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Finite size of hadrons and Bose–Einstein correlations

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ABSTRACT

It is observed that the finite size of hadrons produced in high energy collisions implies that their positions are correlated, since the probability to find two hadrons on top of each other is highly reduced. It is then shown that this effect can naturally explain the values of the correlation function below one, observed at LEP and LHC for pairs of identical pions.

1. Momentum correlations between identical bosons have been studied for over 50 years [1–3]. In particular for pairs of identical bosons one usually discusses the correlation function

$$C(\mathbf{p}_1,\mathbf{p}_2)=\frac{dN/d^3p_1\,d^3p_2}{dN/d^3p_1\,dN/d^3p_2}. \tag{1}$$

The various corrections which have to be applied when extracting this function from the data are described in the reviews [2,3]. After these corrections have been applied, it is hoped (and usually taken for granted) that the correlation function contains no other correlations than those due to Bose–Einstein statistics. In the following section we recall that this implies the inequality

$$C(\mathbf{p}_1,\mathbf{p}_2)\geqslant 1. \tag{2}$$

Actually the data from LEP [4,5] and also recent measurements at LHC [6,7] show consistently a broad minimum where the correlation function takes values below one This means that the produced particles are correlated. It is of course interesting to look for possible origins of these correlations.

The suggestion that such a minimum results from non-resonant, strong final-state $\pi^\pm$–$\pi^\pm$ interactions, was made by Bowler [8]. To estimate the effect he used as input the measured S-wave, $I=2$ π–π phase shift. Later he pointed out, however, that presence of many particles tends to cancel the effects of final-state interactions [9].

More recently, the importance of the observation of such a minimum in e^+-e^- data was realized and studied in [10], using the τ-model [11]. This was continued by W.J. Metzger in several contributions [12].

In this note we observe that there is a natural source of inter-hadron correlations following from the fact that non-interacting hadrons, being composite, extended objects, cannot be located in space too close to each other.[1] The point is that, when the two pions are too close to each other, their constituents start to mix and they cannot be considered as pions subjected to Bose–Einstein statistics. Note that this fact is related neither to strong interactions nor to position–momentum correlations.[2] This observation obviously implies that the hadron positions, as measured by quantum interference, are correlated. Using a simple model we discuss consequences of this kind of correlations on the measurements of the quantum interference and show that they naturally lead to values below one for the correlation function, as observed in data [4–7].

In the following two sections we introduce the notation and give some basic formulae. In Section 4 we propose a simple model implementing the requirement that two hadrons cannot be too close to each other. A summary and some comments are given in the last section.

2. Let us denote by ρ the single-particle density matrix in the momentum representation. Then the single-particle momentum distribution is

$$P(\mathbf{p};t)=E\frac{dN}{d^3p}(t)=\rho(\mathbf{p},\mathbf{p};t). \tag{3}$$

Note that the density matrix is here normalized to the total number of particles N and not to unity. As seen from this formula, the diagonal elements of the density matrix are non-negative. For uncorrelated particles the two-particle density matrix is a product of the single-particle ones. For two identical

[1] In statistical models this is often taken into account as the so-called excluded volume effect, first considered in [13] and then studied by many authors [14,15].

[2] In particular, this is not a hard core effect: the two quarks and two antiquarks can be contained in a small volume, but this means that for them the hadronization process leading to a pair of pions has not yet been completed.

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bosons, however, this product has to be symmetrized and one finds

$$P(\mathbf{p}_1,\mathbf{p}_2;t)=\rho(\mathbf{p}_1,\mathbf{p}_1;t)\rho(\mathbf{p}_2,\mathbf{p}_2;t)+\rho(\mathbf{p}_1,\mathbf{p}_2;t)\rho(\mathbf{p}_2,\mathbf{p}_1;t). \tag{4}$$

This yields

$$C(\mathbf{p}_1,\mathbf{p}_2;t)=1+\frac{|\rho(\mathbf{p}_1,\mathbf{p}_2;t)|^2}{\rho(\mathbf{p}_1,\mathbf{p}_1;t)\rho(\mathbf{p}_2,\mathbf{p}_2;t)}\geqslant 1, \tag{5}$$

where we have used the hermiticity of the density matrix which implies that $\rho(p_2,p_1;t)=\rho^*(p_1,p_2;t)$.

3. In order to discuss the geometry of the interaction region it is necessary to express the density matrix by an emission function which can be interpreted as the distribution of particles in momentum *and* position. For the single-particle density matrix, the standard choice [3] is

$$\rho(p,p')=\int d^4x\,S(x,P)e^{iqx}, \tag{6}$$

where $P=\frac{1}{2}(p+p')$, $q=p-p'$.

Similarly, for the two-particle density matrix one has [16]

$$\rho(p_1,p_2;p_1',p_2')=\int d^4x_1\,d^4x_2\,S(P_1,P_2;x_1,x_2)e^{iq_1x_1+q_2x_2}. \tag{7}$$

Note that the popular interpretation of emission functions as probability distributions makes sense only if all the momenta satisfy the on-shell condition $p^2=m^2$. The assumption that the four-momentum P can be modified to satisfy this condition without significantly distorting the results is known as the smoothness assumption [3].

Using these formulae one obtains for the correlation function

$$C(p_1,p_2)=\Bigl(\int d^4x_1\int d^4x_2\,S(p_1,p_2;x_1,x_2)+\int d^4x_1\int d^4x_2\,\cos\bigl[q(x_1-x_2)\bigr]S(P,P;x_1,x_2)\Bigr)\Big/\Bigl(\int d^4x_1\,S(p_1,x_1)\int d^4x_2\,S(p_2,x_2)\Bigr). \tag{8}$$

If particles are uncorrelated, $S(p_1,p_2;x_1,x_2)=S(p_1,x_1)S(p_2,x_2)$ and the first term on the right-hand side equals 1. As shown above, in this case the second term is non-negative. In order to explain the motivation of the present work let us make the admittedly unrealistic assumption that the particles are correlated so that x_1-x_2 is constant. Then the cosine can be taken outside the integral and the second terms gets a cosinusoidal dependence on q, taking both positive and negative values. The excluded volume approach is a somewhat more realistic way of realizing a qualitatively similar scenario.

4. As already mentioned, taking into account the finite size of hadrons implies that the positions of hadrons in space are correlated. In this section we show, using a very simple model, how these correlations may lead to a minimum (below 1) in the correlation function of identical particles.

We assume that the correlations affect neither the single-particle momentum distribution nor the unsymmetrized two-particle momentum distribution.[3] Therefore we have

$$\int d^4x_1\,d^4x_2\,S(p_1,x_1,p_2,x_2)=\int d^4x_1\,S(p_1,x_1)\int d^4x_2\,S(p_2,x_2). \tag{9}$$

As our illustrative example we choose

$$S(p_1,x_1;p_2,x_2)=\mathcal{N}S(p_1,x_1)S(p_2,x_2)\Theta\bigl[\mathbf{x}_-^2+t_-^2-r_0^2\bigr], \tag{10}$$

where $x_-=x_1-x_2$. The normalizing constant $\mathcal{N}$ is necessary to enforce condition (9). For $t_1=t_2$ the θ-function just excludes the configurations with $|\mathbf{x}_-|<r_0$. The time difference in the argument corrects for the fact that this exclusion is not needed when one particle is far in time from the other.

For our illustrative calculation we choose

$$S(p,x)=e^{-\mathbf{x}^2/R^2}e^{-t^2/\tau^2}f(p), \tag{11}$$

where the momentum dependent factor is left unspecified. Including the correlations as in (10), substituting into (8) and performing the Gaussian integrations over $x_+=(x_1+x_2)/2$ in the numerator and over x_1 and x_2 in the denominator, one is left with the four-dimensional integral over x_-. Rewriting the integral d^3x_- in spherical coordinates the integrations over angles can be done. Thus there are only two integrations left and one gets

$$C(p_1,p_2)-1\sim\int_0^\infty dt\int_0^\infty dr\,r^2e^{-t^2/2\tau^2}e^{-r^2/2R^2}\frac{\sin Qr}{Qr}\times\cos(q_0t)\theta\bigl(r^2+t^2-r_0^2\bigr), \tag{12}$$

where $r=|\mathbf{x}_-|$, $t=|t_-|$ and $Q=|\mathbf{q}|$. The proportionality coefficient, skipped on the right-hand side, is positive and the normalization is fixed by (9) which implies $C(p_1=p_2)=2$. The integral in (12) can be easily evaluated numerically.

Since $2qP=m_1^2-m_2^2=0$ (the on-shell condition) one has $q_0=\mathbf{qP}/P_0$. Therefore the result depends on the direction of the vector $\mathbf{q}$. For the "side" direction $\mathbf{qP}=0$ For the "out" direction $\mathbf{qP}/P_0=|v|Q\approx Q$ where the approximate equality holds when the particle pair is highly relativistic.

As an example, Fig. 1 shows the (normalized to 1 at $Q=0$) integrals from (12) corresponding to the out and side directions, evaluated at $r_0=R=\tau=1$ fm. In both cases one clearly sees a range where the values of $C(p_1,p_2)-1$ are negative.

Without pretending that this description is realistic, we can thus conclude that the result proves the main point of our Letter: the values below one of the correlation function describing the quantum interference of two identical pions, find a natural explanation as a consequence of the "excluded volume effect", i.e. finite size of the produced hadrons.

5. To conclude: starting from the observation that non-interacting hadrons, being extended objects, cannot be located too close to each other, we have investigated the implications of this fact for the quantum interference of identical bosons. This "excluded volume" effect implies obviously specific correlations between particle positions, which are not included in the standard analyses. We have found that these correlations can lead to the existence of a region where the observed two-particle correlation function falls below one. This gives, in our opinion, a natural explanation of the observations reported in several experiments in e^+–e^- and p–p collisions [4–6].

Our treatment of the problem is, admittedly, rather crude, as it involves several approximations which are used to avoid complications and thus to focus on the two main points of this Letter, that is

[3] This condition is satisfied, in particular, when there is no correlation between the momenta and positions of the produced particles. In elementary collisions this seems a reasonable approximation, although it may be questioned for heavy ion collisions.

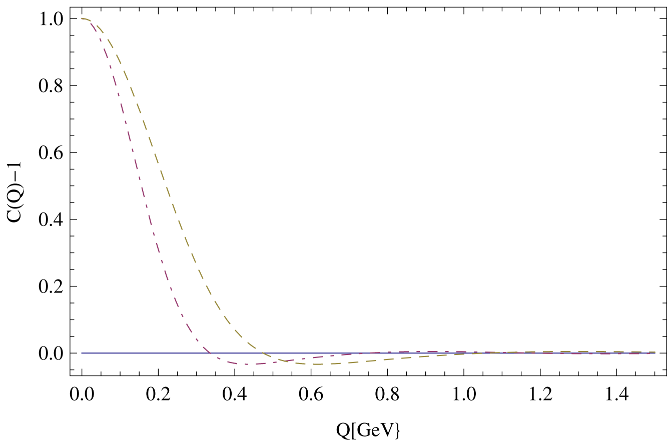

Fig. 1. $C(p_1, p_2) - 1$ plotted vs Q in GeV. Dashed line: "side" direction; Dashed–dotted line: "out" direction. $r_0 = R = \tau = 1$ fm.

(i) to emphasize the role of inter-hadron correlations in the explanation of the observed negative values of $C(p_1, p_2) - 1$ and
(ii) to point out that a natural source of such inter-hadron correlations can be provided by the finite sizes of the produced hadrons.

Several comments are in order.

(i) Our use of the Θ-function to parametrize the excluded volume correlations is clearly only a crude approximation. For a precise description of data almost certainly a more sophisticated parametrization of the effect will be needed. In particular, note that with our parametrization the correlation in space–time does not affect the single-particle and two-particle non-symmetrized momentum distributions. The same comment applies to our use of Gaussians.

(ii) It has been recently found [6,7] that in *pp* collisions at LHC, the volume of the system (as determined from the fitted HBT parameters) depends weakly on the multiplicity of the particles produced in the collision. This suggests that large multiplicity in an event is due to a longer emission time. If true, this should be also reflected in the HBT measurements and it may be interesting to investigate this aspect of the problem in more detail.

(iii) To investigate further the space and/or time correlations between the emitted particles more information is needed. It would be interesting to study the minima in the correlation functions separately for the "side", "out" and "long" directions. Such studies may allow to determine the size of the "excluded volume" and compare it with other estimates [14,15]. We also feel that with the present accuracy and statistics of data, measurements of three-particle B–E correlations represent the potential to provide some essential information helping to understand what is really going on.

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