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Resonance Vibrations of an Elastic System Supported on a Textile Layer

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Abstract
The vibrations of a flexible system resting on a layer of fibres are investigated in this paper. A mathematical model of the system excited by an electromagnet supplied with an alternating current is formulated. Two resonance regions are found. In one region the frequency of the varying contact force is twice the supply electric current frequency. In the other region the frequency of vibration is equal to that of the current. At the resonances the system loses contact with the layer at the moment when the reaction force becomes equal to zero.

Key words: *nonlinear vibration, layer, fibres, electromagnet excitation.*

Introduction

The properties of textiles subjected to the action of compressive forces were investigated in works reported in papers [1 - 3]. The dynamic properties of compressed textiles were studied numerically in paper [4] by subjecting them to vibrations excited by electromagnet. The mathematical model of a textile layer subjected to compression was proposed in paper [5]. The formulas were derived on the assumption that the properties of the layer were determined by the bending elasticity of fibres and by the resistance to fluid flow that is squeezed out of the layer. The problem of the vibration of a rigid object supported on a textile layer was studied in paper [6]. In this paper, the elasticity of the vibrating object is included. The paper is a continuation of works [4, 6].

Equations of motion

A schematic diagram of the system considered is shown in ***Figure 1***. The system is used for measuring the fatigue of textiles subjected to oscillating compression forces. A similar system was previously studied in paper [4]. The element of mass m_2 is connected to that of mass m_1 by a flat spring of stiffness k_2. The elements are attracted to one another by the force of the electromagnet with coil of inductance L and resistance R. The coil is powered by voltage $u = u(t)$, being a function of time. Element m_1 rests on textile layer k_1.

The relationship between the compressive force and contraction of the textile layer was assumed to be presented in ***Equation 1***. Parameters (k_1, c_1, L_1, H_1) are material constants of the textile layer and are defined in paper [5].

Summing up all the forces acting on mass m_1 and m_2, one obtains a set of equations governing their motion (2).

In ***Equations 2*** (k_2, c_2) are the stiffness and damping coefficients of the flat spring. Electromagnet force F_E is defined by relationships (3).

In ***Equations 3***, i is the intensity of the current, R the circuit resistance, u the feed voltage; coordinate x specifies the position of the movable core centre and has its origin in the centre of the coil; δ denotes the distance from the centre of the coil to that of the core at rest. In order to determine the inductance L, the coil was supplied with a voltage $u = U_m \sin\omega t$ for several stationary positions x of the core and the current I was measured. The values of inductance $L = L(x)$ were then calculated as a function of the mutual

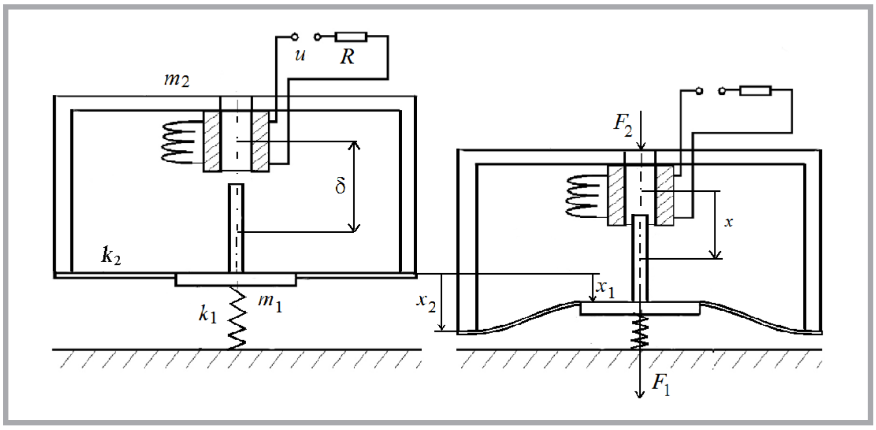

Figure 1. *Schematic diagram of an elastic system resting on a textile layer and subjected to an electromagnetic oscillating force.*

$$F_k = \frac{k_1 x_1}{\left(1-\frac{x_1}{L_1}\right)^3}, \quad F_c = \frac{c_1 \operatorname{sgn}\left(\frac{dx_1}{dt}\right)\left(\frac{dx_1}{dt}\right)^2}{\left(1-\frac{x_1}{H_1}\right)^3}, \quad y_1 < H_1, \; y_1 < L_1, \qquad (1)$$

$$F = F_k + F_c \;\; for \;\; F_k + F_c \geq 0, \; F = 0 \;\; for \;\; F_k + F_c < 0.$$

$$F_1 = -m_1 \frac{d^2 x_1}{dt^2} - \frac{k_1 x_1}{\left(1-\frac{x_1}{L_1}\right)^3} - \frac{c_1 \operatorname{sgn}\left(\frac{dx_1}{dt}\right)\left(\frac{dx_1}{dt}\right)^2}{\left(1-\frac{x_1}{H_1}\right)^3} + c_2\left(\frac{dx_2}{dt} - \frac{dx_1}{dt}\right) +$$

$$+ k_2(x_2 - x_1) - F_E + m_1 g = 0, \qquad (2)$$

$$F_2 = -m_2 \frac{d^2 x_2}{dt^2} - c_2\left(\frac{dx_2}{dt} - \frac{dx_1}{dt}\right) - k_2(x_2 - x_1) + F_E + m_2 g = 0.$$

Equations 1 and 2.

position x of the core and coil (4). The discrete function $L(x)$ (4) was then approximated by continuous function (5).

$$L(x)=\frac{1}{\omega}\sqrt{\left(\frac{U}{I}\right)^2-R^2} \quad (4)$$

It was found that the shape of the function describing the intensity of the magnetic field of a stationary solenoid suited for this purpose if the constant parameters were properly chosen. In ***Equation 5*** L_{max} is the maximum inductance of the coil measured when the centre of the core coincides with that of the coil; L_{min} is the minimum inductance of the coil measured when the core is in the end position; $2l$ denotes the computational length of the coil equal to the distance between the maximum and minimum of its first derivative dL/dx, and r_0 denotes the computational radius of the coil.

$$L\frac{di}{dt}+\frac{dL}{dx}i+Ri=u, \quad F_E=-\frac{1}{2}i^2\frac{dL}{dx}, \quad x=\delta-(x_2-x_1). \quad (3)$$

$$L(x)=\frac{L_{maz}-L_{\min}}{2l}\sqrt{r_0^2+l^2}\left(\frac{l+x}{\sqrt{r_0^2+(l+x)^2}}+\frac{l-x}{\sqrt{r_0^2+(l-x)^2}}\right)+L_{\min},$$

$$\frac{dL}{dx}=\frac{L_{\max}-L_{\min}}{2l}\sqrt{r_0^2+l^2}\left(\frac{1}{\sqrt{r_0^2+(l+x)^2}}\left(1-\frac{(l+x)^2}{r_0^2+(l+x)^2}\right)+\frac{1}{\sqrt{r_0^2+(l-x)^2}}\left(-1+\frac{(l-x)^2}{r_0^2+(l-x)^2}\right)\right) \quad (5)$$

Equations 3 and 5.

Numerical results and discussion

A computer program was created in C++ and a set of differential ***Equations 2, 3*** was integrated using the Runge-Kutta method. Calculations were carried out for each frequency till the system achieved steady-state vibrations. The following values of system parameters were taken: masses m_1 = 0.2 kg and m_2 = 4 kg, gravity acceleration g = 9.81 m/s^2, stiffness and damping coefficients of the flat spring k_2 = 8257.5 N/m, c_2 = 160 Ns/m, textile layer material constants k_1 = 500 N/m, c_1 = 16 Ns^2/m^2, L_1 = 0.03 m, H_1 = 0.03 m, electromagnet parameters L_{max} = 0.364319 H, L_{min} = 0.04 H, l = 0.028 m, r = 0.032 m, R = 40 Ω, distance from the centre of the coil to that of the core at rest $\delta = l$, voltage $u = U_m\sin(\omega t)$, U_m = 220 V, initial conditions $x_1(0) = 0$, $x_2(0) = 0$, $x_3(0) = 0$, $x_4(0) = 0$, $i(0) = 0$.

The maximum amplitudes of vibrating masses (x_1/L_1, x_2/L_1) and the textile layer reaction force $F/(m_2g)$ versus the frequency ω of the supply voltage relative to the constant $\omega_2 = (k_2/m_2)^{0.5}$ are shown in ***Figure 2***. Here two regions of the resonance amplification of vibrations can be observed. The time history of the contact reaction force F, the current i and the electromagnet core displacement with respect to the coil centre x at the resonance regions are shown in ***Figure 3***. At the resonance regions the system loses contact with the layer and as a result the reaction force vanishes for a moment. Comparing the frequency of the force with that of the current, one can see that the frequency of the force is two times higher than that of the current for ω/ω_2 = 0.3, while for ω/ω_2 = 0.7 they are equal in a steady state. For both cases, however, the number of electromagnet core oscillations is doubled compared to the number of current oscillations. For ω/ω_2 = 0.7, one core oscillation takes place in contact with the layer and one when the system is above the layer, not touching it.

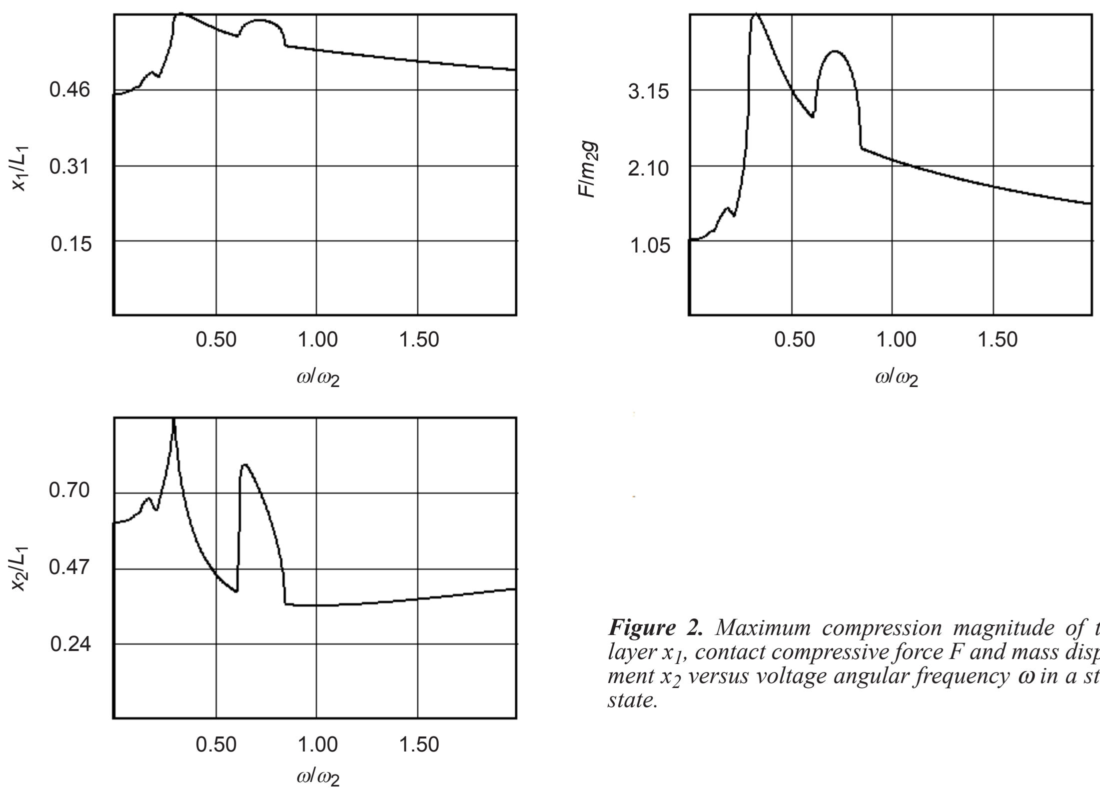

Figure 2. *Maximum compression magnitude of textile layer x_1, contact compressive force F and mass displacement x_2 versus voltage angular frequency ω in a steady-state.*

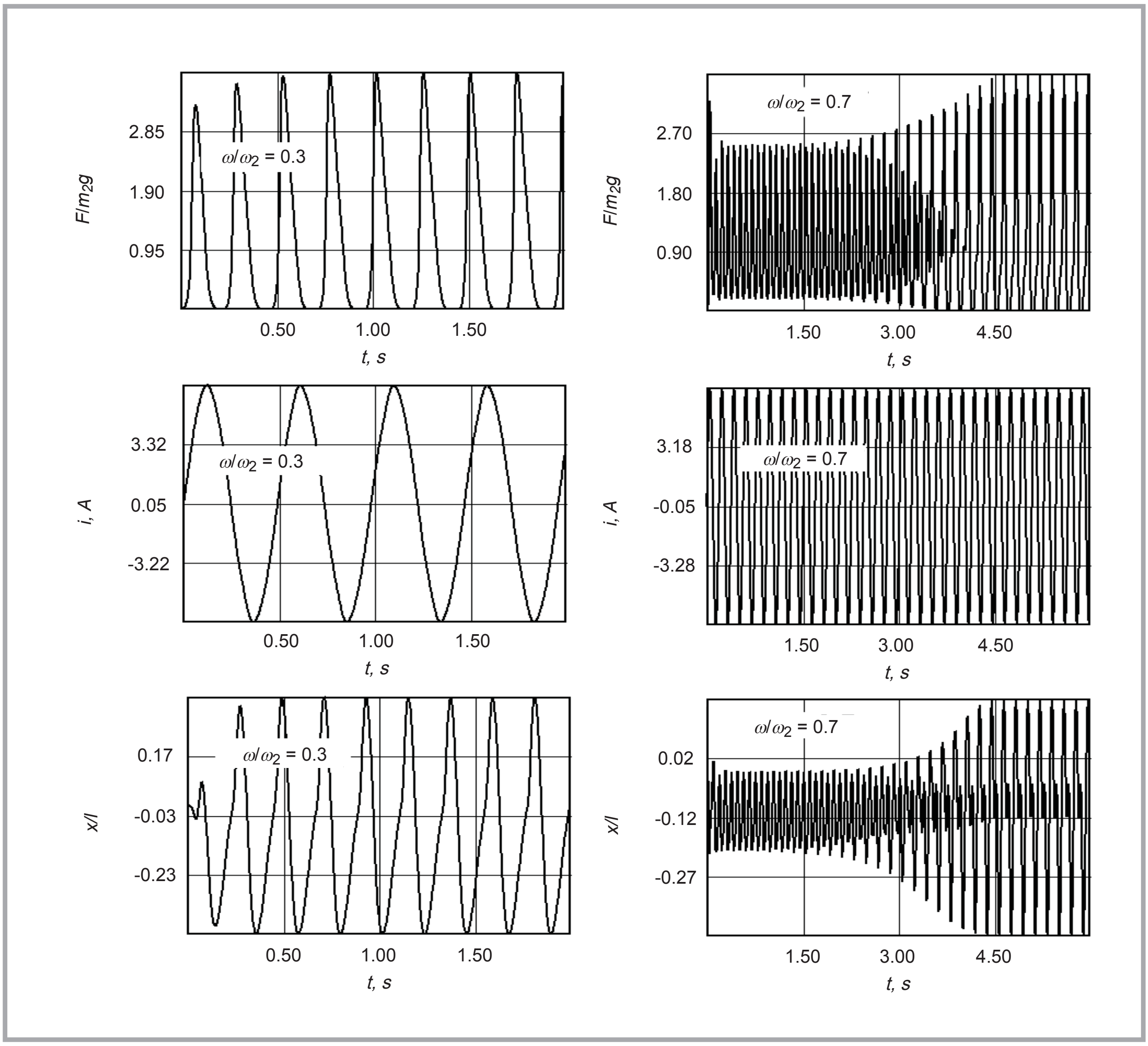

Figure 3. *Contact reaction force F relative to the gravity force of mass m_2, current and electromagnet core displacement x relative to the half length l of the coil in the resonance regions.*

Conclusions

1. The system composed of two masses connected by an elastic flat spring and supported on a layer of fibres exhibits two resonances: a peak and local maximum.
2. In the resonance regions the elastic system loses contact with the fibre layer and as a result the reaction force of the layer drops to zero for a short while.
3. In the neighbourhood of the peak resonance the frequency of the varying contact force is twice that of the supply voltage. This is because the excitation force depends on the square power of the electrical current.
4. In the neighbourhood of the resonance associated with the local maximum, the frequency of the varying contact force is equal to that of the supply voltage in a steady state, while the number of oscillations of the electromagnet core is twice that of the oscillations of the supply voltage in a steady state. This phenomenon is a result of the textile layer losing contact with the vibrating object and its free motion.

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