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Advances in Variational and Hemivariational Inequalities

Theory, Numerical Analysis, and Applications

Springer

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Preface

The theory of variational inequalities is a relatively young mathematical discipline. One of the bases for its development was the contribution of Fichera [5], who coined the term "Variational Inequality" in his paper on the solution of the frictionless contact problem between a linearly elastic body and a rigid foundation posed by Signorini [15]. The foundations of the mathematical theory of elliptic variational inequalities were laid by Stampacchia [16], Hartman and Stampacchia [7], Lions and Stampacchia [11], and others. Evolutionary variational inequalities have been preliminary treated by Brézis [2] who also connected the notion of variational inequality to convex subdifferential and maximal monotone operators. The theory of variational inequalities can be viewed as an important and significant extension of the variational principle of virtual work or power in inequality form, the origin of which can be traced back to Fermat, Euler, Bernoulli brothers, and Lagrange. The theory of variational inequalities and their applications represents the topics of several well-known classical monographs by Duvaut and Lions [4], Glowinski, Lions, and Trémoliéres [6], Kinderlehrer and Stampacchia [10], Baiocchi and Capelo [1], Kikuchi and Oden [9], and so on.

The notion of hemivariational inequality was first introduced by Panagiotopoulos [13] and is closely related to the development of the concept of the generalized gradient of a locally Lipschitz functional provided by Clarke [3]. Interest in hemivariational inequalities originated, similarly as in variational inequalities, in mechanical problems. From this point of view, the inequality problems in Mechanics can be divided into two main classes: that of variational inequalities, which is concerned with convex energy functions (potentials), and that of hemivariational inequalities, which is concerned with nonsmooth and nonconvex energy functions (superpotentials). Through the formulation of hemivariational inequalities, problems involving nonmonotone and multivalued constitutive laws and boundary conditions can be treated successfully mathematically and numerically. The theory of hemivariational inequalities and their applications was developed in several monographs by Panagiotopolous [13], Naniewicz and Panagiotopolous [12], and Haslinger, Mietten, and Panagiotopolous [8], among others.

During the last decades, variational and hemivariational inequalities were shown to be very useful across a wide variety of subjects, ranging from nonsmooth mechanics, physics, and engineering to economics. For this reason, there are a large number of problems which lead to mathematical models expressed in terms of variational and hemivariational inequalities. The mathematical literature dedicated to this field is growing rapidly, as illustrated by the list of references at the end of each chapter of this volume.

The purpose of this edited volume is to highlight recent advances in the field of variational and hemivariational inequalities with an emphasis on theory, numerical analysis, and applications. The theory includes existence and uniqueness results for various classes of nonlinear inclusions and variational and hemivariational inequalities. The numerical analysis addresses numerical methods and solution algorithms for solving variational and hemivariational inequalities and provides convergence results as well as error estimates. Finally, the applications illustrate the use of these results in the study of miscellaneous mathematical models which describe the contact between deformable bodies and a foundation. This includes the modeling, the variational and the numerical analysis of the corresponding contact processes.

This volume presents new and original results which have not been published before and have been obtained by recognized scholars in the area. It addresses to mathematicians, applied mathematicians, engineers, and scientists. Advanced graduate students can also benefit from the material presented in this book. Generally, the reader is expected to have background knowledge on nonlinear analysis, numerical analysis, partial differential equations, and mechanics of continua.

This volume is divided into three parts with 14 chapters. This division of the material is not strict and it is done only for the convenience of the reader. A brief description of each part is the following.

Part I, entitled *Theory*, is devoted to the study on abstract nonlinear evolutionary inclusions and hemivariational inequalities of the first and second order, an approximation method to solve nonsmooth problems and its application to variational–hemivariational inequalities, a bifurcation result for a nonlinear Dirichlet elliptic problem, and variational inequality problems on nonconvex sets.

Part II, entitled *Numerical Analysis*, deals with the numerical approximation of the hemivariational inequalities, extragradient algorithms for solving various classes of variational inequalities, the proximal methods for treating a nonlinear inverse problem in linearized elasticity relating to tumor identification, and discontinuous Galerkin methods for solving an elliptic variational inequality of the fourth order.

Part III, entitled *Applications*, is dedicated to the study of miscellaneous classes of problems issued from Contact Mechanics. Topics include the analysis of a dynamic contact model for Gao beam, an energy-consistent numerical model for the dynamic frictional contact between a hyperelastic body and a foundation, a nonclamped frictional contact problem in thermo-viscoelasticity, the large time asymptotics for contact problems for Navier–Stokes equation and antiplane elasticity, hemivariational inequalities, and history-dependent hemivariational inequalities in dynamic elastic-viscoplastic contact problems.

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