Measuring Diversity of Socio-Cognitively Inspired ACO Search

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Abstract. In our recent research, we implemented an enhancement of Ant Colony Optimization incorporating the socio-cognitive dimension of perspective taking. Our initial results suggested that increasing the diversity of ant population — introducing different pheromones, different species and dedicated inter-species relations — yielded better results. In this paper, we explore the diversity issue by introducing novel diversity measurement strategies for ACO. Based on these strategies we compare both classic ACO and its socio-cognitive variation.

Keywords: Ant colony optimization · Metaheuristics · Diversity · Nature-inspired optimization · Discrete optimization

1 Introduction

In population-based metaheuristics, attaining certain balance between exploration of the search space in general, and exploitation of its most promising parts is a major issue to achieve robust algorithms producing feasible solutions, using rationally available resources (as computing power or time) [16]. Lack of diversity may lead to stagnation and the system may focus on locally optimal solutions (in other words—trapped in a local extremum), needing more randomness to escape [15]. This balance is usually attained by maintaining diversity

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in the population of individuals: different diversity-preserving mechanisms have been introduced to accomplish this, e.g. coevolution and speciation in Evolutionary Algorithms [16]. In this connection, it should be noted that diversity is shown to play a key role in creativity [9,21].

Ant systems have proven to be a popular tool for solving many discrete optimization problems, e.g. Traveling Salesman Problem (TSP), Quadratic Assignment Problem (QAD), Vehicle Routing Problem (VRP), Graph Coloring Problem (GCP) and others [8]. In our previous paper, we considered the ant system as a way to express socio-cognitive behaviors of a population of ants, differentiating them into species and defining their stigmergic interactions, following and enhancing the substantial results presented in [23], getting promising results in TSP optimization.

We claim that the promising results obtained by the socio-cognitive ants, as presented in [23], were, at least partially, due to an enhanced diversity of ants in the population (instead of a homogeneous population). However, to verify this claim we need to devise some way to measure and monitor diversity. One approach is to check the contents of the current solutions of the problem in the population of individuals. In particular, in the case of evolutionary algorithms, one can analyze the center of weight of the solutions, dispersion of genes etc. (see, e.g. [5,17]).

However, though measuring diversity in a real-valued space is relatively easy, it becomes a very difficult problem in a discrete-valued space. For example, looking for a center-of-weight of the strings (e.g. encoding TSP solutions) is an NP-hard problem *per se* [14]), so analysis of the search space is impossible in a feasible time period.

We introduce in this paper a new approach for measuring the diversity in ACO search and evaluate its feasibility on the socio-cognitive ant system compared to the classic ACO. Instead of analyzing solutions we focus on analyzing the pheromone table, treating the information contained therein as a kind of *derivative* of the information contained in the search space.

The rest of the paper is organized as follows. First, ACO and its selected variants including our socio-cognitive ones, are described. Later, dedicated diversity measurement techiques are introduced and appropriate results obtained from comparing the classic and the socio-cognitive ACO are shown. In the final section we present the conclusions and mention some future research issues.

2 Classic and Novel Approaches to Ant Colony Optimization

Ant System, introduced in 1991, applied to solve TSP, is considered to be a progenitor of all ant colony optimization (ACO) algorithms [6]. Because the action of a certain ant during one iteration is completely independent of the actions of other ants during any iteration, the sequential ant algorithm can easily be parallelized.

The ACO algorithm is an iterative process during which certain number of ants (agents) gradually create a solution [7,8]. The problem being solved is usually depicted as a graph, and the main goal of the ants is to traverse this graph in an optimal way. Each move of an ant consists in choosing a subsequent component of the solution (graph edge) with certain probability. This decision may be affected by interaction among the ants based on the levels of *pheromones*, which may be deposited into the environment (on the edges of the graph) by some ants and perceived by other ants. This interaction is guided by stigmergy (environment-mediated communication among individuals instead of direct contact) rules proposed in [6]). The iteration process is finished when a feasible solution is reached through the cooperative efforts of all the ants.

Recently, new interesting modifications of ACO-related techniques have been introduced. For example, multi-type ACO [19,24] define many species of ants and allow complex stigmergic interactions such as attraction to the pheromone of the same species and repulsion from that of the others. These algorithms have been successfully applied to problems such as edge disjoint path problem [19] and light path protection [24].

There are other modifications of the classic ACO, such as hierarchical ACO, where additional means of control are introduced to manage the output of particular ants or ant species [22]. In another approach, ants are endowed with different skills (e.g. sight, speed) in order to realize global path-planning for a mobile robot [13]. In a successfull approach to solve TSP, the authors propose to use two types of ants: classic and exploratory (creating 'short routes', moving according to some predefined conditions like near some selected cities, etc.) [12]. In [4], the authors introduce different ant sensitivity to pheromones such that ants with higher sensitivity follow stronger pheromone trails, while ants with lower sensitivity behave more randomly. This model strives to sustain a balance between exploration and exploitation.

Taking inspiration from these approaches, especially the ones proposed by Nowé et al. [19] (many species of ants with detailed stigmergic interactions), and by Chira et al. (different sensivity of the ants to the pheromones) [4], we proposed a novel method of simulation and analysis of socio-cognitive properties of individuals of a certain population, at the same time being an efficient optimization algorithm that already produced encouraging results [23].

Measuring of diversity in ACO has been tackled by Nakamichi et al. [18], who constructed elitist ACO and examined the number of paths found by elite ants. In other papers, one can find mostly visible enhancements of diversity (though not a measurement techniques themselves), see, e.g. [1,20]. Therefore, we chose our research goal to work out a universal diversity measurement techniques for ACO.

3 From Perspective Taking to Enhancing Diversity in ACO

In [23] we have explored the effect of incorporating socio-cognitive mechanism, namely perspective taking, on the search capabilities of ACO. We assumed that such an approach would promote the diversity of the ant colony, however we were not able to validate this claim: hence we undertook the research presented here. We briefly summarize below the main ideas from [23], which provide the context in which we address the problem of characterising diversity parameters.

Typically, perspective taking is seen as a one-dimensional ability: the degree to which an agent can take another one's perspective. But recent research has shown that human's variability in terms of perspective-taking performance can be better explained if one considers two dimensions [2]: the ability of an agent to handle conflict between its own and the other agent's perspectives, and the relative priority that an agent gives to his own perspective relative to the other's perspective. Individuals endowed with good cognitive skills to manage conflicting information are usually better perspective-takers [10]. In addition, the less a person focuses on her own perspective and the more that person will be motivated to engage in perspective taking [2]. Experimental research has also shown that situational factors such as someone's emotional state can selectively impact on one of the two perspective-taking dimensions (conflict handling or perspective priority), which further shows that both dimensions are important to characterize human diversity in perspective taking [3]. This two-dimensional approach to perspective taking inspired us to define four types of individuals.

Let us consider these four types of individuals and their possible interactions [23]:

- Egocentric individuals: Focus on their own perspective and can become creative by finding their own new solutions to a given task. They do not pay attention to others and do not get inspired by others' actions (or these inspirations do not become a main factor of their work).
- Altercentric individuals: Focus on the perspective of others and thus follow the mass of others. They are less creative but can end up supporting good solutions by simply following them.
- Good-at-conflict-handling individuals: Get inspired in a complex way by the actions of other individuals by considering different perspectives and choosing the best.
- Bad-at-conflict-handling individuals: Act purely randomly, following sometimes one perspective, sometimes another without any inner logic.

Now let us work on incorporating of these ideas by constructing a population consisting of different species of ants. These species will search for the solution not only using their own expertise, but also getting stigmergic inspirations from other ones (by the analysis and combination of different pheromones left by these species in the environment).

These four types of individuals are directly inspired by socio-cognitive phenomena. It is to note that a dedicated paper, reviewing different configurations of ACO populations (exceeding the above-mentioned inspirations) is under review.

4 Socio-Cognitive Ant Colony Optimization

In this section we recall the description of classic and socio-cognitive ACO (after [23]). The reason to do this is to prepare a simple formalism that will help in proper introducing different methods of diversity measuring in the next section.

Starting from the definition of classic ACO, we consider optimization of a combinatorial problem (e.g. to find a Hamiltonian cycle in a graph as in Travelling Salesman Problem). The method is based on agents, namely ants, that roam along the edges of a graph, searching for cycles and leaving trails of pheromones behind them.

4.1 Classic ACO

In the classical ACO algorithm, the ants are deployed in a graph consisting of vertices $V = \{i : i \in \mathbb{N}\}$ and edges $E = \{e_{ij} : i, j \in V\}$, where each edge is associated with the cost of moving along it. Each ant gets a randomly chosen starting graph node. Beginning from this node, the ant searches for a cycle, in a step-by-step manner, by moving from one node to another, choosing the next one and not coming back. While considering which node to visit next, the ant has to compute attractiveness for all possible paths that can be taken from the present node. The attractiveness n_{ij} of the edge ij starting from the node i where the ant is currently at is the basis for computing the probability of choosing a particular path:

$$\mu_{ij} = \frac{n_{ij}}{\sum_j n_{ij}} \tag{1}$$

where j is computed only for nodes that have not yet been visited by the ant.

The exact values of n_{ij}, which is the attractiveness computed for the next edges constituting the constructed path, for classic ants and all the introduced modifications are given below in details.

Finally, the ant randomly selects a path based on the previously computed probabilities: paths with higher attractiveness are more likely to be chosen. After visiting all the nodes exactly once, the ant finishes its trip and returns the found cycle as a proposed solution, and then retreats depositing certain amount of pheromone on the path of its current cycle. The amount of pheromone deposited on an edge e_{ij} is denoted by π_{ij}, and the deposition algorithm of ant a_k retreating along cycle c_{a_k} is as follows:

$$\pi'_{ij} \leftarrow \pi_{ij} + \frac{\pi_d}{\sum_{e \in c_{a_k}} cost(e)} \tag{2}$$

where the default pheromone deposit π_d is 1, e_{ij} denotes an edge in the cycle, and $cost(e_{ij}) : E \rightarrow \mathbb{R}$ is a function that assigns a cost to each edge.

The pheromone evaporates in each iteration (in each edge of the graph) according to this formula:

$$\pi'_{ij} = (1 - \pi_e) \cdot \pi_{ij} \tag{3}$$

Default pheromone evaporation coefficient π_e is 0.01.

Classic ants. They consider both pheromone and distance while choosing their direction by computing *path attractiveness* in order to complete the cycle. So an ant at node i will choose the next edge according to the following attractiveness:

$$n_{ij} = \frac{\pi_{ij}^{\alpha}}{cost(e_{ij})^{\beta}} \tag{4}$$

Default factors are, pheromone influence $\alpha = 2.0$, distance influence $\beta = 3.0$.

Each type of ant in Sect. 4.2 below uses Eq. 1 to calculate probabilities for the subsequent paths, differing only in **attractiveness** to various types of pheromones.

4.2 Multi-pheromone ACO

In socio-cognitive ACO, the idea of multiple pheromones is implemented by introducing different 'species' of ants and enabling their interactions (similar to the approach taken in [19]). The interaction is considered as a partial inspiration or perspective taking, realized by a particular ant reacting to the decisions taken by ants belonging to other species. This is made possible by having ants of different species leave different 'smells' (see Fig. 1). Different ants use different rules (consider different properties of the path) for computing attractiveness; and looking for inspirations or perspective taking, they utilize the smells of pheromones left by other species in a predefined way. Therefore different species may be treated as organisms with selective smelling capabilities (reacting to different combinations of the smells that are present).

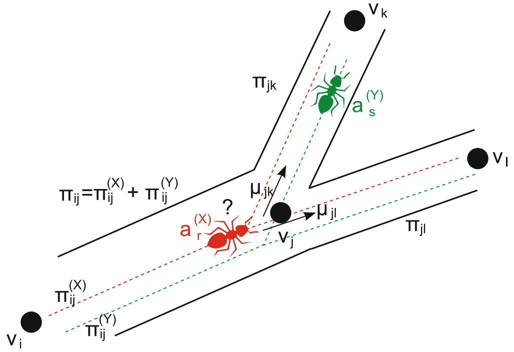


Fig. 1. Multi-pheromone ACO setting

Different ant species leave pheromones that 'smell' different, so the pheromone left at a particular edge is described as a sum of the following components:

$$\pi_{ij} = \pi_{ij}^{(EC)} + \pi_{ij}^{(AC)} + \pi_{ij}^{(GC)} + \pi_{ij}^{(BC)} \tag{5}$$

Other ants may react to different combinations of these pheromones. Of course, more species and more pheromones may be introduced into the system as needed.

Based on this framework, details of the actions undertaken by various ant species are described below.

Egocentric ants (EC). They are creative in trying to find a new solution and finding their own way. They care less about other ants and about the pheromone trail. Instead, they focus mostly on the distance as a way to determine their next directions. An ant at node i will choose the next edge with the following attractiveness:

$$\frac{1}{cost(e_{ij})^{\beta}} \tag{6}$$

Default distance influence $\beta = 3.0$, again.

Altercentric ants (AC). They follow the majority of other ants, thereby focusing on the pheromone, without caring for the distance. So an ant at node i will choose the next edge with the following attractiveness:

$$\pi_{ij}^{\alpha} \tag{7}$$

Default pheromone influence $\alpha = 2.0$.

Good-at-conflict-handling ants (GC). They wait and observe the others, thereby caring for all existing pheromones (the particular weights are to be determined experimentally). So an ant at node i will choose the next edge with the following attractiveness:

$$\left(14 \cdot \pi_{ij}^{(EC)} + 2 \cdot \pi_{ij}^{(AC)} + 2.5 \cdot \pi_{ij}^{(GC)} + 0.5 \cdot \pi_{ij}^{(BC)}\right)^{\alpha} \tag{8}$$

Default pheromone influence $\alpha = 2.0$.

Bad-at-conflict-handling ants (BC). They behave impulsively (in effect randomly), irrespective of the pheromone or the distance. So an ant at node i will choose the next edge with the following attractiveness:

$$\frac{1}{\sum_{e_{ik}, k \in V \setminus \{i\}}} \cdot 100\,\% \tag{9}$$

5 Measuring the Diversity of ACO Search

As it was mentioned in the introduction, direct measuring of the diversity in the discrete space is very difficult. However, a "derivative" of information contained in the search space, especially connected with the search abilities of the ants, resulting in their behavior (exploration—when they actively search for new solutions and exploitation—when they fine-tune the already found, good ones) is contained in the pheromone table. Let us consider the following measures based directly on the analysis of the pheromone table:

- The number of pheromone-marked edges of the graph should directly show the diversity of the search, as when a small number of the edges is marked, the ants will only travel using these edges, possibly getting stuck in a local extremum. Otherwise, when a large number of edges is marked, the ants will roam through the graph. Therefore we propose to treat pheromone dispersion as a first measure of diversity for ACO. In other words, this measure is based on the ratio of the pheromone-marked edges count to all edges count:

$$PR = 100\,\% \cdot \frac{\#\{e_{ij} : \pi_{ij} > 0\}}{\#\{e_{ij}\}}, \forall i, j \tag{10}$$

PR standing for Pheromone Ratio.

– The second measure is based on the attractiveness, as the ants are directly driven by this parameter during their travels. In the extreme case, when the ants roam everywhere randomly, choosing directions with equal probability, the edges would have equal attractiveness. Therefore one can compute the attractiveness of each edge, and measure its dispersion throughout the whole graph. If the dispersion (measured e.g. by the means of standard deviation) is low, the attractiveness is equally distributed. If it is high, only part of the graph is marked with high attractiveness. Therefore the second measure of diversity, based on the attractivenesses of edges is as follows:

$$AD = \sigma(\{n_{ij}\}), \forall i, j \tag{11}$$

AD standing for Attractiveness Dispersion.

– The third measure is also based on attractiveness, however the rationale for it is different compared to the second measure. If the ants have chosen only one solution (they have totally lost the diversity), they will travel only along one hamiltonian in the graph, and the edges belonging to this hamiltonian will have non-zero attractiveness, while the attractiveness of the other edges is zero. Therefore one can compute the sum of attractivenesses of the best edges belonging to the best solution, and divide it by the sum of attractivenesses of the other edges. Thus the third diversity measure, also based on attractiveness, is given as follows:

$$AR = 100\,\% \cdot \frac{\sum_{i,j} n_{ij}}{\sum_{k,l} n_{kl}}, k \neq i \wedge l \neq j, \forall i, j, k, l \tag{12}$$

Where n_{ij} belong only to the currently best individual, and AR stands for Attractiveness Ratio.

6 Experimental Results

The experimental results were obtained from a dedicated software developed in Python[1], run on Zeus supercomputer[2] We considered the Travelling Salesman Problem: find a Hamiltonian in a graph defined by a network of cities, with the goal being a cycle with the least cost (distance) [11]. The instances used in the experiments were taken from TSPLIB library[3].

[1] www.python.org.
[2] http://plgrid.pl.
[3] http://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/.

6.1 Configuration and Infrastructure

Zeus cluster, which is a supercomputer consisting of different kinds of 2-processor servers with different processor frequencies, number of cores, number of cores per node and RAM memory per node. Experiments were run on machine with the following technical parameters: **Model**: HP BL2x220c G5, G6, G7, **Total number of cores**: 17516, **Processors**: 2x Intel Xeon L5420, L5640, X5650, E5645, **Number of cores per node**: 8–12, **Processor frequency**: 2,26-2,66 GHz, **RAM memory per node**: 16–24 GB.

The following platform configuration was assumed for each experimental run:

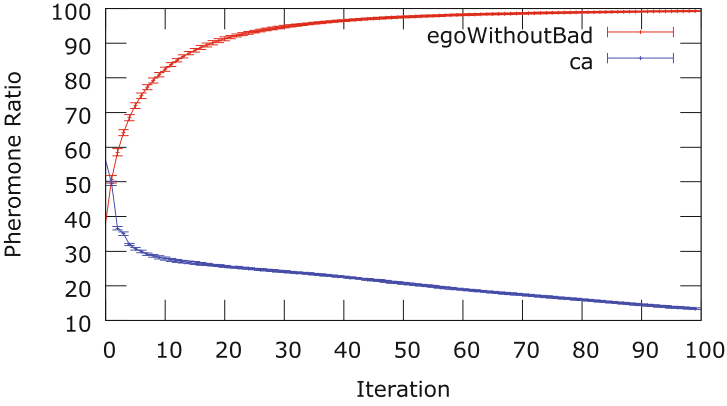
- Number of ants: 100.
- Number of iterations: 100.
- Number of trials for each experiment: 30. Final data is the avarage of these 30 trials.
- Tested data taken from TSPLIB: berlin52, rat195, ts225. These instances were taken to exemplify easy, medium and difficult TSP problems. Of course this evaluation is subjective, and a dedicated publication reviewing efficiency of socio-cognitive ACO for different TSP instances is under review.

During the experiment, the following compositions (with respect to proportions of different ant species) of the simulated population were considered:

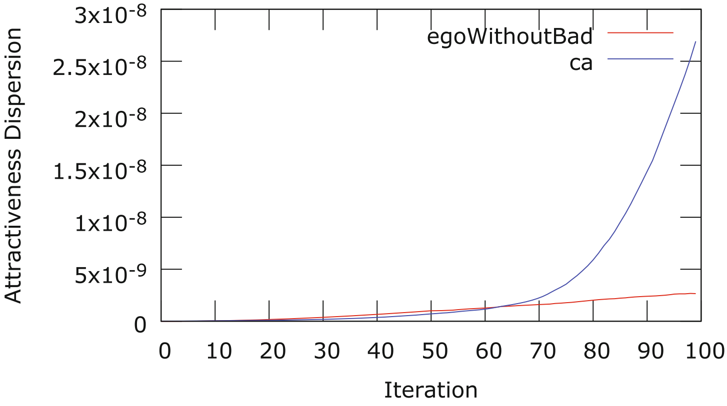
- Classic Ant Population: Only ants acting as in classic ACO.
- Modification based on Human-inspired sample populations: Egocentrical without bad at conflict handling **(egoWithoutBad)** population: 60 % egocentric, 20 % altercentric, 20 % good at conflict handling, 0 % bad at conflict handling. The proportions were chosen arbitrarily as an exemplification of the socio-cognitive ACO algorithm. Publication of a paper focused on review of configurations of socio-cognitive ACO populations is pending.

PR diversity measure points out, that diversity of the ants belonging to socio-cognitive species is much higher than the one observed in ACO (see Figs. 2a, 3a, 4a). It is necessary to keep in mind, that the pheromone ratio was averaged for socio-cognitive ants, in order to be able to compare it with the classic ACO. Though it may seem that pheromone is located everywhere in the pheromone table, it is probably not equally distributed. Therefore the ants are able to explore the search space, by traveling from time to time along the edges with lower attractiveness, and exploit the search space by traveling along the highly attractive edges. It is also to note, that in the case of **ts225** problem (see Fig. 4a), classic ants performed so badly (cf. Fig. 5e), probably unable to mark any reasonable part of the graph with the pheromone, that the Pheromone Ration diversity measure outcome was zero.

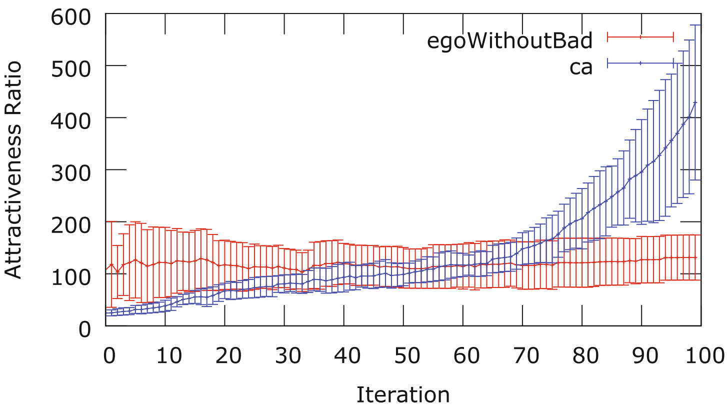
In Figs. 2b, 3b, 4b presenting standard deviation of the average attractiveness depending on iteration. Its value becomes visibly bigger in **berlin52** and **rat195** (see Figs. 2b, 3b) problems for classic ACO when compared to socio-cognitive system. In the case of the experiment **ts225** (Fig. 4b) one should remember about the size of the problem (225 cities, it means maximally 25200 edges to be



(a) Diversity measure: Pheromone Ratio

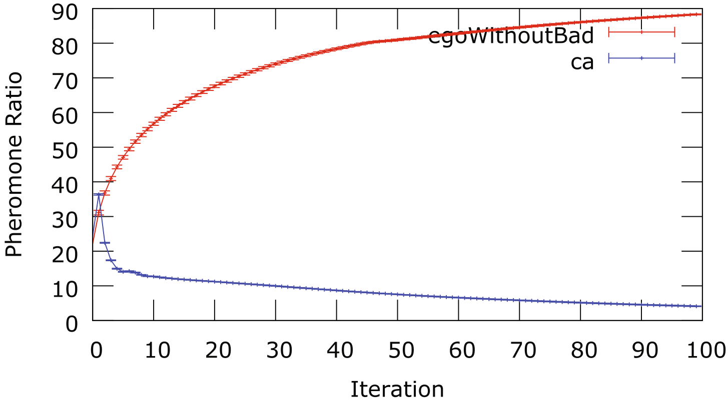


(b) Diversity measure: Attractiveness Dispersion

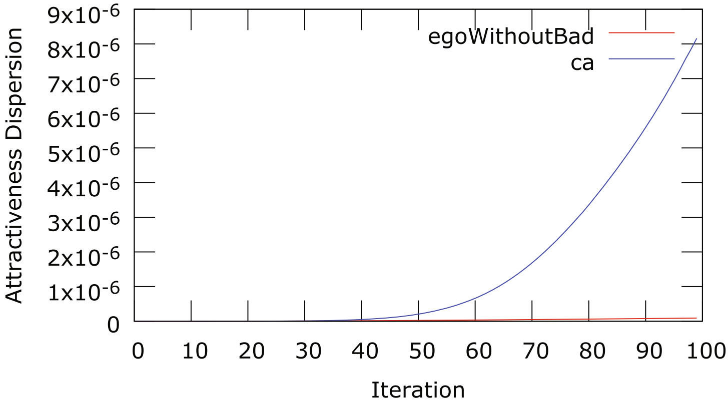


(c) Diversity measure: Attractiveness Ratio

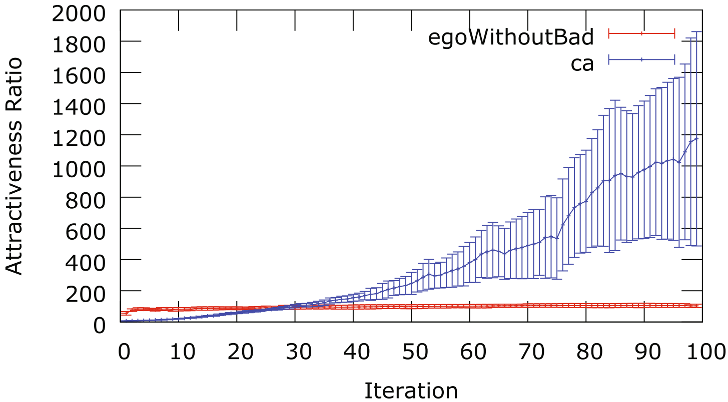
Fig. 2. Problem: **berlin52**, iterations: **100**, ants: **100**, different diversity measures for classic and socio-cognitive ACO



(a) Diversity measure: Pheromone Ratio

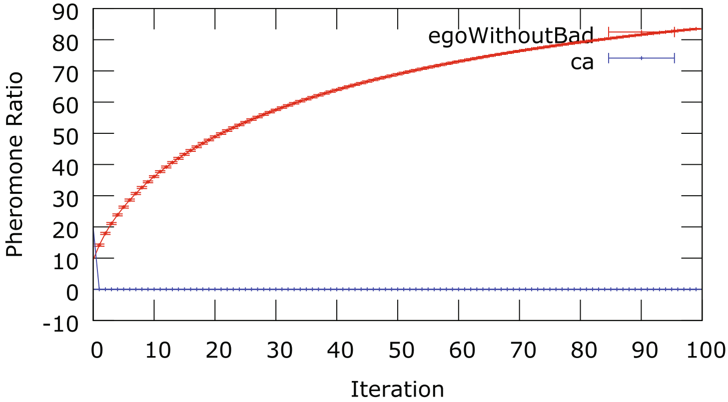


(b) Diversity measure: Attractiveness Dispersion

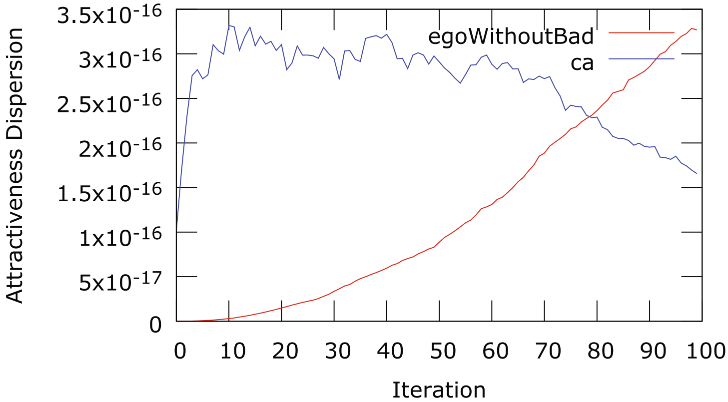


(c) Diversity measure: Attractiveness Ratio

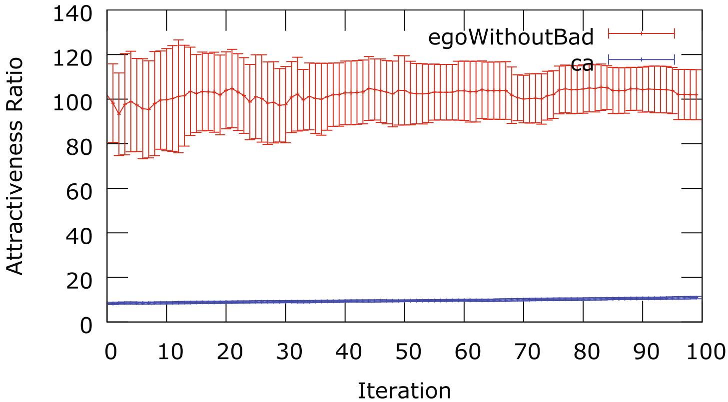
Fig. 3. Problem: **rat195**, iterations: **100**, ants: **100**, different diversity measures for classic and socio-cognitive ACO



(a) Diversity measure: Pheromone Ratio

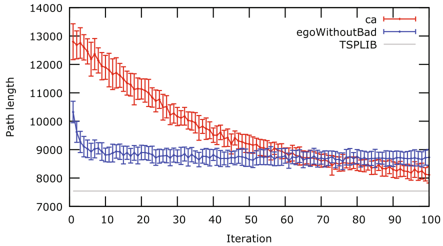


(b) Diversity measure: Attractiveness Dispersion

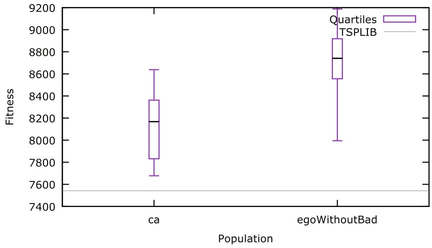


(c) Diversity measure: Attractiveness Ratio

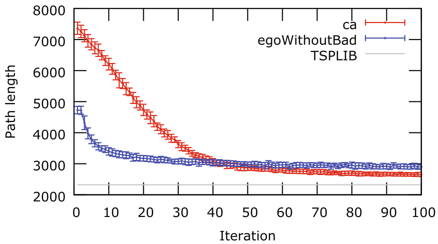
Fig. 4. Problem: **ts225**, iterations: **100**, ants: **100**, different diversity measures for classic and socio-cognitive ACO



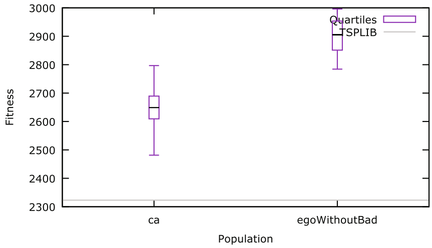
(a) Best fitness for **berlin52** dependent on iteration



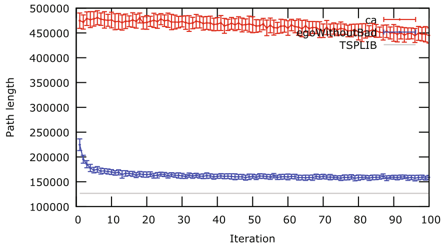
(b) Final results for **berlin52** in the form of box and whiskers plot



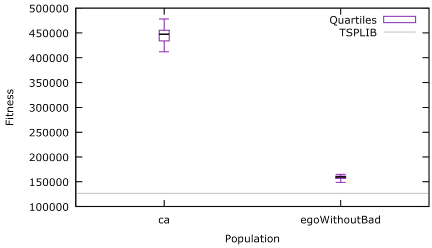
(c) Best fitness for **rat195** dependent on iteration



(d) Final results for **rat195** in the form of box and whiskers plot



(e) Best fitness for **ts225** dependent on iteration



(f) Final results for **ts225** in the form of box and whiskers plot

Fig. 5. Comparison of **classic ACO** and **egoWithoutBad** populations fitnesses

marked by the pheromone, and only 100 ants in this experiment). The diversity measurement techniques based on attractiveness are not reliable here, as classic ants are unable to generate any reasonable solution in this case. The pheromone table for classic ants seems to be quite chaotic, thus the observation of the higher dispersion of their attractiveness. Anyway, this measure becomes the second one supporting the assumption of higher diversity in socio-cognitive ACO search.

Figures 2c, 3c, show clearly the dynamic nature of the search conducted in both systems. Classic ACO maintains high diversity in the beginning, exploring the space, later focusing on promising area, going into exploitation. At the same time, the socio-cognitive system maintains quite stable balance between exploitation and exploration. Figure 4c corresponds to the above-mentioned problems of classic ants in this setting—the diversity is practically zero, as the best solution in this case is probably not attractive enough in comparison with other edges marked with pheromones, to contribute to this fraction to a visible extent.

Finally one should check the actual outcome of the search, namely the evolution and final result obtained for **berlin52, rat195, ts225** problems in Fig. 5a, c, e. It is easy to see, that socio-cognitive system produced very quickly a very promising, sub-optimal result, while ACO needed at least 5 times more iterations to attain a similar result (Fig. 5a, c) (even though finally classic ACO was better than socio-cognitive one). However considering the hardest problem, **ts225**, classic ACO acted significantly poorer during all the observed iterations (Fig. 5e).

The curves of best fitness shown in Fig. 5a will behave in a similar way (reach suboptimal result of a similar quality as in Fig. 5c), as these two experiments were conducted with the same set of parameters.

7 Conclusions

We have presented here three novel diversity measurement strategies for ACO search: instead of using the search-space features, it is based on the information contained in the pheromone table. This strategies were applied to compare the diversity of socio-cognitive ant system and the classic one. The results obtained show that socio-cognitive ACO has a visibly higher diversity compared with the classic ACO in terms of the proposed diversity measures.

In the future, we plan to extend the analysis of the diversity measures and their impact on the ACO algorithms by getting a better insight on the pheromone table, distribution of attractiveness and pheromones, verification using other ACO-based metaheuristics and problems other than TSP. Moreover, the proposed measurement techniques will be used to automatically adapt the parameters of the ACO search. We would like also to further tune-up the parameters of our search in order to finally escape the local extrema that withhold us from reaching the global optima (found as best so far for TSPLIB), even though they are quite close to them.

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