\bar{\nu}_e$ beam is the mixture of the left-chirality $\bar{\nu}_e$'s detected in the standard c_V^L, c_A^L weak interactions and right-chirality ones detected in the exotic $c_S^R, c_T^R, c_V^R, c_A^R$ weak interactions. The result of the calculation performed with the above amplitude takes the form:

$$\begin{aligned} \frac{d^2\sigma}{dy_e d\phi_e} &= \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(V-A)} + \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(V+A)} \\ &+ \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(S,T)} + \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(VS)} \\ &+ \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(AT)}, \end{aligned} \quad (14)$$

$$\begin{aligned} \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(V-A)} &= B \left\{ (1 + \hat{\eta}_{\bar{\nu}} \cdot \hat{\mathbf{q}}) \left[(c_V^L - c_A^L)^2 \right. \right. \\ &+ (c_V^L + c_A^L)^2 (1 - y_e)^2 \\ &\left. \left. - \frac{m_e y_e}{E_\nu} ((c_V^L)^2 - (c_A^L)^2) \right] \right\}, \end{aligned} \quad (15)$$

$$\begin{aligned} \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(V+A)} &= B \left\{ (1 - \hat{\eta}_{\bar{\nu}} \cdot \hat{\mathbf{q}}) \left[(c_V^R - c_A^R)^2 \right. \right. \\ &+ (c_V^R + c_A^R)^2 (1 - y_e)^2 \\ &\left. \left. - \frac{m_e y_e}{E_\nu} ((c_V^R)^2 - (c_A^R)^2) \right] \right\}, \end{aligned} \quad (16)$$

$$\begin{aligned} \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(S,T)} &= B (1 - \hat{\eta}_{\bar{\nu}} \cdot \hat{\mathbf{q}}) \left\{ \frac{1}{2} y_e \left(y_e + 2 \frac{m_e}{E_\nu} \right) |c_S^R|^2 \right. \\ &+ \left((2 - y_e)^2 - \frac{m_e}{E_\nu} y_e \right) |c_T^R|^2 \\ &\left. - y_e (y_e - 2) \operatorname{Re}(c_S^R c_T^{R*}) \right\}, \end{aligned} \quad (17)$$

$$\begin{aligned} \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(VS)} &= B \left\{ -4 \sqrt{y_e \left(y_e + 2 \frac{m_e}{E_\nu} \right)} \right. \\ &\times [\eta_{\bar{\nu}}^\perp \cdot (\hat{\mathbf{p}}_e \times \hat{\mathbf{q}}) \operatorname{Im}(c_V^L c_S^{R*}) \\ &\left. + (\eta_{\bar{\nu}}^\perp \cdot \hat{\mathbf{p}}_e) \operatorname{Re}(c_V^L c_S^{R*}) \right\}, \end{aligned} \quad (18)$$

$$\begin{aligned} \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(AT)} &= B \left\{ -2 \sqrt{y_e \left(y_e + 2 \frac{m_e}{E_\nu} \right)} \right. \\ &\times [\eta_{\bar{\nu}}^\perp \cdot (\hat{\mathbf{p}}_e \times \hat{\mathbf{q}}) \operatorname{Im}(c_A^L c_T^{R*}) \\ &\left. + (\eta_{\bar{\nu}}^\perp \cdot \hat{\mathbf{p}}_e) \operatorname{Re}(c_A^L c_T^{R*}) \right\}, \end{aligned} \quad (19)$$

Table 2
Parameters of assumed detector configuration.

| | |
|--|------------------------|
| Detector threshold T_e^{th} | 10 eV |
| Minimal value of initial antineutrino energy E_ν^{min} | 1603.44 eV |
| Number of target-electrons N_e (75 Kton of Fe) | $2.097 \cdot 10^{34}$ |
| Number of muons decaying per one year N_μ | 10^{21} |
| Efficiency of detector for antineutrino energies above threshold ϵ | 1 |
| Inner radius of detector that is equal to distance Between antineutrino source and detector $R = L$ | 2205 cm |
| δ | 0.01 |
| $S_D = 4\pi R^2 \sin \delta$ | 610970 cm ² |

Table 3
Flux of $\bar{\nu}_e$ beam and the number of events for mixture of the left- and right-chirality $\bar{\nu}_e$, when time reversal symmetry is violated, i.e. $\alpha_{VS} = \pi/2, \alpha_{VT} = 3\pi/2, \phi - \beta_{SV} = \pi/2, \phi - \beta_{TA} = \pi/2$.

| Case | Φ_ν^\perp [cm ⁻² s ⁻¹] | $\frac{dN_e(t)}{d\phi_e}$ |
|--|--|--------------------------------|
| V-A | $6.21 \cdot 10^{18}$ | $2.39 \cdot 10^9$ |
| Squares of exotic couplings | $5.93 \cdot 10^{16}$ | $3.96 \cdot 10^4$ |
| Interferences between standard and exotic couplings | $2.02 \cdot 10^{17} \sin(\phi)$ | $5.51 \cdot 10^5 \sin(\phi_e)$ |

$$y_e \equiv \frac{T_e}{E_\nu} = \frac{m_e}{E_\nu} \frac{2 \cos^2 \theta_e}{1 + \frac{m_e}{E_\nu} - \cos^2 \theta_e} \quad (20)$$

is the ratio of the kinetic energy of the recoil electron T_e to the incoming antineutrino energy E_ν ; $B \equiv (E_\nu m_e / 4\pi^2) (G_F^2 / 2)$; θ_e is the angle between the direction of the outgoing electron momentum $\hat{\mathbf{p}}_e$ and the direction of the incoming $\bar{\nu}_e$ momentum $\hat{\mathbf{q}}$ (recoil electron scattering angle); m_e is the electron mass; ϕ_e is the angle between the production plane and the reaction plane spanned by the $\hat{\mathbf{p}}_e$ and $\hat{\mathbf{q}}$ (the azimuthal angle of outgoing electron momentum).

We see that the interference terms, Eqs. (18), (19), between the standard $c_{V,A}^L$ and exotic $c_{S,T}^R$ couplings do not depend on the $\bar{\nu}_e$ mass and they pertain in the massless $\bar{\nu}_e$ limit. It can be noticed that the interferences include only the contributions from the transverse components of the $\bar{\nu}_e$ polarization, both T-even and T-odd:

$$\begin{aligned} & \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(VS)} + \left(\frac{d^2\sigma}{dy_e d\phi_e} \right)_{(AT)} \\ &= B |\eta_{\bar{\nu}}^\perp| \sqrt{\frac{m_e}{E_\nu}} y_e \left[2 - \left(2 + \frac{m_e}{E_\nu} \right) y_e \right] \\ & \cdot \{ 4 |c_V^L| |c_S^R| \cos(\phi - \beta_{SV} - \phi_e) \\ & + 2 |c_A^L| |c_T^R| \cos(\phi - \beta_{TA} - \phi_e) \}, \quad (21) \end{aligned}$$

where $\beta_{SV} \equiv \beta_S^R - \beta_V^L$, $\beta_{TA} \equiv \beta_T^R - \beta_A^L$ – the relative phases between the c_S^R , c_V^L and c_T^R , c_A^L couplings respectively.

Using the available data, we calculate both the flux of $\bar{\nu}_e$ beam and the number of events predicted by the SM for $c_V^L = 1 - 0.0398$, $c_A^L = 1 - 0.5064$, [19]. $(\Phi_\nu^\perp)_{SM} = 7.48 \cdot 10^{18}$ cm⁻² s⁻¹ is the number of the $\bar{\nu}_e$ passing through S_D in the direction perpendicular to the $\hat{\eta}_\mu$.

$(dN_e(t)/d\phi_e)_{SM} = 3.07 \cdot 10^9$ is the number of the outgoing electrons corresponding to the $(\Phi_\nu^\perp)_{SM}$. In this case there is no dependence on the ϕ_e , it means that the azimuthal distribution is symmetric.

In the case of the mixture of the left- and right-chirality $\bar{\nu}_e$, we get the upper limits on the Φ_ν^\perp and $dN_e(t)/d\phi_e$, Table 3:

We see that the number of events coming from the interference terms depends on the azimuthal angle ϕ_e . The above values are determined for the normalized couplings in the DPMaR (Table 1) and experimental values of the couplings in the $\bar{\nu}_e e^-$ scat-

tering: $c_V^L = 1 + (-0.04 \pm 0.015)$, $c_A^L = 1 + (-0.507 \pm 0.014)$ [19]. The interference effects can be significantly enlarged by assuming 10 years of experiment and using ~ 750 kt of iron detector. It is worth noticing that in the case of non-vanishing electron mass, new interference between the g_{LL}^V and g_{RL}^V couplings enters the angle-energy $\bar{\nu}_e$ distribution and increases the right-chirality $\bar{\nu}_e$ contribution. Our analysis is made in the limit of vanishing electron mass.

5. Conclusions

In this Letter, we have shown that the admittance of the exotic vector, scalar and tensor charged weak interactions in addition to the standard vector interaction in the DPMaR generates the interference terms between the standard left- and exotic right-chirality $\bar{\nu}_e$ in the energy-angle distribution of $\bar{\nu}_e$. These interferences are independent of the $\bar{\nu}_e$ mass. If the interacting left- and right-chirality $\bar{\nu}_e$ are produced, the $\bar{\nu}_e$ polarization vector may acquire a transversal component, both T-even and T-odd. As the direct measurement of interference effects in the energy-angle $\bar{\nu}_e$ distribution is extremely difficult, one proposes to scatter the transversely polarized $\bar{\nu}_e$ beam produced in the DPMaR on the unpolarized electron target. Observation of the azimuthal asymmetry in angular distribution of the recoil electrons would be a positive evidence for the existence of the right-chirality $\bar{\nu}_e$ and would allow to search for the CP-breaking phases. According to the SM, such a distribution should be azimuthally symmetric.

We have demonstrated that the eventual effects connected with the $\bar{\nu}_e$ mass and mixing in the spectral function are totally inessential, of the order 10^{-19} .

Using the available experimental data, we have calculated the flux of $\bar{\nu}_e$ and the expected event number, both for the SM prediction and for the case of mixture of the L- and R-chirality $\bar{\nu}_e$.

The DPMaR may also be used to measure the dependence of the $\bar{\nu}_e$ energy spectrum on the $\hat{\eta}_\mu \cdot \hat{\mathbf{q}}$.

It is worth stressing that searching for the new effects coming from the exotic weak interactions requires the very intense polarized (anti)neutrino sources (10^{21} muons decaying at rest per year or more) and large unpolarized target of electrons (10^{34} or more), and also long time duration of experiment (more than one year). In this context, there is the interesting proposal of the experiment with an artificial tritium source of $\bar{\nu}_e$ giving the flux density $\sim 10^{14}$ cm⁻² s⁻¹ [20]. Moreover, the (anti)neutrino beams must be well understood (shape and normalization). In addition, the detectors should have a low threshold and measure both polar angle and azimuthal angle of the outgoing electron momentum with a high resolution. They must also distinguish the electrons from various potential background sources; for example, the electron produced by neutrino-nucleon scattering can give a final state that is often consistent with a single recoil electron coming from neutrino-electron scattering. This background may be reduced by the precise measurement of transverse electron momentum. It is worthwhile mentioning the silicon cryogenic detectors based on the ionization-into-heat conversion effect and the high purity germanium detectors with the internal amplification of a signal in the electric field.

One should point out that the observation of the right-chirality current interaction is also important for interpreting of results on the neutrinoless double beta decay [21].

We plan to search for the other polarized (anti)neutrino beams, which could be interesting from the aspect of observable effects caused by the exotic interacting right-chirality states. We expect some interest in the neutrino laboratories working with polarized muon decay and artificial polarized (anti)neutrino sources, and neutrino beams, e.g. KARMEN, PSI, TRIUMF, BooNE.

The experiments measuring neutrino observables will be a real challenge for experimental groups, but could establish the full Lorentz structure of the charged current weak interaction, detect the existence of the right-chirality (anti)neutrinos and of the non-standard T-violating phases.

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Appendix A. Four-vector antineutrino polarization and density operator

The formula for the spin polarization 4-vector of massive antineutrino S' moving with the momentum \mathbf{q} is as follows:

$$S' = (S'^0, \mathbf{S}'), \quad (22)$$

$$S'^0 = \frac{|\mathbf{q}|}{m_{\bar{\nu}}} (\hat{\boldsymbol{\eta}}_{\bar{\nu}} \cdot \hat{\mathbf{q}}), \quad (23)$$

$$\mathbf{S}' = - \left(\frac{E_{\bar{\nu}}}{m_{\bar{\nu}}} (\hat{\boldsymbol{\eta}}_{\bar{\nu}} \cdot \hat{\mathbf{q}}) \hat{\mathbf{q}} + \hat{\boldsymbol{\eta}}_{\bar{\nu}} - (\hat{\boldsymbol{\eta}}_{\bar{\nu}} \cdot \hat{\mathbf{q}}) \hat{\mathbf{q}} \right), \quad (24)$$

where $\hat{\boldsymbol{\eta}}_{\bar{\nu}}$ – the unit 3-vector of the antineutrino polarization in its rest frame. The formula for the density operator of the polarized antineutrino in the limit of vanishing antineutrino mass $m_{\bar{\nu}}$ is given by:

$$\lim_{m_{\bar{\nu}} \rightarrow 0} \Lambda_{\bar{\nu}}^{(s)} = \lim_{m_{\bar{\nu}} \rightarrow 0} \frac{1}{2} \{ [(q^\mu \gamma_\mu) - m_\nu] [1 + \gamma_5 (S'^\mu \gamma_\mu)] \} \quad (25)$$

$$= \frac{1}{2} \{ (q^\mu \gamma_\mu) [1 - \gamma_5 (\hat{\boldsymbol{\eta}}_{\bar{\nu}} \cdot \hat{\mathbf{q}}) - \gamma_5 S'^\perp \cdot \boldsymbol{\gamma}] \}, \quad (26)$$

where $S'^\perp = (0, \boldsymbol{\eta}_{\bar{\nu}}^\perp = \hat{\boldsymbol{\eta}}_{\bar{\nu}} - (\hat{\boldsymbol{\eta}}_{\bar{\nu}} \cdot \hat{\mathbf{q}}) \hat{\mathbf{q}})$. We see that in spite of the singularities $m_{\bar{\nu}}^{-1}$ in the polarization four-vector S' , the density operator $\Lambda_{\bar{\nu}}^{(s)}$ remains finite including the transverse component of the antineutrino spin polarization [15].

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