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Hydrodynamics of the Polyakov line in SU(N_c) Yang–Mills

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ABSTRACT

We discuss a hydrodynamical description of the eigenvalues of the Polyakov line at large but finite N_c for Yang–Mills theory in even and odd space-time dimensions. The hydro-static solutions for the eigenvalue densities are shown to interpolate between a uniform distribution in the confined phase and a localized distribution in the de-confined phase. The resulting critical temperatures are in overall agreement with those measured on the lattice over a broad range of N_c, and are consistent with the string model results at $N_c = \infty$. The stochastic relaxation of the eigenvalues of the Polyakov line out of equilibrium is captured by a hydrodynamical instanton. An estimate of the probability of formation of a Z(N_c) bubble using a piece-wise sound wave is suggested.

1. Introduction

Lattice simulations of Yang–Mills theory in even and odd dimensions show that the confined phase is center symmetric [1,2]. At high temperature Yang–Mills theory is in a deconfined phase with broken center symmetry. The transition from a center symmetric to a center broken phase is non-perturbative and is the topic of intense numerical and effective model calculations [3] (and the references therein). Of particular interest are the semi-classical descriptions and matrix models.

In the semi-classical approximations, the confinement–deconfinement transition is understood as the breaking of instantons into a dense plasma of dyons in the confined phase and their reassembly into instanton molecules in the deconfined phase [4,5]. This mechanism is similar to the Berezinskii–Kosterlitz–Thouless transition in lower dimensions [6], and to the transition from insulators to superconductors in topological materials [7]. In matrix models, the Yang–Mills theory is simplified to the eigenvalues of the Polyakov line and an effective potential is used with parameters fitted to the bulk pressure to study such a transition [8,9], in the spirit of the strong coupling transition in the Gross–Witten model [10].

Matrix models for the Polyakov line share much in common with unitary matrix models in the general context of random matrix theory [11]. The canonical example is Dyson circular unitary ensemble and its analysis in terms of orthogonal polynomials or a one-component Coulomb plasma. The Dyson circular unitary ensemble relates to the one-dimensional Calogero–Sutherland model [12] which is an effective model for quantum Luttinger liquids.

A useful analysis of one-dimensional interacting electron systems relies on hydrodynamics which does not require an exact solution of the many-body problem. The method treats the system in the continuum limit as a fluid, and allows for the understanding of both small amplitude collective phenomena (phonons) as well as large amplitude effects (solitons, schocks) [13,14]. A reduction of the many-body Hamiltonian onto the hydrodynamical collective degrees of freedom makes use of the collective quantization method developed in the context of quantum field theory [15] and extended to problems in condensed matter physics [16].

In this letter we develop a hydrodynamical description of the gauge invariant eigenvalues of the Polyakov line for an SU(N_c) Yang–Mills theory at large but finite N_c. We will use it to derive the following new results: 1/ a hydrostatic solution for the eigenvalue density that interpolates between a confining (uniform) and de-confining (localized) phase; 2/ explicit critical temperatures for the Yang–Mills transitions in $1+2$ and $1+3$ dimensions; 3/ a hydrodynamical instanton for the density distribution that captures the stochastic relaxation of the eigenvalues of the Polyakov line; 4/ an estimate of the fugacity or probability to form a Z(N_c) bubble using a piece-wise sound-wave.

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2. Polyakov line in 1 + 2 dimensions

The matrix model partition function for the eigenvalues of the Polyakov line for SU(N_c) in 1 + 2 dimensions was discussed in [8]. If we denote by diag$(e^{i\theta_1},\dots,e^{i\theta_{N_c}})$ with $\sum_i\theta_i = 0$ the gauge invariant eigenvalues of the Polyakov line, then [8]

$$Z[\alpha,\beta] = \int \prod_{i=1}^{N_c} d\theta_i \prod_{i<j}^{N_c} |z_{ij}|^{\beta(T)} e^{-\alpha(T)\sum_{i<j} V(|z_{ij}|)} \tag{1}$$

with $z_{ij} = z_i - z_j$ and $z_i = e^{i\theta_i}$. The perturbative potential $V(z_{ij})$ is center symmetric and quadratic in leading order or $V(|z_{ij}|) \approx |z_{ij}|^2$, with $\alpha(T) = T^2 V_2/2\pi$ and V_2 the spatial 2-volume [8]. The mass expansion of the one-loop determinant gives $\beta(T) = m_D^2 V_2/\pi$ [8]. The Debye mass is self-consistently defined as $m_D^2 = N_c g^2 T(\ln(T/m_D) + C)/2\pi$ [17] to tame all infra-red divergences, with $C \approx 1.3$ from lattice simulations [18,19].

(1) can be regarded as the normalization of the squared and real many-body wave-function $\Psi_0[z_i]$ which is the zero-mode solution to the Shrödinger equation $H_0\Psi_0 = 0$ with the self-adjoint squared Hamiltonian

$$H_0 \equiv \sum_{i=1}^{N_c} (-\partial_i + \mathbf{a}_i)(\partial_i + \mathbf{a}_i) \tag{2}$$

with $\partial_i \equiv \partial/\partial\theta_i$ and the pure gauge potential $\mathbf{a}_i \equiv \partial_i S$. Here $S[z] = -\ln\Psi_0[z]$ is half the energy in the defining partition function in (1). In (2) the mass parameter is 1/2.

3. Hydrodynamics

We can use the collective coordinate method in [15] to re-write (2) in terms of the density of eigenvalues as a collective variable $\rho(\theta) = \sum_{i=1}^{N_c}\delta(\theta-\theta_i)$. For that, we re-define $H_0 \to H$ through a similarity transformation to re-absorb the diverging 2-body part induced by the Vandermond contribution $\Delta = \prod_{i<j}|z_{ij}|^{\beta(T)}$, i.e. $\Psi = \Psi_0/\sqrt{\Delta}$ and $\sqrt{\Delta}H = H_0\sqrt{\Delta}$. Now H is of the general form discussed in [15] and is amenable after some algebra to

$$H = \int d\theta\,(\partial_\theta\pi\rho\,\partial_\theta\pi + \rho\mathbf{u}[\rho]) \tag{3}$$

with the potential-like contribution

$$\mathbf{u}[\rho] = \left(A(\theta) - \frac{\pi\beta(T)\rho_H}{2} + \frac{1}{2}\partial_\theta\ln\rho\right)^2 \equiv \mathbf{A}^2 \tag{4}$$

Here

$$A(\theta) = \frac{1}{2}\alpha(T)\int d\theta'\rho(\theta')\,\partial_\theta V\left(2\sin\left(\frac{\theta-\theta'}{2}\right)\right) \tag{5}$$

and ρ_H is the periodic Hilbert transform of ρ

$$[\rho]_H \equiv \rho_H(\theta) = \frac{1}{2}\frac{\mathrm{P}}{\pi}\int \rho(\theta')\,\mathrm{cotan}\left(\frac{\theta-\theta'}{2}\right) \tag{6}$$

As conjugate pairs, $\pi(\theta)$ and $\rho(\theta)$ satisfy the equal-time commutation rule $[\pi(\theta),\rho(\theta')] = -i\left(\delta(\theta-\theta') - 1/2\pi\right)$. We identify the collective fluid velocity with $v = \partial_\theta\pi$ and re-write (3) in the more familiar hydrodynamical form

$$H \approx \int d\theta\,\rho(\theta)\left(v^2 + \mathbf{u}[\rho]\right) \approx \int d\theta\,\rho(\theta)\,|v + i\mathbf{A}|^2 \tag{7}$$

modulo ultra-local terms. The Heisenberg equation for ρ yields the current conservation law $\partial_t\rho = -2\partial_\theta(\rho v)$, and the Heisenberg equation for v gives the Euler equation

$$\partial_t v = i[H,v] = -\partial_\theta\left(v^2 + \mathbf{A}^2 - \partial_\theta\mathbf{A} - \mathbf{A}\partial_\theta\ln\rho + \pi\beta[\mathbf{A}\rho]_H - 2\alpha[\mathbf{A}\rho]_S\right) \tag{8}$$

with the sine-transform $[\mathbf{A}\rho]_S = \int\sin(\theta-\theta')\mathbf{A}(\theta')\rho(\theta')$. Note that all the relations hold for large but finite N_c.

4. Hydro-static solution

The static hydrodynamical density follows from the minimum of (6) with $v(\theta) = 0$,

$$\beta(T)\pi\rho_H(\theta) - \partial_\theta\ln\rho(\theta) = 2A(\theta) \tag{9}$$

To solve (9), we insert the leading quadratic contribution $A(\theta) \approx 2\alpha(T)\sin^2(\theta/2)$ in (9)

$$\rho\rho_H - a\partial_\theta\rho = bc_1\rho\sin(\theta) \tag{10}$$

with $a \equiv 1/\pi\beta(T)$, $b \equiv 2\alpha(T)/\beta(T)$ and c_1 the first moment of the density or $\pi c_1 \equiv \int_0^{2\pi}\rho(\theta)\cos\theta d\theta$. Let $\rho_0 = N_c/2\pi$ be the uniform eigenvalue density and $\rho_1 = \rho - \rho_0$ its deviation. Consider the Cauchy transform

$$G(z) = \frac{1}{\pi i}\int_{\mathcal{C}}\frac{\rho_1(\eta)}{\eta - z}d\eta \tag{11}$$

with $\eta = e^{i\theta}$. The contour $\mathcal{C}$ is counter-clockwise along the unit circle. $G(z)$ is a holomorphic function in the complex z-plane. Let G^+ and G^- be its realization inside and outside $\mathcal{C}$ respectively, so that

$$G^{\pm}(z \to e^{i\theta}) = \pm\rho_1(\theta) + i\rho_H(\theta) \tag{12}$$

We now carry the Hilbert transform on both sides of (10). Setting $G(z) = G^+(z)$ and using $2[\rho_1\rho_H]_H = \rho_H^2 - \rho_1^2$, we have for (10)

$$\frac{1}{2}G^2 + (\rho_0 - \frac{1}{2}bc_1(z - z^{-1}))G + az\partial_z G = bc_1\rho_0 z + \frac{1}{2}bc_1^2 \tag{13}$$

on the boundary $\mathcal{C}$, thus within the circle. Here, we should require $G(z=0) = 0$ to ensure that ρ_1 integrates to zero.

$a \approx 1/V_2$ is subleading and will be dropped. Thus (13) is algebraic in $G(z)$. Since $\rho(\theta) = \rho_0 + \mathrm{Re}\,G^+(z = e^{i\theta})$, careful considerations of the singularity structures of the quadratic solutions to (13) yield (Θ is a step function)

$$\rho(\theta) = \sqrt{bc_1}(\cos\theta + 1)^{\frac{1}{2}}(\cos\theta - \cos\theta_0)^{\frac{1}{2}}\,\Theta(|\theta_0| - |\theta|) \tag{14}$$

The analytic properties of $G(z)$ fix $c_1/\rho_0 = 1 + (1 - 1/b)^{\frac{1}{2}}$ and θ_0 at $\cos\theta_0 = 1 - 2\rho_0/bc_1$. For $b < 1$ the non-uniform solution with $\rho_1 \neq 0$ is absent. For $b \gg 1$, $c_1 \to 2\rho_0$ and

$$\rho(\theta) \to \frac{N_c}{2\pi}\sqrt{8b - 4b^2\theta^2} \tag{15}$$

Therefore (14) interpolates between a uniform density distribution ρ_0 (confined phase) and a Wigner semi-circle (deconfined phase) with a transition at $b = 1$ or $T_c = m_D$. In 1 + 2 dimensions the fundamental string tension is given to a good accuracy by $\sqrt{\sigma_1}/g^2N_c = ((1 - 1/N_c^2)/8\pi)^{\frac{1}{2}}$ [22]. Thus the ratio in 1 + 2 dimensions

$$\frac{T_c}{\sqrt{\sigma_1}} = \frac{C}{2\pi}\left(\frac{8\pi}{1 - 1/N_c^2}\right)^{\frac{1}{2}} \to \sqrt{\frac{2}{\pi}}C \tag{16}$$

with $C \approx 1.3$ [18,19]. In Fig. 1 we show the behavior of (16) (upper curve) versus N_c, in comparison to the numerical fit $T_c/\sqrt{\sigma_1} = 0.9026 + 0.880/N_c^2$ to the lattice results (lower curve) in [23]. Amusingly, (16) at large N_c is consistent with $\sqrt{3/\pi}$ in the string model [20].

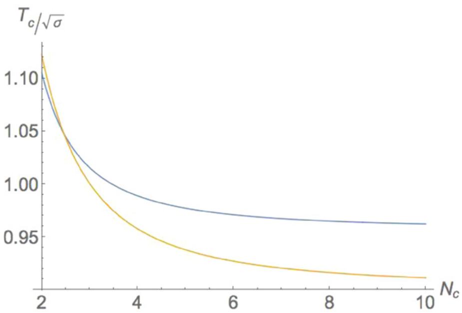

Fig. 1. $T_c/\sqrt{\sigma_1}$ versus N_c in (16) (upper curve) compared to the numerical fit to the lattice results (lower curve) from [23].

5. Dyson Coulomb gas

We note that (9) coincides with the saddle point equation to (1) by re-writing it using Dyson charged particle analogy on S^1 with the energy $2S[z] = \sum_{i<j} G(z_{ij})$ and the pair interaction

$$G(z_{ij}) = -\ln|z_{ij}|^{\beta(T)} + \alpha(T)V(|z_{ij}|) \equiv \mathbf{G}(\theta_i - \theta_j) \tag{17}$$

At large N_c the ensemble described by (1) is sufficiently dense to allow the change in the measure. Following Dyson [11] we obtain

$$Z[\alpha, \beta] \to \int D\rho\, e^{-\Gamma[\alpha,\beta;\rho]} \tag{18}$$

with the effective action

$$\Gamma[\alpha, \beta; \rho] = \frac{1}{2}\int \rho(\theta)\mathbf{G}(\theta-\theta')\rho(\theta') - \left(\frac{\beta(T)}{2} - 1\right)\int d\theta\,\rho(\theta)\ln\left(\frac{\rho(\theta)}{\rho_0}\right) \tag{19}$$

The β contribution is the self-Coulomb subtraction and is consistent with the subtraction in the Hilbert transform. The saddle point equation $\delta\Gamma/\delta\rho = 0$ following from (18)–(19) is in agreement with the hydro-static equation (9),

$$\frac{d}{d\theta}\frac{\delta\Gamma[\alpha, \beta; \rho]}{\delta\rho(\theta)} = 2\mathbf{A} = 0 \tag{20}$$

6. Hydrodynamical instanton

The fixed time zero energy solution to (7) is an instanton with imaginary velocity $v = -i\mathbf{A}$. We have checked that this is a solution to (8) for all times. The current $j \equiv \rho v = -i\rho\mathbf{A}$ is conserved. Thus $\partial_\tau\rho - 2\partial_\theta(\rho\mathbf{A}) = 0$ or

$$\partial_\tau\rho + \beta(T)\partial_\theta(\pi\rho\rho_H) = \partial_\theta^2\rho + 2\partial_\theta(\rho A(\theta)) \tag{21}$$

for Euclidean times $\tau = it$. For $A = 0$ and $\beta(T) = 2$, (21) agrees with the viscid Burger's equation describing large Wilson loops in $1+1$ dimensions [21]. Following [11] we identify τ with the stochastic (Langevin) time. (21) describes the stochastic relaxation of the eigenvalue density of the Polyakov line (out of equilibrium) to its asymptotic (in equilibrium) hydro-static solution.

7. Sound waves

The hydrodynamical action follows from standard procedure. The momentum $\pi(\theta) = (1/\partial_\theta)v$ is canonically conjugate to the density ρ, and the Lagrange density is $\mathbf{L} = \pi\partial_t\rho - H$. Thus the action $\mathbf{S} = \int dt d\theta\,\rho(\theta)\left(v^2 - \mathbf{u}[\rho]\right)$, which is linearized by

$$\rho \approx \rho_0(\theta) + 2\partial_\theta\varphi \qquad \text{and} \qquad \rho v \approx -\partial_t\varphi \tag{22}$$

Inserting (22) into $\mathbf{S}$ yields

$$\mathbf{S}_2 = \int dt\frac{d\theta}{\rho_0(\theta)}\left((\partial_t\varphi)^2 - \rho_0^2(\theta)W^2[\varphi]\right) \tag{23}$$

with the potential

$$W[\varphi] = 2\alpha(T)[\partial_\theta\varphi]_S - \pi\beta(T)[\partial_\theta\varphi]_H + \partial_\theta\left(\frac{\partial_\theta\varphi}{\rho_0(\theta)}\right) \tag{24}$$

For constant ρ_0 and large N_c, (23) simplifies to

$$\mathbf{S}_2 \approx m_D^2 V_2\int dt d\theta\left((\partial_t\varphi)^2 - (\partial_\theta\varphi)^2\right) \tag{25}$$

after the rescaling $v_s t \to t$ with $v_s = \pi\rho_0\beta(T)$. (25) describes sound waves in the large N_c space of holonomies.

8. Z(N_c) bubble

In a de-confined phase of infinite volume, the Yang–Mills ground state settles in one of the degenerate Z(N_c) vacua. In a finite volume, bubbles of different vacua may form [25]. Consider a de-confined bubble of volume $\mathbb{V}_2$ immersed in a confined volume $\overline{\mathbb{V}}_2$. In $\mathbb{V}_2$ all the eigenvalues are localized initially within a small $\Delta\theta$ around the origin with $\rho(\tau = 0, \theta) = N_c/\Delta\theta \equiv \rho_B$, and zero otherwise.

Using this piece-wise wave as an initial condition we solve (21) with $A = 0$ for simplicity. For large times τ, the result is

$$\rho(\tau, \theta) \approx \rho_0 - \left(\frac{2}{\pi}\rho_B\sin\left(\frac{\Delta\theta}{2}\right)\right)\cos\theta\, e^{-v_s\tau} \tag{26}$$

which shows the relaxation of the piece-wise wave over a time $\tau \approx 1/v_s$ set by the speed of sound. Using (26) in $\mathbf{S}$ yields the Euclidean action estimate for small $\Delta\theta$

$$\mathbf{S}_E(\mathbb{V}_2) \approx \mathbb{V}_2\left(\pi m_D\rho_B\sin\left(\frac{\Delta\theta}{2}\right)\right)^2 \to \mathbb{V}_2\left(\frac{\pi}{2}N_c m_D\right)^2 \tag{27}$$

The bubble formation probability or fugacity is $e^{-\mathbf{S}_E(\mathbb{V}_2)}$.

9. Polyakov line in 1 + 3 dimensions

To extend our analysis to $1+3$ dimensions, we approximate the Yang–Mills thermal state by a dense plasma of dyons and anti-dyons [4,5]. This semi-classical description reproduces a number of key features of the Yang–Mills phase both in the confined (center-symmetric) and de-confined (center-broken) phase. There are two key differences with the $1+2$ dimensional partition function in (1). First the many-body energy $2S[z] = -2\ln\Psi_0[z]$ in (1) is now shifted

$$2S[z] \to 2S[z] - \gamma(T)\prod_i^{N_c}(\theta_{i+1} - \theta_i)^{\frac{1}{N_c}} \tag{28}$$

with $\gamma(T) = 4\pi N_c f V_3$ and $f = 4\pi\Lambda^4/Tg^4$ the dyon fugacity [4]. Second and more importantly $\beta(T) = 2$ and is not extensive with the spatial 3-volume V_3. Finally, $\alpha(T) = T^3V_3/3$. Since $(\theta_{i+1} - \theta_i) \approx 1/2\pi\rho(\theta_i)$, then in the continuum the additional string of factors in (28) is

$$\prod_i^{N_c}(\theta_{i+1} - \theta_i)^{\frac{1}{N_c}} \to e^{\frac{1}{N_c}\int d\theta\rho(\theta)\ln(1/2\pi\rho(\theta))} \tag{29}$$

With this in mind, a re-run of the preceding arguments yields the Hamiltonian in (3)–(4) with the shifted potential

$$A \to A + \frac{\gamma(T)}{4\pi N_c^2}e^{-\gamma_0[\rho]}\partial_\theta\ln\rho(\theta) \tag{30}$$

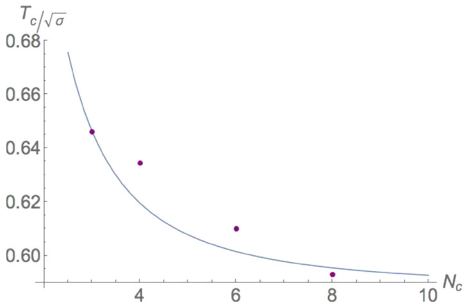

Fig. 2. $T_c/\sqrt{\sigma_1}$ versus N_c in (34). The dots are the lattice results from [24].

and $N_c \ln\gamma_0[\rho] = \int d\theta\rho(\theta)\ln(\rho(\theta)/N_c)$. The hydro-static equation (9) now reads

$$\beta\pi\rho_H(\theta) - 2A(\theta) = \left(1 + \frac{\gamma(T)}{4\pi N_c^2}e^{-\gamma_0[\rho]}\right)\partial_\theta \ln\rho(\theta) \tag{31}$$

The $\beta = 2$ contribution is now sub-leading and can be dropped. The corresponding solution to (31) is a localized density for $\pi c_1 = \int_0^{2\pi} d\theta\rho(\theta)\cos\theta \neq 0$, and a uniform density $\rho_0 = N_c/2\pi$ for $c_1 = 0$. Specifically

$$\frac{\rho(\theta)}{\rho_0} = \frac{e^{\frac{8\pi\alpha\gamma_0}{\gamma'}c'\cos\theta}}{I_0(\frac{8\pi\alpha\gamma_0}{\gamma'}c')} \tag{32}$$

with $c' = c_1/N_c$ and $\gamma' = \gamma/N_c^3$. The two parameters $\eta = 8\pi\alpha(T)/\gamma'$ and $x = c'\eta\gamma_0$ are fixed by the transcendental equations

$$\frac{I_1(x)}{I_0(x)} = \frac{\pi x}{\eta\gamma_0} \quad \text{and} \quad \frac{I_1(x)}{I_0^2(x)}e^{x\frac{I_1(x)}{I_0(x)}} = \frac{2\pi^2 x}{\eta} \tag{33}$$

A solution exists only for $\gamma' < 2\alpha(T)/\pi$. Else the density is uniform. Thus the transition temperature from center symmetric (confining) to center-broken (deconfining) occurs for $\alpha(T_c)/\gamma(T_c) = \pi/2N_c^3$ or $T_c^4 = \frac{3}{8\pi}\frac{\Lambda^4}{\lambda^2}$ with $\lambda = g^2N_c/8\pi^2$. For the dyon model, the fundamental string tension is given by $\sigma_1 = (N_c/\pi)\sin(\pi/N_c)\,\Lambda^2/\lambda$ [4]. Thus the model independent ratio in $1+3$ dimensions

$$\frac{T_c}{\sqrt{\sigma_1}} = \left(\frac{3\pi}{8N_c^2\sin^2(\pi/N_c)}\right)^{\frac{1}{4}} \to \left(\frac{3}{8\pi}\right)^{\frac{1}{4}} \tag{34}$$

(34) compares favorably to the lattice results [24] even for small N_c as shown in Fig. 2. At large N_c, (34) is consistent with the value of $\sqrt{3/2\pi}$ in the string model [20].

10. Conclusions

The hydrodynamical description of the Polyakov line captures aspects of the center dynamics in Yang–Mills theory in terms of the gauge invariant density of eigenvalues. The hydro-static equations yield solutions that interpolate between a center symmetric (confining) and a center-broken (de-confining) phase. The transition temperatures normalized to the string tension compare well to the lattice results over a broad range of N_c, and asymptote the string model results at $N_c = \infty$. The hydrodynamical set-up supports a hydrodynamical instanton that describes the stochastic relaxation of the eigenvalues of the Polyakov line viewed as a fluid. The fluid supports sound waves that can be used to estimate the probability of formation of $Z(N_c)$ bubbles. The relaxation of a fluid of holonomies across the critical temperature may prove useful for understanding the onset of equilibration in a Yang–Mills plasma.

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