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# **A Cognitive Perspective on Norms**

## Abstract

Norms are ideals that serve as guiding beacon in many human activities. They are considered to transcend accepted social and cultural practices, and reflect some universal, moral principles. In this chapter, we will show that norms are cognitive constructs by considering several examples in the domains of language, art and aesthetics, law, science and mathematics. We will argue that, yes, norms are ideals that we posit, so in this respect they go beyond current social and cultural values. However, norms are posited using cognitive mechanisms and are based on our existing knowledge and wisdom. In this sense, norms are what we, as an individual or as a society, strive for, but they show the horizon effect in that they recede and transform as we progress towards them, and sometimes this transformation can be radical.

# 1. Introduction

Plato argued that moral values like justice, and aesthetic values like beauty, are not a matter of social convention or practical advantage, but are grounded in objectivity, just like mathematical concepts.<sup>1</sup> This

<sup>&</sup>lt;sup>1</sup> See, for instance, *Republic*.

sentiment was echoed relatively recently by M.K. Gandhi: "An error does not become truth by reason of multiplied propagation, nor does truth become error because nobody sees it. Truth stands, even if there be no public support. It is self sustained." These objective, mind-in-dependent values or truths are called *norms*.

Norms play a key role in various human endeavours and institutions. In law, for example, concepts of justice, right and wrong, are considered objective, and not a matter of social acceptance. So even though slavery was widely practiced and accepted at one time, one argues, it was morally and objectively wrong. Similar arguments underlie heated debates on issues such as abortion, gay marriages, and so on.

In language, it has been argued, proper names and natural kinds point to the essences of their referents, and any descriptions that a cognitive agent may associate with them are incidental. Scientific concepts are even more so: gravity, in this view, is a mind-independent aspect of reality, which was *discovered* by Newton, as someone finds a set of lost keys. Mathematical concepts are perhaps extreme in this regard, for numbers, geometric figures such as triangles, mathematical operations are all considered pure, abstract and mind-independent.

In art, even though one admits of cultural variations, true beauty is considered to transcend social practices and customs. A mind-independent view of aesthetics can account for cases when previously obscure works come to be highly regarded both by art experts and general public – there aesthetic value is considered to be *discovered*, like Newton's discovery of gravity.

In this chapter, my aim is to critically examine this mind-independent view of norms from a cognitive perspective. I will consider five domains: language, art, law, science, and mathematics; and in each domain I will present a few examples to illustrate how norms change. Based on these examples, I will argue that norms are cognitive constructs: they have an objective component, but are nonetheless shaped by cognitive interaction initiated by the cognitive agent. Norms can change as individuals and societies grow and evolve; moreover, this evolution is non-monotonic in that it allows Kuhnian revolutions – creative insights often manifest such revolutions. Though fully acknowledging that norms are not based on accepted social practices, I will argue that norms are *ideals* that a society posits for itself as utopian goals to be achieved. However, these ideals are based on the existing beliefs of the cognitive agents and the society. As the knowledge of the cognitive agents grows, and their society evolves, these ideals (and, therefore, the norms) also change. In this regard, I will conclude, norms show the horizon effect.

In the next five sections, I will analyse examples from the domains of language, art, law, science, and mathematics, in this order. As the issue of a norm is quite complex in each of these domains, the discussion here is intended to highlight the cognitive aspects underlying normativity, rather than a deep and thorough discussion of normativity in the respective domains.

## 2. Normativity in language

Perhaps the most significant manifestation of normativity in language is Kripke's theory of proper names,<sup>2</sup> and Putnam's theory of natural kinds.<sup>3</sup> In order to illustrate the cognitive aspects of normativity in language, I will focus on Kripke's theory in this section.

In *Naming and Necessity*, Kripke<sup>4</sup> argued that proper names are rigid designators; that is, there is a direct and rigid (across all possible worlds) association between a proper name and its referent. Any descriptions associated with the name are incidental. Descriptions do not define the referent, and names are not a label for a cluster of descriptions.

Let us examine what are considered to be the epistemic arguments that Kripke puts forth in support of the rigid designator theory. Kripke notes that descriptions can turn out to be in error. So, for

<sup>&</sup>lt;sup>2</sup> S. Kripke, *Naming and Necessity*, Harvard University Press, Cambridge (MA), 1980.

 <sup>&</sup>lt;sup>3</sup> H. Putnam, "The meaning of 'meaning'", in *idem, Mind, Language and Reality. Philosophical Papers, vol. 2*, Cambridge University Press, Cambridge (UK), 1975, pp. 215–272.
<sup>4</sup> S. Kripke, *Naming and Necessity, op. cit.*

instance, for most people, Kurt Gödel is the person who proved the incompleteness theorem, so this feature might be taken as the defining description of who Kurt Gödel was. However, it may well turn out that Kurt Gödel did not prove the incompleteness theorem. In this case, we would say that we were mistaken in assuming that Kurt Gödel proved the incompleteness theorem, and not that we have been using the name Kurt Gödel to refer to the wrong person.

Kripke goes on to argue that even prototypical descriptions associated with a name may turn out to be in error; most of the descriptions associated with the name may turn out to be in error; even *all* of descriptions may turn out to be in error. Though one can take this position philosophically, if we consider it from a cognitive point of view, it raises a serious question: if *all* the descriptions associated with a name turn out to be in error, how is the error discovered?

To appreciate this point, let us look at the details of an actual case of error discovery: Han van Meegeren's (1889–1947) forgeries of Johannes Vermeer (1632–1675). In 1930s, van Meegeren painted a number of works in the style of Vermeer. One of them, *Christ with the Adulteress*, was bought by an art dealer in 1942, and eventually sold to Reichsmarschall Herman Göring for 1.65 million guilders. After Germany's surrender in May 1945, van Meegeren was arrested and charged with being a Nazi collaborator. Facing the bleak prospect of a long prison sentence, or possibly the death penalty, van Meegeren confessed to having forged the painting. People did not believe him at first – they assumed that he was just saying this to save himself from the harsh penalty, for art experts had authenticated the so-called Vermeers. To convince them, he painted another 'Vermeer' in front of witnesses. After that, an *expert panel* concluded that the painting sold to Göring was not a genuine Vermeer but a forgery.<sup>5</sup>

<sup>&</sup>lt;sup>5</sup> E. Dolnick, *The Forger's Spell: A True Story of Vermeer, Nazis, and the Greatest Art Hoax of the Twentieth Century*, Harper, New York 2008; J. Lopez, *The Man Who Made Vermeers: Unvarnishing the Legend of Master Forger Han Van Meegeren*, Houghton Mifflin Harcourt, New York 2008; E. Morris, "Bamboozling ourselves (Part 1 – Part 7)", *The New York Times*, May 27– June 4, 2009. http://opinionator.blogs.nytimes.com/ category/bamboozling-ourselves/ (accessed on Dec. 25, 2011.)

Even after all this, one Brussels-based art expert disagreed with the expert panel, and argued (in 1951) that two of the so-called fake Vermeers were actually genuine. One art collector sued the head of the expert panel because it had brought down the value of his 'Vermeers'. Even though the court sided with the expert panel (in 1955), it was only much later that *Pb-210-dating* (1967) and *gas chromatography* (1977) confirmed beyond doubt that those 'Vermeers' were forgeries, for the paint pigment used in them did not exist in Vermeer's time.

There are two points that I would like to highlight in this story. One is that even though one can take the normative view that the works of Vermeer are the works actually painted by him, and any properties, like the style etc. are incidental, in order to determine whether a painting is indeed a Vermeer or not, we have to rely on some other descriptions to access the referent so that its authenticity (whether it satisfies the normativity criteria) can be checked. If some researcher were trying to establish that Kurt Gödel did not actually prove the incompleteness theorem, she or he would have to use some other descriptions to identify the referent of the name Kurt Gödel, and then establish, using some descriptions again, that this person could not have or did not prove the incompleteness theorem. So even though most of the descriptions associated with Kurt Gödel might turn out to be in error, it is *logically impossible to have all the descriptions turn* out to be incorrect, for if that were so, we would have no way of finding this out.

The second instructive point concerning this Vermeer story is that when new techniques like *Pb-210-dating* or *gas chromatography* come into being, new attributes (descriptions) to existing objects (like the disputed Vermeers) were added. In this process, it is the (previously) existing descriptions of the object that ensure continuity. In other words, we access the referent through existing descriptions, apply the new technique (or perceptual/cognitive process) on it, and then enrich the description of the referent based on the observed results. This allows us to say that we have *discovered* new properties of the referent. To sum up this discussion, though we may posit a normative account of meaning, so that a name rigidly points to the same referent across all possible worlds, the descriptions associated with the name are not mere spectators. The descriptions provide the only cognitive mechanism for an agent to access the referent. So, though it quite possible that each available description, no matter how critical or important it may be deemed, can turn out to be in error, it is not possible (from a cognitive point of view) that *all* of the descriptions are in error. Notice here that the fact that an object was named such-and-such in a naming ceremony itself becomes a description. Later, if there is a dispute whether the object named such-and-such has a certain attribute, we have to access the object via its descriptions related to the naming ceremony (using historical research.)

Thus, norms in language (at least for proper names) are ideals that we posit, but those ideals in their true sense are not cognitively accessible. Because each of the description can be in error, we cannot be sure that the description we used to access the referent is itself not in error. As we interact with the referent more and more, and our knowledge about it grows (as was the case with the fake Vermeers), our normative ideals itself can change.

Similar arguments have been made for the normative status of natural kind words like water or whale,<sup>6</sup> but we will discuss them later in Section 5 while considering normativity in science.

## 3. Normativity in art and aesthetics

One of the fake Vermeers by van Meegeren, *The Supper at Emmaus*, was praised highly by art experts, and hung in a well-known museum in Rotterdam in the late 1930s. Many people must have passed through the museum, looked and admired the painting, and had some

<sup>&</sup>lt;sup>6</sup> H. Putnam, "The meaning of 'meaning", op. cit.

kind of aesthetic experience. Are there any normative aspects of such aesthetic experiences?

There are a number of parameters that affect aesthetic experience, one of which is the context effect. A striking example of this is provided by the case of Joshua Bell, a world-renowned violinist, when he played six classical pieces in a Metro stop in Washington DC during rush hour, and hardly anyone stopped by to listen to him; while people pay over \$100 a seat to listen to him in a concert hall.<sup>7</sup> This illustrates a phenomenon that Nelson Goodman has dubbed When is Art?.<sup>8</sup> The same object (or the same performance) can be perceived (aesthetically speaking) quite differently depending on the context. So when Marcel Duchamp put an upside down urinal in a museum (Fountain, 1917), it served a different purpose, exemplified different attributes, and evoked a different aesthetic experience, compared to when it is placed in a men's toilet. However, for our purpose here, we can fix the context so that we consider the difference in the aesthetic experiences of viewing an authentic Vermeer and a fake Vermeer, when both are hung ceremoniously in an art museum.

Another parameter is the difference between aesthetic value and artistic value, as articulated by Kulka.<sup>9</sup> The artistic value refers to the appreciation attached to a work of art by the community of art experts (and people as well), as reflected in the awards, monetary value, etc.

<sup>&</sup>lt;sup>7</sup> G. Weingarten, "Pearls before breakfast: Can one of the nation's great musicians cut through the fog of a D.C. rush hour? Let's find out", *Washington Post*, April 8, 2007. http://www.washingtonpost.com/lifestyle/magazine/pearls-before-breakfast-can-one-of-the-nations-great-musicians-cut-through-the-fog-of-a-dc-rush-hour-lets-find-out/2014/09/23/8a6d46da-4331-11e4-b47c-f5889e061e5f\_story.html (accessed on 30 September, 2014); see also J. Contrera, "Joshua Bell is playing in the Metro again. This time, maybe you won't pass it up", *Washington Post*, September 23, 2014. http://www.washingtonpost.com/lifestyle/style/joshua-bell-is-playing-in-the-metro-again-this-time-you-can-plan-to-be-there/2014/09/23/7a699e28-4282-11e4-9a15-137aa0153527\_story.html (accessed on 30 September, 2014).

<sup>&</sup>lt;sup>8</sup> N. Goodman, *Ways of Worldmaking*, Hackett, Indianapolis 1978.

<sup>&</sup>lt;sup>9</sup> T. Kulka, "Forgeries and art evaluation: An argument for dualism in aesthetics", *Journal of Aesthetic Evaluation* 39(3), 2005, 58–70.

Though these two values clearly influence one another, we would like to focus here on the aesthetic value.

A recent case of art forgeries perhaps best exemplifies the problem of normativity of aesthetic experiences. Wolfgang Beltracchi made a career in creating fakes purported to be from such well-known artists as Heinrich Campendonk, Max Ernst, and Fernand Léger. When a long-lost Max Ernst [fake], *La Forêt* was 'discovered', Dorothea Tanning, the widow of Max Ernst, supposedly exclaimed that it was the most beautiful picture that her late husband [Max Ernst] had ever painted. When the scam was exposed and Wolfgang and his wife Helene were arrested and sentenced to prison, their daughter Franziska remarked, "What [my parents] did was criminal–it's a fact. But I think they didn't really hurt anybody. They took money for pictures that people wanted. Maybe now they're not worth anything, but they still got the picture. I don't think it's fair that they went to jail."<sup>10</sup>

Franziska's comments illustrate the view that a painting is essentially *you get what you see*: the aesthetic experience is the totality of the visual experience, and nothing more. So, in this view, what is the difference (aesthetically) between viewing an authentic work and a fake? For instance, now even average art students can 'see' that the fake Vermeers are of rather poor quality. After the Beltracchi fakes were revealed, some experts described them as crude fakes. So how does the aesthetic experience change from one time to another?

Nelson Goodman has addressed this issue in an insightful essay *Art and Authenticity*.<sup>11</sup> His explanation is that our aesthetic perception, individually and also as a social or cultural group, changes as we interact with more and more artworks. So every time one views a work of art, their aesthetic criteria (which can be considered akin to norms) change a little bit. Over time, these little bits can add up to a big macro shift. Modern art students are exposed to authentic Ver-

<sup>&</sup>lt;sup>10</sup> J. Hammer, "The greatest fake-art scam in history?", *Vanity Fair (Art)*, October 10, 2012. http://www.vanityfair.com/unchanged/2012/10/wolfgang-beltracchi-helene-art-scam (accessed on 15 September 2014).

<sup>&</sup>lt;sup>11</sup> N. Goodman, *Languages of Art*, Bobbs-Merrill, Indianapolis 1968, Chap. III.

meers and fake Vermeers, and they learn to distinguish their aesthetic values perceptually (without needing the advanced technology of *gas chromatography*), and in this sense we can say that their aesthetic norms have changed.

Mark Sagoff<sup>12</sup> proposed an alternative explanation for why there might be a difference in the aesthetic experience when viewing an original as opposed to a fake. He argued that one cannot consider aesthetic experience with an artwork in isolation, but it has to be considered in the context of other related artworks. So (in his view), it is meaningless to compare the aesthetics of an original with that of a forgery, as they do not belong to a meaningful semantic class. Even though this approach is usually contrasted with that of Goodman,<sup>13</sup> I submit that the two approaches are quite consistent. What Goodman points out is that when the first fake Vermeer or Max Ernst appears in the market, it is compared with the existing set of Vermeers and Max Ernsts. But as the scam is exposed, and more fakes are unmasked, the class of paintings that the fakes are compared to is changed, thereby changing their aesthetic experience.

In the search for universal aesthetic norms, it is worth mentioning the proposal of Ramachandran and Hirstein.<sup>14</sup> They put forth eight laws of aesthetic experience that are grounded in our neural and perceptual processes. For example, one law is that of peak shift, according to which if certain perceptual attributes (of form, posture, colour, etc.) are preferable for us (for whatever evolutionary reasons), then to emphasize them further, even to an unrealistic extent, is more attractive. There is much supporting evidence for this principle from animal research. For instance, Tinbergen<sup>15</sup> noted that a seagull chick pecks at its mother's beak to get food. Similar to Kon-

<sup>&</sup>lt;sup>12</sup> M. Sagoff, "The aesthetic status of forgeries", *Journal of Aesthetics and Art Criticism* 35 (2), 1976, 169–180.

<sup>&</sup>lt;sup>13</sup> For example, see T. Kulka, "Forgeries and art evaluation...", op. cit.

<sup>&</sup>lt;sup>14</sup> V. S. Ramachandran, W. Hirstein, "The science of art: A neurological theory of aesthetic experience", *Journal of Consciousness Studies* 6 (6–7), 1999, 15–51.

<sup>&</sup>lt;sup>15</sup> N. Tinbergen, Curious Naturalists, Basic Books, New York 1954.

rad Lorenz's famous experiments with goslings that demonstrated biological imprinting, Tinbergen found that seagull chicks respond equally well to a brown stick with a red dot at the end. But surprisingly, they found that the chicks respond even more effectively to a longer brown stick with three red stripes at the end, even though this does not correspond to any natural stimuli. Ramachandran and Hirstein use this peak shift principle to explain occurrences of exaggerated female forms found in ancient Indian artwork, in medieval art, in Picasso, and so on.

While acknowledging that there are some neurological factors that constrain our aesthetic experience, there is much evidence that higher-level cognitive factors can override these constraints and significantly alter our aesthetic experience. In the examples of fake Vermeers and Max Ernsts discussed above, the belief whether one is looking at an authentic artwork or a fake can affect her or his aesthetic appreciation. Morris,<sup>16</sup> echoing the direct reference theory of Kripke,<sup>17</sup> points out that for visual arts, the history of the work is an essential component of its identity, and thereby affects its aesthetic potential. But history of an artwork is a matter of our current state of knowledge, and the background and beliefs of the viewer.

Many studies in different domains support the view that our beliefs and expectations have a remarkable influence on our perception and behaviour. For example, Lang *et al.*<sup>18</sup> showed that the aggressive behaviour of participants after drinking was affected by what they thought they were drinking rather that what they actually drank. Brochet<sup>19</sup> found similar effect with wine tasting: the partic-

<sup>&</sup>lt;sup>16</sup> E. Morris, "More bamboozling", *The New York Times*, June 17, 2009. http://opinionator.blogs.nytimes.com/2009/06/17/more-bamboozling/#more-10885 (accessed on Dec. 25, 2011.)

<sup>&</sup>lt;sup>17</sup> S. Kripke, Naming and Necessity, op. cit.

<sup>&</sup>lt;sup>18</sup> A. R. Lang, D. J. Goeckner, V. J. Adesso, G. A. Marlatt, "Effect of alcohol on aggression in male social drinkers", *Journal of Abnormal Psychology* 84 (5), 1975, 508–518.

<sup>&</sup>lt;sup>19</sup> F. Brocher, *Tasting: Study of chemical representations of objects in the field of consciousness*, PhD thesis, Faculty of Oenology of Bordeaux, Laboratory of General Oenology 2001.

ipants' perception of taste depended on the bottle from which the wine was poured, even though it was the same wine.<sup>20</sup> All this points to the fact that our expectations, past experience, and understanding influences our perception and aesthetic experience. More importantly, as our experience grows, these expectations change and so do our aesthetic norms.

Martindale<sup>21</sup> provides an interesting study on how aesthetic norms can change over a long period of time. He started by formulating a framework for creativity in art, according to which societies require novelty, but in small doses. When a novel style is introduced, artworks in the new style have a high arousal potential. But over time, due to habituation, novelty wears off, and another shot of novelty is necessary, which requires that a new style be introduced. He tested his theory by analysing many genres of art, including British poetry over a span of 650 years (from 1290 to 1949), and found clear evidence of cyclical changes in the style.<sup>22</sup>

To conclude, we see that aesthetic experience is essentially a cognitive process. Though there are some biological constraints, but by and large cultural and social conventions, background knowledge and beliefs of the viewer, expectations of the viewer etc. dominate the aesthetic experience related to a particular artwork in a given context. Moreover, these expectations change over time resulting in a shift of aesthetic norms.

<sup>&</sup>lt;sup>20</sup> See also R. Goldstein, J. Almenberg, A. Dreber, J. W. Emerson, A. Herschkowitsch, J. Katz, "Do more expensive wines taste better? Evidence from a large sample of blind tastings", *Journal of Wine Economics* 3 (1), 2008, 1–9; L. Lee, S. Frederick, D. Ariely, "Try it, you'll like it – the influence of expectation, consumption, and revelation on preferences for beer", *Psychological Science* 17 (12), 2006, 1054–1058.

<sup>&</sup>lt;sup>21</sup> C. Martindale, *The Clockwork Muse: The Predictability of Artistic Change*, Basic Books, New York 1990, Chap. 4.

<sup>&</sup>lt;sup>22</sup> See also R. K. Sawyer, *Explaining Creativity: The Science of Human Innovation*, Oxford University Press, Oxford (UK) 2006, Chap. 9.

#### 4. Normativity in law

The extent to which law is normative is a somewhat controversial issue.<sup>23</sup> On one extreme there are natural law thinkers who conflate legal norms with moral norms. Legal positivists, on the other hand, consider legal norms to be distinct from moral norms. Sidestepping this debate, in this section I would like to present a few examples of how legal norms change.

Consider a very basic legal concept, that of *personhood*. The Fifth Amendment to the US Constitution states: "No *person* shall be held to answer for a capital, or otherwise infamous crime, unless on a presentment or indictment of a grand jury..."<sup>24</sup>

Now who qualifies to be a 'person'? The amendment was ratified in 1791, when slavery was an accepted practice. Slaves were regarded as property, so they clearly were not 'persons.' For instance, in the case of *Dred Scott v. Sandford* [60 U.S. 393 (1857)], the US Supreme court ruled that slaves, including those freed by their masters, were not US citizens and could not sue in federal court.

Another twenty-two years later, there was another case before the courts: *Standing Bear vs. General Crook* [United States *ex rel.* Standing Bear v. Crook, 25 F.Cas. 695 (C.C.D.Neb. 1879) (No. 14,891)]. Standing Bear was a Ponca Native American chief. When Europeans were settling in America, Native Americans were forcibly resettled into reservations, and were prohibited from venturing out. Standing Bear, with a group of followers left the reservation to bury his eldest son. Brigadier General George Crook was sent to arrest the Poncas and bring them back to the reservation. General Crook, however, was sympathetic to the plight of Native Americans, and encouraged them to seek legal redress.

<sup>&</sup>lt;sup>23</sup> See for a recent discussion: T. Spaak, "Kelsen and Hart on the Normativity of Law", in *Perspectives on Jurisprudence: Essays in Honour of Jes Bjarup*, P. Wahlgren (ed.) pp. 397–414. Available at SSRN: http://ssrn.com/abstract=922755.

<sup>&</sup>lt;sup>24</sup> Emphasise added. As we are interested in the concept of person, we can skip the rest of the text.

During the trial, the main issue was whether a Native American is a person, so that the Fifth Amendment and habeas corpus can be applied to them. The District Attorney relied on the *Dred Scott* decision to argue that Native Americans, because they were not US citizens, cannot file a writ of habeas corpus. The lawyers for the Poncas argued that citizenship is not an issue but personhood. In a very dignified and moving short speech, after the closing arguments had ended, Standing Bear said (through an interpreter), "That hand is not the color of yours, but if I pierce it, I shall feel pain. If you pierce your hand, you also feel pain. The blood that will flow from mine will be of the same color as yours. I am a man. The same God made us both."

In his decision, Judge Dundy used Webster's definition of 'person' and issued an order releasing the Poncas. In summarizing his decision, he wrote, "an *Indian* is a PERSON within the meaning of the laws of the United States, and has therefore the right to sue out a writ of habeas corpus in a federal court."<sup>25</sup>

We can also see the evolution of the legal concept of personhood in two other strands. One concerns voting rights in the US. Though there were some variations among different states, initially the voting rights were extended only to white males who owned property and paid taxes. Slowly, it was relaxed so that all white males were eligible. The 15<sup>th</sup> Amendment in 1870 gave the voting rights to black males, but the women had to wait until 1920 (19th Amendment). In 1971, the 26th Amendment lowered the voting age to 18 (from 21), which was largely due to the social pressure from the Vietnam war: it seemed anomalous that people in the age group 18–21 were required to serve in the military and could die for their country, but were not eligible to vote for their government representative.

The other strand related to personhood concerns abortion laws. Almost everyone can agree on two extremes: before an egg is ferti-

<sup>&</sup>lt;sup>25</sup> J. Starita, "The Case of Standing Bear: Establishing personhood under the law", *Court Review: The Journal of the American Judges Association*, Vol. 45, Paper 287, 2009, 4–11, http://digitalcommons.unl.edu/ajacourtreview/287.

lised, there is no person and no rights, but when the baby is born, the baby has certain rights (well, perhaps not the voting rights). Now if we look the foetal development: at about four weeks the fertilised cell implants itself in the uterus; heart, brain, spinal cord begin to form at week 5; around week 6 spinal cord connects to the brain and the heart starts to pump, facial features begin to appear; in week 7 baby's head develops; in week 8 eyes become visible; toes form in week 9; neck developed in week 10; genitals develop in week 11; and fingernails start to develop in week 12; and so on. The point is that the development of foetus from the conception to the birth is a gradual process. However, as long as the baby is inside the womb, there is a potential conflict between the rights of the mother and the rights of the foetus. Abortion laws are aimed to address this issue. In the US, for instance, Roe v Wade is considered a landmark ruling drawing a line at the 1st trimester (so 12 weeks), which seems quite arbitrary. The complexity of this issue is evident in that this is still a very emotional and disturbing topic for many people, violent debate rages on, and a social and legal solution that will satisfy everyone does not seem to be in sight. A very thorough and thought-provoking discussion is provided in Ford.<sup>26</sup> Interestingly, he argues that personhood (or soul) appears after 14 days, which is based on the criteria that identical twinning is not possible after that.

Let us look at another example from Australian law, which concerns whether Aborigines had sovereignty and land rights. Until recently, the indigenous people of Australia had few if any proprietary rights in Australian land. When one considers that the Australian indigenous people had settled the land some 40,000 years prior to the English invasion, this seems unfair; and even more so when one considers that under English law the aborigines should have been granted limited sovereignty over Australia. At the time of the settlement of Australia, English law drew the distinction between lands that were

<sup>&</sup>lt;sup>26</sup> N. Ford, *When Did I Begin?: Conception of the Human Individual in History, Philosophy and Science*, Cambridge University Press, Cambridge (UK) 1991.

colonised where there was an existing population of people, and lands that were settled where there were no people. Where the land was colonised, the indigenous laws of the people remained, but where the land was empty – in Latin *terra nullius* – English law landed at the same moment as the first foot of the British seafarers. Under British colonial rule, Australia was held to be *terra nullius* at the time of white settlement. This was nothing more than a patent fiction, as the evidence of its falsity – the native people, their settlements, their tools, their culture – was present everywhere. Nonetheless the fiction remained and it was held that the only property laws in Australia were those stemming from the introduction of the white rule; laws which were less than generous in their grant of land to the Aborigines.

The original cases – created during the 1800s in an era of *laissez faire* capitalism and blatant racism – set the precedent to limit aboriginal holdings of land, except as a consequence of the English property law. Subsequent cases merely adopted the principle that Australia was 'empty land' even though the fiction was always obvious. This illustrates what in psychology is called *einstellung* effect:<sup>27</sup> when people are used to solving problems in a certain way, they continue to use the same schema without exploring alternate possibilities that may be more efficient (or, as in this case, more fair.) It is inconceivable that no judge in these cases – whether at trial or during any of the numerous appeals that they entailed – never perceived the term 'empty land' to be at odds with their eventual decision to uphold the white rule.

However, as with *Standing Bear vs. General Cook*, one court finally reconsidered the whole doctrine. This happened in the case of *Mabo v Queensland* (No.2). [(1992) 175 CLR 1]. In *Mabo*, the Australian High Court held that previous decisions – holding that Australia was *terra nullius* at settlement – were wrong at law. This is an interesting decision in that the court did not decide to change the law to accommodate modern developments, in the way we see this done

<sup>&</sup>lt;sup>27</sup> A. S. Luchins, "Mechanisation in problem solving: the effect of Einstellung", *Psychological Monographs* 54 (Whole No. 248), 1942.

in fields as diverse as homicide (including a new defence for 'battered wives') or tax (making modern-day tax evasion illegal) or discrimination law (adding age or sexual-preference as grounds for anti-discrimination suits). Instead, the court went back to the basic *terra nullius* formulation at the time of white settlement, and concluded that the previous courts were wrong according to the law at the time. Notwithstanding prior cases to this effect, the High Court said that Australia could not have been an empty land at settlement, since the Aboriginal presence meant that, according to the law of the time, it was a colonised country. Aboriginal law had thus remained in force for the 200 years that the white courts had declared that it never existed. This is a remarkable example of a norm shift, though similar processes occur all the time as judges adapt laws to social needs.<sup>28</sup>

As a final example, I would like to present a case discussed by Susan Haack.<sup>29</sup> She examines a 1927 US Supreme Court ruling on *Buck v. Bell*, where the court argued that Carrie Buck's constitutional rights would not be violated if the State of Virginia were to sterilize her against her will under the state's eugenics law. According to the law, the State of Virginia could sterilize the Epileptic and the Feeble Minded. The justification for the law was that this was in the interest of the larger society.

In writing the majority opinion, Justice Oliver Wendell Holmes, Jr. compared this case to that of vaccinating individuals, even against their will, because it is in the larger interest of the society and the risk to the individual is miniscule. In her detailed analysis of Holmes's background and earlier legal and philosophical writings, Haack argues that the decision took into account the prevailing scientific view at that time, namely that feeble-mindedness is hereditary. It is only later, when the horrors of the widespread application of eugenics un-

<sup>&</sup>lt;sup>28</sup> See also D. Hunter, B. Indurkhya, "'Don't Think, but Look' A Gestalt Interactionist Approach to Legal Thinking", *Proceedings of the Workshop on Advances in Analogy Research*, Sofia, Bulgaria, 1999, 345–353.

<sup>&</sup>lt;sup>29</sup> S. Haack, "Pragmatism, law, and morality: The lessons of Buck v. Bell", *European Journal of Pragmatism and American Philosophy* III, 2, 2011, 65–87.

der the Nazi regime were experienced, courts became more cautious. For instance, in 1942, in *Skinner v. Oklahoma*, the Supreme Court decided on the constitutional status of an Oklahoma legislation that allowed the state to sterilize a person who committed three felonies involving 'moral turpitude.' In rendering the decision, Justice Douglas wrote, "The power to sterilize, if exercised, may have subtle, far reaching and devastating effects. In evil or reckless hands it can cause races or types which are inimical to the dominant group to wither and disappear."<sup>30</sup>

To conclude, we see that even though law tends to be conservative in the sense that legal concepts are regarded with stability – their interpretations are not changed so lightly, and they are considered to transcend existing social practices; nonetheless, legal concepts also evolve as our scientific knowledge progresses and social norms change. We can see this process clearly in the current legal wrangling concerning gay marriages.

#### 5. Normativity in science

It is difficult to dispute that scientific concepts change and evolve over time. As early as 1866, Charles Sanders Pierce quipped, "Perception is the possibility of acquiring information, meaning more; now a word may learn. How much more the word *electricity* means now than it meant in the days of Franklin; how much more the term *planet* means now than it did in the time [of] Hipparchus. These words have acquired information; just as a man's thought does by further perception."<sup>31</sup>

The issue we would like to focus on here is whether there is a normative component (ontologically speaking) to scientific concepts. For

<sup>&</sup>lt;sup>30</sup> S. Haack, "Pragmatism, law, and morality...", op. cit., p. 83.

<sup>&</sup>lt;sup>31</sup> C. S. Peirce, *Collected Papers, Vol. 7: Science and Philosophy*, A. W. Burks (ed.), Harvard University Press, Cambridge (MA) 1958–66, par. 587.

instance, at about the same time that Saul Kripke was arguing for his direct reference theory of proper names, Hilary Putnam<sup>32</sup> argued that natural kind terms like 'tiger' and 'water' have a mind-independent ontology. So, for example, 'water' refers to this substance here, and is it is connected to all other such substances by virtue of its essence (or chemistry), even though we may not yet know all aspects of this essence. Putnam argued that this view is necessary to maintain our intuition that we *discover* the chemical structure of water, and not that we posit a new substance that includes chemical structure as one of its attributes.

Notwithstanding such arguments, I would like to present here some examples to illustrate the cognitive aspects of this normativity. There are interesting historical accounts of how and when whales were classified as mammals,<sup>33</sup> and how Pluto was declassified as a planet.<sup>34</sup> They reveal how our concepts of mammals and planets evolved over the years and generations as our scientific knowledge grew. But for our discussion here, let us consider the concept of *vacuum*.

In 1643, when Torricelli filled a tube, closed at one end, with mercury, and inverted it into a tub filled with mercury, he found that the mercury in the tube did not all fall out. The mercury level dropped somewhat, but then the remaining column of mercury stayed suspended inside the tube. There was a wide-ranging debate among the scientists and philosophers of those days about an explanation of this phenomenon, and on the nature of the empty space between the top of the tube and the level of mercury. There were vacuists who posited that there was vacuum in this space, and there were plenists, who claimed that the idea of vacuum is simply logically inconsistent with that of space, and were seeking alternate explanations.

<sup>&</sup>lt;sup>32</sup> H. Putnam, "The meaning of 'meaning", op. cit.

<sup>&</sup>lt;sup>33</sup> A. Romero, "When Whales Became Mammals: The Scientific Journey of Cetaceans From Fish to Mammals in the History of Science", in *New Approaches to the Study of Marine Mammals*, A. Romero and E.O. Keith (eds.), InTech, Rijeka (Croatia) 2012, pp. 3–30.

<sup>&</sup>lt;sup>34</sup> D. A. Weintraub, *Is Pluto a Planet? A Historical Journey through the Solar System*, Princeton University Press, Princeton (NJ) 2008.

To explore these ideas, Robert Boyle in England devised an air pump and, what some consider to have been the first physics experiments, conducted a number of tests to check out various hypotheses. For instance, in one experiment, he immersed the Torricelli apparatus in a big chamber, and sucked the air out of the chamber using his air pump. He found that the mercury level in the tube falls rapidly but does not quite reach the level in the tub. Several of his contemporaries, including well-known natural philosopher Thomas Hobbes, refused to accept the results of these experiments or their interpretations. Hobbes, for example, raised doubts about the working of the pump itself, and questioned if the air is indeed being sucked out. He also made several methodological arguments against Boyle's experimental set up.<sup>35</sup>

The arguments raised against the notion of vacuum were quite interesting. Linus, for instance, argued that because we can see through this space, it could not be vacuum. This was based on the assumption that no visible species can proceed from or through vacuum. (It is interesting to note that this conception of vacuum is similar to our modern concept of a black hole.) He also used another experiment to counter Boyle's theory. If you close the upper opening of the Torricelli tube with your finger, you can feel a downward suction when the experiment is performed. Linus argued that this contradicts Boyle's explanation that the mercury is being pushed up the tube by the pressure of external air. He posited a substance, which he called *funiculus*, in the Torricellian space that holds the column of mercury in position, and pulls our finger down.

This brief discussion shows that the concept of vacuum is cognitively constructed based on experimental observations and available theories and models that explain those observations. We cannot see it, but can only perceive its manifestation indirectly: mediated by some theoretical model.

<sup>&</sup>lt;sup>35</sup> See for an excellent account of Boyle-Hobbes debate: S. Shapin, S. Schaffer, *Levia-than and the Air Pump: Hobbes, Boyle and the Experimental Life*, Princeton University Press, Princeton (NJ) 1985/2011.

Let us consider a related example where the referent turned out to be non-existent: namely that of *aether*. Sir Isaac Newton, while formulating his gravitation theory, felt that there has to be some medium through which gravitational fields can travel and distant objects can interact, so he posited *aether*, which permeates all space.<sup>36</sup> Later on, when Huygens proposed the wave theory of light, he required a medium for the light waves to travel (as longitudinal waves), and so *aether* was conveniently adapted. It was only in 1887, when Albert A. Michelson and Edward W. Morley performed their famous experiment in Cleveland, Ohio, that raised serious doubts about the existence of *aether*. It is interesting to note that Michelson was a strong proponent of the *aether* theory, and continued to look for the evidence supporting it with another colleague Dayton Miller.<sup>37</sup>

These examples illustrate that the referents of the scientific concepts are not something that are sitting in the external world waiting for us to discover their essences. We access the referents through our cognitive models or theories, which dictate what aspects of these referents we observe, what possible actions we can take on them, and what are the possible outcomes.<sup>38</sup> All scientific discussion assumes some norms, for norms dictate the methodology that is acceptable. However, this methodology evolves, and changes drastically from time to time.

To illustrate this point, consider Francesco Sizzi's argument against Galileo discovery of satellites of Jupiter. "There are seven windows in the head, two nostrils, two eyes, two ears, and a mouth; so in the heavens there are two favourable stars, two unpropitious, two luminaries, and Mercury alone undecided and indifferent. From which and many other similar phenomena of nature, such as the seven metals, etc., which it were tedious to enumerate, we gather that the

<sup>&</sup>lt;sup>36</sup> L. Rosenfeld, "Newton's views on aether and gravitation", *Archive for History of Exact Sciences* 6 (1), 1969, 29–37.

<sup>&</sup>lt;sup>37</sup> See G. Johnson, *The Ten Most Beautiful Experiments*, Vintage, London 2009 Chap. 8.

<sup>&</sup>lt;sup>38</sup> See also S. Hawking, L. Mlodinow, *The Grand Design*, Bentam Books, New York 2010, Chap. 3; B. Indurkhya, "Rationality and reasoning with metaphors", *New Ideas in Psychology* 25, 2007, 16–36; B. Indurkhya, "Thought experiments, models, and the heuristic power of metaphors in science", to appear.

number of planets is necessarily seven."<sup>39</sup> Nowadays we consider this argument quite irrational, and almost humorous, but Francesco Sizzi was a respected astronomer of his time: he was credited with discovering the annual movement of sunspots. Moreover, he was not alone in expressing these sentiments, for another well-respected scientist of that time, Francis Bacon, who also had a great regard for empirical observations, made very similar arguments. This shows that Sizzi and Bacon were working with a different set of norms. For them, God made the universe in harmony, and there are some universal laws about it. In particular, human design reflects cosmos design, so from the seven windows of the head we may infer seven planets.<sup>40</sup> Another example is provided by Einstein, who initially rejected quantum mechanics, famously quipping, "God doesn't play dice with the world." It was inconceivable, given his normative view of physics, that nature would determine the outcome of the events randomly.

To sum up, we see that scientific norms are not a rigid set of standards laid out once and for all. On the contrary, they are constantly being redefined as our scientific knowledge grows and new methodologies evolve. To be sure, these norms transcend the existing scientific practices, so one appeals to these norms (as Hobbes, Linus, Sizzi and Bacon did) to support or critic particular scientific claims, but as these claims are accepted (or rejected), the norms themselves change as a result of this debate.

## 6. Normativity in mathematics

In a best-seller science fiction novel *Contact* by Carl Sagan (1985), alien intelligence hides meaningful messages in the expansion of  $\pi$ , the transcendental number that is the ratio of the circumference of a circle to its

<sup>&</sup>lt;sup>39</sup> Quoted in J. L. Christian, *Philosophy: An Introduction to the Art of Wondering* (11th ed.) Cengage Learning 2011, p. 570.

<sup>&</sup>lt;sup>40</sup> See also F. Leavitt, *An Even Greater Scandal: I'm a Liar but You're a Bigger One*, Strategic Book Publishing, Houston 2012.

diameter. The premise of this plot element is that the number  $\pi$  is mindindependent, so any intelligence, no matter what form it takes, will be able to find this pattern and decode the message. This, in essence, exemplifies the normativity of mathematics. Numbers, mathematical operations, mathematical theorems etc. are all considered by many to refer to mind-independent, Platonic, ideal forms. However, in this section, I would like to present some examples to suggest otherwise.

Before delving into a couple of detailed examples, I would like to mention a few general works that address the cognitive aspects underlying mathematics. A well-known algebraist and one of the founders of category theory, Saunders Mac Lane, has analyzed different branches of mathematics, like geometry, algebra, topology to show how they evolved from human activities and the needs related to those activities:

I assert that subjects of Mathematics are *extracted* from the environment; that is, from activities, phenomena, or science – and that they are then later applied to that – or other – environments. Thus number theory is 'extracted' from the activity of counting, and geometry is extracted from motion and shaping. The exact mechanism of this 'extraction' has not been described in detail here; it will clearly vary considerably from case to case. I have deliberately chosen this work 'extraction' to be close to the more familiar word 'abstraction' – and with the intent that the Mathematical subject resulting from an extraction is indeed abstract. Mathematics is not 'about' human activity, phenomena, or science. It is about the extractions and formalization of ideas – and their manifold consequences.<sup>41</sup>

Lakoff and Núñez<sup>42</sup> take on the audacious task of explaining the Euler's equation:  $e^{i\pi}$  = -1. They quote a Harvard University Mathematics professor who tells his freshman students that he can show how to

<sup>&</sup>lt;sup>41</sup> S. Mac Lane, *Mathematics: Form and Function*, Springer, Berlin 1986, p. 418, emphasis autor's.

<sup>&</sup>lt;sup>42</sup> G. Lakoff, R. E. Núñez, *Where Mathematics Comes from: How the Embodied Mind Brings Mathematics Into Being*, Basic Books, New York 2000.

derive the equation, but not what it means. Lakoff and Núñez take us on a journey where they explain logarithms, what is *e*, what is *i*, what is  $\pi$ , what it means to multiply *i* with  $\pi$ , and what it means to raise *e* to the power of this product, and why, at the end of it, we get a concrete number -1.

Another interesting work is by the neuroscientist Stanislas Dehaene,<sup>43</sup> which focuses on the neurocognitive basis of numbers and arithmetic. He takes us on a fascinating tour detailing research on the number cognition in animals, babies, prodigies, people with brain damage, and so on, to conclude:

The evolution of mathematics is a fact. Science historians have recorded its slow rise, through trial and error, to greater efficiency. It may not be necessary, then, to postulate that the universe was designed to conform to mathematical laws. Isn't it rather our mathematical laws, and the organizing principles of our brain before them, that were selected according to how closely they fit the structure of the universe?<sup>44</sup>

Now let us look at two examples in some detail. The first one is about the concept of infinity. Wallace<sup>45</sup> provides a good overview of the history of infinity, and here we will focus only on one aspect of it, namely the cardinality or the size of infinite sets. Intuitively, for finite sets, a set *A* is bigger than another set *B*, if *A* has more elements than *B*. This is the traditional way in which Euclid, and later Galileo, characterized cardinality of sets. According to this view, the whole cannot be of the same size (or same cardinality) as a part of it.

However, for infinite sets, this creates a paradox. The set of even numbers is clearly a proper subset of natural numbers, so is a part of it, but both are infinite. Do they have the same cardinality? Or, is the

<sup>&</sup>lt;sup>43</sup> S. Dehaene, *The Number Sense: How the Mind Creates Mathematics. Revised, Expanded Edition*, Oxford University Press, Oxford (UK) 2011.

<sup>&</sup>lt;sup>44</sup> *Ibidem*, p. 232.

<sup>&</sup>lt;sup>45</sup> D. F. Wallace, *Everything and More: A Compact History of Infinity*, W.W. Norton, New York 2010.

set of natural numbers bigger than the set of even numbers? This issue becomes even more complex when we consider rational numbers, for they are dense. In other words, between any two rational numbers, no matter how close, there is another rational number. If we apply this rule recursively, we can get infinitely many rational numbers between any two rational numbers, no matter how close together. So, intuitively, it seems obvious that there ought to be more rational numbers than natural numbers.

In the later half of the 19th century, Richard Dedekind and Georg Cantor worked out a technique for assigning cardinality to infinite sets. Dedekind's idea was to take the intuition from the finite sets: if a set A can be mapped in a one-to-one fashion into another set B, then A's cardinality is smaller than or equal to B's. And then if B can also be mapped in a one-to-one fashion into A, then B's cardinality will be smaller than or equal to B's. So if both conditions are satisfied, the sets A and B are deemed to have the same cardinality.

This works quite well for finite sets, but an advantage of defining it like this is that it can now be applied to infinite sets. So, for example, if we consider the set of natural numbers N, and the set of even numbers E, we can have a one-to-one mapping from E to N by mapping every even number to itself, and a one-to-one mapping from N to E by mapping every number n to 2\*n. Ergo, E and N have the same cardinality.

Using this technique, Georg Cantor produced two surprising theorems: 1) rational numbers have the same cardinality as natural numbers, and 2) real numbers are more numerous (have a higher cardinality) than natural numbers. Now, even a high-school student can understand these proofs. However, at the time they were proposed, it was considered a radical idea. Many leading mathematicians at that time refused to accept this formalization of cardinality and its implications for infinite sets.<sup>46</sup>

<sup>&</sup>lt;sup>46</sup> See E. T. Bell, *Men of Mathematics*, Simon & Schuster, New York 1937; N. Calkin, H. Wilf, "Recounting the rationals", *American Mathematical Monthly* 107 (4), 2000, 360–363; J. W. Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite*, Princeton University Press, Princeton (NJ) 1979.

Cantor's intuition was that there is only one level of infinity. (Perhaps he equated the one infinity with one God.) So he was quite excited when he was able to show that rational numbers, which are dense, have the same cardinality as natural numbers. Invigorated by his success, Cantor tried to show that real numbers are countable as well. Instead, what he found was a way to show that real numbers cannot be put in a one-to-one correspondence with natural numbers. In particular, what he found was that if you assume any way to put real numbers into a one-to-one correspondence with natural numbers, then you can always come up with a real number that is not included in this enumeration. The details of Cantor's original proof, which he spelled out in a letter to Dedekind, can be found in Dauben.<sup>47</sup>

The second example is from Imre Lakatos,<sup>48</sup> who championed the view that mathematics is a dynamic process in which we actively construct ontology of the mathematical objects as much as discovering their properties via lemmas and theorems. This essay is set as a classroom discussion with a teacher and a few pupils. We start with Euler's theorem concerning polyhedra, namely V - E + F = 2, where V is the number of vertices, E is the number of edges and F is the number of faces. First, the students verify the conjecture with some examples, and then construct a formal proof. The proof is construction-based: it requires cutting and deforming the polyhedra. But they go carefully through each step of the proof, examining alternatives and questions if they can indeed do the required cutting and stretching. Then, they try to construct counter examples to refute the proof. Lakatos distinguishes between local and global counter examples: local ones question some specific step in the proof, but the global ones try to refute the main conjecture. For example, in the case of Euler's theorem, a cube with a cube-shaped hole inside it refutes the theorem.

<sup>&</sup>lt;sup>47</sup> J. W. Dauben, Georg Cantor..., op. cit., pp. 50–54.

<sup>&</sup>lt;sup>48</sup> I. Lakatos, *Proofs and Refutations: The Logic of Mathematical Discovery*, J. Worrall, & E. Zahar (eds.), Cambridge University Press, Cambridge (UK) 1976.

Lakatos then discusses two mechanisms that are used to fix such problems. One is *monster barring*, which works by redefining the concepts (polyhedra in this case) so as to exclude the counter example (the monster). The other is *weakening the theorem*, so that the counter example is included. As we apply these two mechanisms, we partly change the ontology of the mathematical concepts, and partly discover new relationships among them. Donald Knuth<sup>49</sup> also captured this kind of dynamic interaction very elegantly in a monograph on surreal numbers.

We can see the evidence of Lakatosian mechanisms in science as well. For example, Pluto's status as a planet became questionable because a number of bodies of similar size were identified in the same region of the solar system. It seems that Pluto was one of a cluster of bodies constituting the Kuiper belt. This situation is very similar to what Lakatos discusses in *Proofs and Refutations*. Because of this discovery, astronomers felt a need to define the concept of planet formally, and these new definitions excluded Pluto from the planet category.

These examples provide a glimpse of the cognitive processes underlying mathematical norms. I would like to close this section with a long quote from Jean Piaget, who has also emphasized biological and cognitive underpinning in science and mathematics.<sup>50</sup> The quote is from a short monograph based on the lectures he gave at Columbia University in 1968:

It is agreed that logical and mathematical structures are abstract, whereas physical knowledge - the knowledge based on experience in general - is concrete. But let us ask what logical and mathematical knowledge is abstracted from. There are two possibilities. The first is that, when we act upon an object, our knowledge is derived from the

<sup>&</sup>lt;sup>49</sup> D. Knuth, *Surreal Numbers: How Two Ex-Students Turned on to Pure Mathematics and Found Total Happiness*, Addison Wesley, Reading (Mass.) 1974.

<sup>&</sup>lt;sup>50</sup> J. Piaget, *Biology and Knowledge*, B. Walsh (trans.), University of Chicago Press, Chicago 1971.

object itself. This is the point of view of empiricism in general, and it is valid in the case of experimental or empirical knowledge for the most part. But there is a second possibility: when we are acting upon an object, we can also take into account the action itself, or operation if you will, since the transformation can be carried out mentally. In this hypothesis the abstraction is drawn not from the object that is acted upon, but from the action itself. It seems to me that this is the basis of logical and mathematical abstraction.

In cases involving the physical world the abstraction is abstraction from the objects themselves. A child, for instance, can heft objects in his hands and realize that they have different weights - that usually big things weigh more than little ones, but that sometimes little things weigh more than big ones. All this he finds out experientially, and his knowledge is abstracted from the objects themselves. But I should like to give an example, just as primitive as that one, in which knowledge is abstracted from actions, from the coordination of actions, and not from objects. This example, one we have studied quite thoroughly with many children, was first suggested to me by a mathematician friend who quoted it as the point of departure of his interest in mathematics. When he was a small child, he was counting pebbles one day; he lined them up in a row, counted them from left to right, and got ten. Then, just for fun, he counted them from right to left to see what number he would get, and was astonished that he got ten again. He put the pebbles in a circle and counted them, and once again there were ten. He went around the circle in the other way and got ten again. And no matter how he put the pebbles down, when he counted them, the number came to ten. He discovered here what is known in mathematics as commutativity, that is, the sum is independent of the order. But how did he discover this? Is this commutativity a property of the pebbles? It is true that the pebbles, as it were, let him arrange them in various ways; he could not have done the same thing with drops of water. So in this sense there was a physical aspect to his knowledge. But the order was not in the pebbles; it was he,

the subject, who put the pebbles in a line and then in a circle. Moreover, the sum was not in the pebbles themselves; it was he who united them. The knowledge that this future mathematician discovered that day was drawn, then, not from the physical properties of the pebbles, but from the actions that he carried out on the pebbles. This knowledge is what I call logical mathematical knowledge and not physical knowledge.

The first type of abstraction from objects I shall refer to as simple abstraction, but the second type I shall call reflective abstraction, using this term in a double sense.<sup>51</sup>

This articulates the position that mathematical objects and theorems reveal as much about our cognitive operations as about the external world. The implication of this view for norms is that even mathematical norms are not mind-independent as they are purported to be, but are actively constructed by the cognitive agent.

## 7. Conclusions

In the course of the previous five sections, we have looked at several examples that illustrate cognitive aspects of norms in the domains of language, art, law, science and mathematics. When we consider these historical examples, some may take a view that, ok, now we know that those theories, models and their underlying norms were wrong, but we have better theories now. We know vacuum exists but *aether* does not; we know eugenics is a bad social policy; we know there are eight planets in our solar system (after Pluto was plutoed out); and so on. But such a view misses the whole point. Hobbes and his colleagues also were sure that vacuum was impossible. Michelson was

<sup>&</sup>lt;sup>51</sup> J. Piaget, *Genetic Epistemology*, E. Duckworth (trans.), W.W. Norton, New York 1971, pp. 15–17.

sure that *aether* had to be there. Justice Holmes was sure that eugenics is justified in the interest of the society for feeble-minded people. Sizzi and Bacon were sure that there could not possibly be an eighth planet. What we know now, and what we feel sure about now, can change, and can change drastically in the future.

This can be illustrated by another comment by Francesco Sizzi. Later on, as a part of the same argument against the discovery of Jupiter's moon by Galileo, he argued: "Furthermore, the [alleged] satellites [of Jupiter] are invisible to the naked eye and therefore can have no influence on the earth, and therefore would be useless, and therefore do not exist."<sup>52</sup> Analogous arguments are made by many respected philosophers these days in arguing why empirical data from brain-imaging studies is irrelevant for understanding emotions, free will, consciousness, and so on.

We did not discuss norms in ethics and morality directly here, but I would like to note that views presented here resonate with the treatment of moral norms articulated by Jesse Prinz:

I will argue that morality derives from us. The good is that which we regard as good. The obligatory is that which we regard as obligatory. The 'we' here refers to the person making a moral claim and the cultural group with which that individual affiliates. If the good is that which we regard as good, then we can figure out what our obligations are by figuring what our moral beliefs commit us to. Figuring out what we believe about morality is a descriptive task *par excellence*, and one that can be fruitfully pursued empirically. Thus, normative ethics can be approached as a social science.<sup>53</sup>

To conclude, we claim that norms are ideals that a cognitive agent (or a society) strives for. But they are based on its current un-

<sup>&</sup>lt;sup>52</sup> Quoted in Quoted in J. L. Christian, *Philosophy..., op. cit.*, p. 570.

<sup>&</sup>lt;sup>53</sup> J. J. Prinz, *The Emotional Construction of Morals*, Oxford University Press, Oxford (UK) 2007, p. 1.

derstanding of itself and the world. As the agent (and the society) evolves, norms are also revised. Norms stay at a distance, so they go beyond the current landscape of social and cultural conventions, but they show the horizon effect: they recede and change as we approach them, sometimes radically.