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INFORMATICS

Higher order fuzzy logic in controlling selective catalytic reduction systems

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Abstract. This paper presents research on applications of fuzzy logic and higher-order fuzzy logic systems to control filters reducing air pollution [1]. The filters use Selective Catalytic Reduction (SCR) method and, as for now, this process is controlled manually by a human expert. The goal of the research is to control an SCR system responsible for emission of nitrogen oxide (NO) and nitrogen dioxide (NO_2) to the air, using SCR with ammonia (NH_3). There are two higher-order fuzzy logic systems presented, applying interval-valued fuzzy sets and type-2 fuzzy sets, respectively. Fuzzy sets and higher order fuzzy sets describe linguistically levels of nitrogen oxides as the input, and settings of ammonia valve in the air filter as the output. The obtained results are consistent with data provided by experts. Besides, we show that the type-2 fuzzy logic controllers allows us to obtain results much closer to desired parameters of the ammonia valve, than traditional FLS.

Key words: Selective Catalytic Reduction (SCR), air pollution, nitrogen oxides, adjustable air filters, ammonia valve, interval-valued fuzzy logic system, fuzzy controlling of air filter adjustments, type-2 fuzzy logic system, fuzzy implications.

1. Fuzzy management and processing data on air pollution

1.1. Characteristics of the problem. Nitrogen oxide (NO) and nitrogen dioxide (NO_2) are gases that are side products of chemical processes, that can be, in general, characterized as a combustion of different fuels. These gases must be exhausted to the atmosphere, though they are very dangerous for human life and for the nature. That is why the amount of nitrogen oxides must be meaningfully reduced, e.g. with ammonia (NH_3), before they are released. The so-called *Selective Catalytic Reduction* (SCR), is one of the most popular way of reduction nitrogen oxides in exhaust gases. The main idea of an SCR system is to add an appropriate amount of ammonia to exhaust gases to cause the reduction reaction. The amount of ammonia is adjusted by a valve, and adjustments (settings) of the valve are dependent on the amount of nitrogen oxides. Since the process is non-linear, no traditional control system excluding human supervision has been applied. As for now, the process, in particular, adjustments of ammonia valve (its opening angle), are still supervised by a human expert. Therefore, we propose to apply fuzzy logic systems (FLS), especially, higher-order fuzzy logic systems to control efficiently the SCR processes, and to limit, at least partially, human participation in the process, see Fig. 1.

1.2. Related work. Publication [2] describes some types of uncertainty that must be handled in power management systems. The authors show numerous problems encountered when trying to apply optimization methods. Despite the great interest in the world of science-based solutions, the majority of expert knowledge rather than optimization algorithms is well visible. Traditional methods of control lead to a compromise to reach acceptable solutions. Fuzzy logic provides a framework for discussion of modeling such solutions. Examples of fuzzy logic systems applications are broadly known, e.g. [3–5]. Another work on FLSs in the industry and the environment is [6]. Because of non-linearity of those processes, fuzzy logic is proposed as one of the most hopeful methods, and the authors postulate to use it when indexing industries, in terms of the level of air and water pollution. Publications that deal with the general problem of fuzzy sets and fuzzy controllers are [7–10]. The authors [11] present rules and ideas for Interval type-2 fuzzy logic systems. They introduce the concept of upper and lower membership functions. Higher order fuzzy logic systems are also described in [12–14].

2. Selective Catalytic Reduction (SCR) system and dedicated fuzzy controller

2.1. Architecture of SCR system and its control signals. DeNOx filtration system relies on chemical reactions occurring between nitrogen oxides (NO_x) and ammonia (NH_3). As a result of reaction of these two components, nitrogen and water are obtained. The exhaust gases containing nitrogen oxides are introduced to a reactor. Liquid ammonia is taken from a tank, and it is then converted by heat to gas injected to the reactor. Ammonia gas in the process is a reducer separating molecules of NO_x to nitrogen N_2 and water H_2O. Because ammonia used in the process is both extremely detrimental

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compound and relatively expensive, the appropriate dosage for both ecological and financial reasons is crucial for the architecture of the system. The amount of ammonia entered to the reactor system is determined by opening the ammonia valve. Valve control signals, in particular, the desired angles of opening the valve are computed by a traditional controller and supervised by a human operator, who makes his/her decisions on the base of "NO_x Flow" taken from exhaust gases, in particular, from sensors located in the final part of the exhaust (in the chimney), see Fig. 1.

The main scope of the research is to propose an intelligent computer system based on fuzzy logic, that can replace, or, at least, limit human participation in the process by applying fuzzy logic and higher-order fuzzy logic systems, see Figs. 1 and 2.

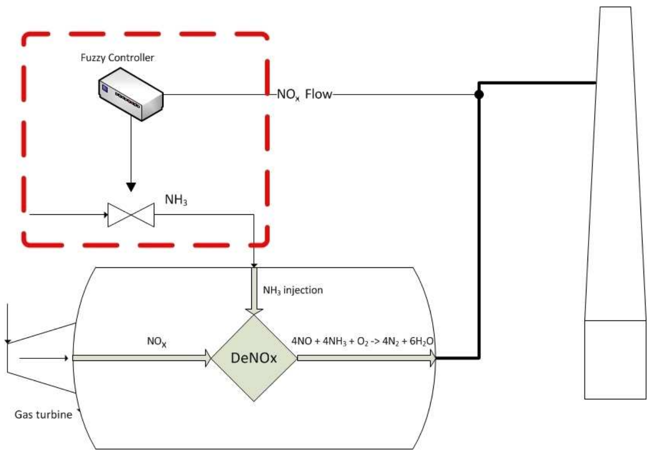

Fig. 1. A general schema of Selective Catalytic Reduction filter system. The red line bounds the part of the system, the fuzzy logic is proposed to be applied

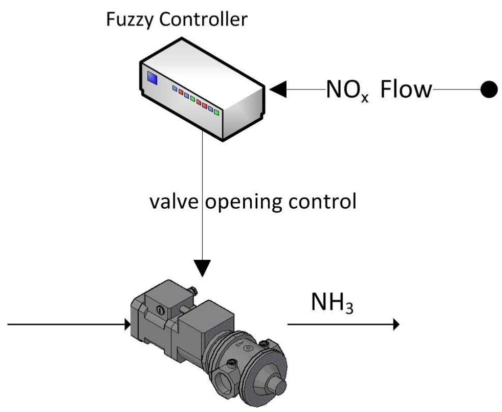

Fig. 2. A part of a general schema of SCR (see Fig. 1). Fuzzy controller output adjusts the valve that is responsible for ammonia injected to exhaust gases

2.2. Design of the higher order fuzzy controller for SCR. We propose to use a fuzzy logic system (in the sense of Mamdani) to control the amount of ammonia (NH_3) injected to the reaction chamber. This decision is taken on the basis of the input data of the level of NO and NO_2. The input data are fuzzified, then IF-THEN rules with fuzzy implication are applied, and finally, the output fuzzy sets are defuzzified to obtain a control value. The controller allows us to configure the method for selecting rule that are fired. The input values are derived from NO and NO_2 sensors located at the output of the SCR system, see "NO_x Flow" and the symbol of chimney in Fig. 1.

The sensors are read every 2 seconds. This period is selected because of limits on emissions of nitrogen oxides and on technical possibilities of controlling valves of the inflow filter of ammonia. According to current regulations limit, the total concentration of nitrogen oxides and dioxides is limited to 400 mg/m^3. The values of NO_x concentration read from sensors are fuzzified to linguistic expressions: *Low, Medium, High, Higher than acceptable*. Interval-valued fuzzy sets representing these labels in Experiment I are given in Fig. 6; analogous type-2 fuzzy sets represent the labels in Experiment II, see Fig. 11. The output of the controller is the desired angle of the ammonia valve opening, and is described by linguistic labels: *Low, Medium, High, Very High*. Interval-valued fuzzy sets representing these labels in Experiment I are depicted in Fig. 7; type-2 fuzzy sets representing the same labels in Experiment II are given in Fig. 12.

2.3. IF-THEN rules. The following sixteen rules are created to specify a linguistic value of the output.

IF (NO IS Low) AND (NO_2 IS Low) THEN Valve opening angle IS Low
IF (NO IS Low) AND (NO_2 IS Medium) THEN Valve opening angle IS Low
IF (NO IS Low) AND (NO_2 IS High) THEN Valve opening angle IS Medium
IF (NO IS Low) AND (NO_2 IS Higher Than Acceptable) THEN Valve opening angle IS High
IF (NO IS Medium) AND (NO_2 IS Low) THEN Valve opening angle IS Low
IF (NO IS Medium) AND (NO_2 IS Medium) THEN Valve opening angle IS Medium
IF (NO IS Medium) AND (NO_2 IS High) THEN Valve opening angle IS High
IF (NO IS Medium) AND (NO_2 IS Higher Than Acceptable) THEN Valve opening angle IS High
IF (NO IS High) AND (NO_2 IS Low) THEN Valve opening angle IS Medium
IF (NO IS High) AND (NO_2 IS Medium) THEN Valve opening angle IS High
IF (NO IS High) AND (NO_2 IS High) THEN Valve opening angle IS High
IF (NO IS High) AND (NO_2 IS Higher Than Acceptable) THEN Valve opening angle IS Very High
IF (NO IS Higher Than Acceptable) AND (NO_2 IS Low) THEN Valve opening angle IS High
IF (NO IS Higher Than Acceptable) AND (NO_2 IS Medium) THEN Valve opening angle IS Very High
IF (NO IS Higher Than Acceptable) AND (NO_2 IS High) THEN Valve opening angle IS Very High
IF (NO IS Higher Than Acceptable) AND (NO_2 IS Higher Than Acceptable) THEN Valve opening angle IS Very High

2.4. Type reduction and defuzzification. The last two element of higher-order fuzzy logic controller are type-reduction block and defuzzification block. Type-reduction is a transformation of higher order fuzzy sets appearing on output to traditional (type-1) fuzzy sets, that are then defuzzified.

For the higher-order fuzzy controller based on interval-valued fuzzy sets, the following method of type reduction is applied [15]:

$$TR_p(A) = \{\langle x, \underline{\mu}_A(x) + p(\overline{\mu}_A(x) - \underline{\mu}_A(x))\rangle : x \in \mathcal{X}\}, \quad (1)$$

where $p \in [0,1]$. In particular, this method for $p = 0.5$ is applied in the interval-valued fuzzy controller described in Section entitled "Experiment I". On the other hand, in the implementation of type-2 fuzzy controller presented in Section "Experiment II, the type reduction operation on a fuzzy set A in a discrete $X = \{x_1, \ldots, x_N\}$, $N \in \mathbb{N}$, is based on the centroid [16]:

$$C_A = \frac{\sum_{i=1}^{N} x_i \mu_A(x_i)}{\sum_{i=1}^{N} \mu_A(x_i)}. \quad (2)$$

Using Extension Principle, centroid of a type-2 fuzzy set $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) \colon x \in X\}$ in X such that $\mu_{\widetilde{A}}(x_i) = \int_{u \in J_{x_i}} f_{x_i}(u)/u$, and $J_x \subseteq [0,1]$ is the set of all primary memberships of x_i to $\widetilde{A}$, is given:

$$C_{\widetilde{A}} = \int_{\theta_1 \in J_{x_1}} \ldots \int_{\theta_N \in J_{x_N}} [f_{x_1}(\theta_1) \overset{T}{*} \ldots \overset{T}{*} f_{x_N}(\theta_N)] \Bigg/ \frac{\sum_{i=1}^{N} x_i \theta_i}{\sum_{i=1}^{N} \theta_i}, \quad (3)$$

where $\theta \in [\underline{J}_x, \bar{J}_x]$, and T is a T-norm.

Defuzzification in both interval-valued and type-2 fuzzy controllers is the Height Method:

$$y^* = \frac{\sum_{i=1}^{m} y_i \mu_{Ci*}}{\sum_{i=1}^{m} \mu_{Ci*}}, \quad (4)$$

where y^* is a real output value, μ_{Ci*} is the value of i-th fuzzy rule activation, and y_i is an element of Y with the highest membership.

3. Type-2 fuzzy implications: Traditional vs. "engineering"

The inference process in type-2 fuzzy controllers is obviously based on fuzzy implications and generalized rule *modus ponens*. Let $\widetilde{A}$, $\widetilde{A}'$ be type-2 fuzzy sets in X, and $\widetilde{B}, \widetilde{B}'$ – in Y. $\widetilde{B}'$ is evaluated as:

$$\widetilde{B}' = \widetilde{A}' \circ (\widetilde{A} \to \widetilde{B}). \quad (5)$$

In typical fuzzy logic systems in the sense of Mamdani, fuzzy implications are based on T-norms:

$$\mu_{A \to B}(x, y) = T(\mu_A(x), \mu_B(y)). \quad (6)$$

However, such an approach does not guarantee that the "implication" can be seen as an extension of traditional implication in classic logic. Mendel calls them "engineering implications" to distinguish them from the implication in its traditional logical meaning [13]. It is illustrated in Table 1.

Table 1
Minimum and product "implications" vs. classic implication. Both T-norm-based implications produce 0, if the value of antecedent is 0, too. This distinguishes them from the classic implication

$\mu_A(x)$	$\mu_B(y)$	$\min\{\mu_A(x), \mu_B(y)\}$	$\mu_A(x)\mu_B(y)$	classic
0	0	0	0	1
0	1	0	0	1
1	0	0	0	0
1	1	1	1	1

On the other hand, some publications, e.g. [17], put strong emphasis on compatibility of fuzzy implications with the classic implication. Conditions that must be fulfilled by operators $I \colon [0,1]^2 \to [0,1]$ to make them compatible with the classic implication are now enumerated [17]:

$\forall_{a,b,c \in [0,1]}$

1. $a \leq c \to I(a,b) \geq I(c,b)$,
2. $b \leq c \to I(a,b) \leq I(a,c)$,
3. $I(0,a) = 1$,
4. $I(a,1) = 1$,
5. $I(1,0) = 0$.

The most known fuzzy implications are presented in Table 2.

Table 2
Fuzzy implications compatible with the classic implication

	Name	Implication
1	Kleene-Dienes	$\max\{1-a, b\}$
2	Łukasiewicz	$\min\{1, 1-a+b\}$
3	Reichenbach	$1-a+ab$
4	Fodor	1, if $a \leq b$ $\max\{1-a, b\}$, otherwise $a > b$
5	Rescher	1, if $a \leq b$ 0, otherwise $a > b$
6	Gödel	1, if $a \leq b$ b, otherwise $a > b$
7	Yager	1, if $a = 0$ b^a, otherwise $a > 0$
8	Zadeh	$\max\{\min\{a,b\}, 1-a\}$
9	Dubois-Prade	$1-a$, if $b = 0$ b, if $a = 1$ 1, otherwise

It must be added, that some of implications enumerated in Table 2 do not satisfy one or more conditions 1–5. An example is the Zadeh implication, that does not meet conditions 1 and 4, (see row 8).

3.1. Interval-valued fuzzy implications. The implications applied in the interval-valued "version" of the designed fuzzy controller are based on T-norm minimum. Let A, B be interval-valued fuzzy sets in X, Y, respectively, $\underline{\mu}_A$, $\overline{\mu}_A \colon X \to [0,1]$ are lower and upper membership functions of A, respectively, and $\underline{\mu}_B$, $\overline{\mu}_B \colon Y \to [0,1]$ – analogously. Implications applied in the interval-valued FLS are in the form of:

$$\mu_{A \to B}(x, y) = \left[T(\underline{\mu}_A(x), \underline{\mu}_B(y)), T(\overline{\mu}_A(x), \overline{\mu}_B(y))\right]. \quad (7)$$

For $T = \min$ we have:

$$\mu_{A\to B}(x,y) = \left[\min\{\underline{\mu}_A(x),\underline{\mu}_B(y)\},\min\{\overline{\mu}_A(x),\overline{\mu}_B(y)\}\right]. \tag{8}$$

An example of applying implication (8) for two of IF-THEN rules:

IF (NO IS Low) AND (NO_2 IS Low) THEN Valve opening angle IS Low

IF (NO IS High) AND (NO_2 IS Low) THEN Valve opening angle IS Medium

presented in Section "Design of the higher order...", is illustrated in Fig. 3. Interval-valued fuzzy sets representing the input, concentration level of nitrogen oxides, are depicted in Fig. 6. Interval-valued fuzzy sets representing the output, the ammonia valve opening angle, are depicted in Fig. 7.

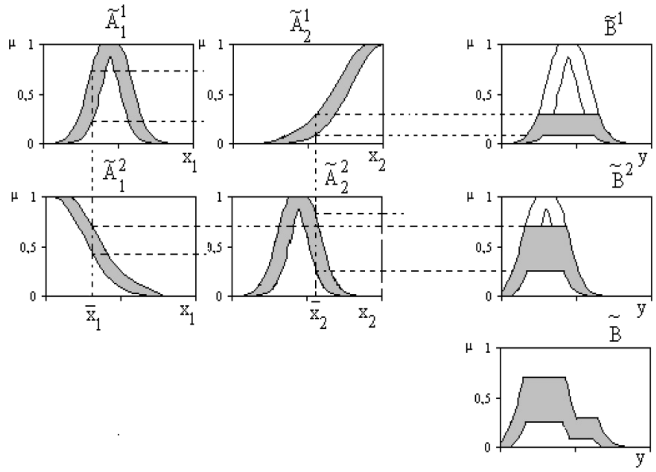

Fig. 3. Fuzzy implication (8) for two of rules fired for chosen values of NO and NO_2. As the result of the inference (on the right), the membership function constructed of membership functions of input interval-valued fuzzy sets (on the left) is evaluated

Other implications for interval-valued fuzzy logic systems are suggested in [7, 18].

3.2. New implications for a type-2 fuzzy logic system. In this paragraph, some modifications of type-2 fuzzy implications are discussed. Let $\widetilde{A}$, $\widetilde{B}$ be type-2 fuzzy sets in X, Y, respectively, denoted as follows:

$$\mu_{\widetilde{A}}(x) = \int_{u\in J_x} f_x(u)/u, \tag{9}$$

$$\mu_{\widetilde{B}}(y) = \int_{v\in J_y} g_y(v)/v, \tag{10}$$

where J_x, $J_y \subseteq [0,1]$ are sets of all primary membership degrees of elements x in $\widetilde{A}$, and of y in $\widetilde{B}$, respectively. A general form of extension of fuzzy implication for type-2 fuzzy sets is given as follows:

$$\mu_{\widetilde{A}\to\widetilde{B}}(x,y) = \int_{u\in J_x}\int_{v\in J_y} f_x(u) \overset{I^*}{*} g_y(v) \Big/ u \overset{I}{*} v, \tag{11}$$

where I^*, I are type-1 fuzzy implications, e.g. min, product, or any given in Table 2. For instance, for $I^* =$ product and $I = \min$, we have:

$$\mu_{\widetilde{A}\to\widetilde{B}}(x,y) = \int_{u\in J_x}\int_{v\in J_y} f_x(u)g_y(v) \Big/ \min\{u,v\}. \tag{12}$$

In the implementations described in the next section, different implications have been tested. Since the goal is to achieve possibly the highest compatibility with expert's opinions, of the designed fuzzy logic system, new implication operators are proposed. In fact, many known fuzzy "implications" do not meet the requirements of the classic implication or even a T-norm. They usually do not meet all the conditions 1.–5. presented for engineering implications either [14]. However, they are applied to model or to reconstruct deduction and/or inference. Therefore, the new implications proposed in the paper are simply another attempt of reconstructing the human way of thinking (to be more precise: human way of inferring consequents from premises), not necessarily having in mind all the formal conditions and requirements for this inference need to be fulfilled (e.g. implication (15) is not a T-norm, but is useful in the inference process, the result of which are given in Table 4, row 7). Their basic forms are presented by Eqs. (13)–(15):

$$I_{K_1}(a,b) = \begin{cases} \dfrac{\sqrt{ab}}{a+b-ab}, & \text{if } a\neq 0 \vee b\neq 0 \\ 0, & \text{otherwise} \end{cases}, \tag{13}$$

$$I_{K_2}(a,b) = \min\left\{1, \frac{\sin\left(\min\{a,b\}\right)}{\cos\left(\min\{a,b\}\right)}\right\}, \tag{14}$$

$$I_{K_3}(a,b) = \begin{cases} \dfrac{ab}{a+b}, & \text{if } a\neq 0 \vee b\neq 0 \\ 0, & \text{otherwise} \end{cases}, \tag{15}$$

where $a, b \in [0,1]$. These new implication operators imply some new type-2 fuzzy implications; examples are given by Eqs. (16)–(18). To be more specific, (13)–(15) define implication operation on primary membership degrees. T-norm min is applied for secondary membership degrees.

$$\widetilde{I}_{K_1}: \mu_{\widetilde{A}\to\widetilde{B}}(x,y) = \begin{cases} \displaystyle\int_{u\in J_x}\int_{v\in J_y} \min\{f_x(u),g_y(v)\} \Big/ \frac{\sqrt{uv}}{u+v-uv}, \\ \quad \text{if } u\neq 0 \vee v\neq 0 \\ 0, \ \text{otherwise} \end{cases}, \tag{16}$$

$$I_{K_2}: \mu_{\widetilde{A}\to\widetilde{B}}(x,y) = \int_{u\in J_x}\int_{v\in J_y} \min\{f_x(u),g_y(v)\} \Big/ \min\left\{1, \frac{\sin\left(\min\{u,v\}\right)}{\cos\left(\min\{u,v\}\right)}\right\}, \tag{17}$$

$$\widetilde{I}_{K_3}: \mu_{\widetilde{A}\to\widetilde{B}}(x,y) = \begin{cases} \int\limits_{u\in J_x}\int\limits_{v\in J_y} \min\{f_x(u), g_y(v)\} \Big/ \dfrac{uv}{u+v}, \\ \quad \text{if } u \neq 0 \vee v \neq 0 \\ 0, \text{ otherwise} \end{cases}. \tag{18}$$

Two examples of inference, using type-2 fuzzy implications (16) and (18), are presented in Figs. 4 and 5, respectively.

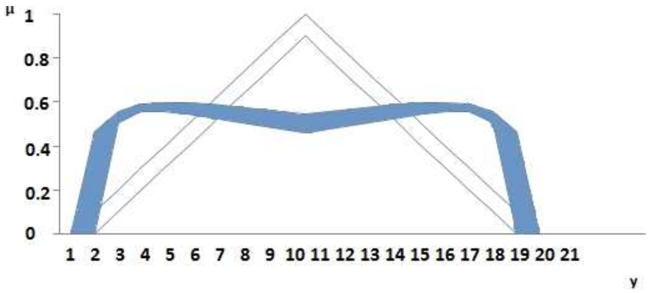

Fig. 4. A sample type-2 fuzzy set represented by white-coloured FOU is an antecedent of a rule. The type-2 fuzzy set represented by blue-coloured FOU is the result of type-2 fuzzy implication (16)

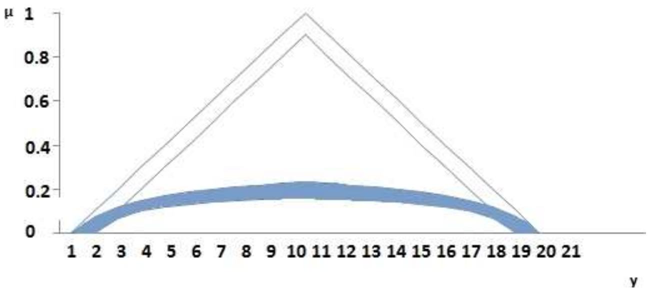

Fig. 5. A sample type-2 fuzzy set represented by white-coloured FOU is an antecedent of a rule. The type-2 fuzzy set represented by blue-coloured FOU is the result of type-2 fuzzy implication (18)

The proposed implication operators and type-2 implications based on them are applied in the implementation of type-2 fuzzy logic system described in Section "Experiment II". Their potential in computing outputs of a FLS is illustrated in Table 4.

4. Experiment I: Interval-Valued Fuzzy Controller for SCR system

4.1. Data preparation and evaluation of output. The interval-valued fuzzy controller evaluates the output: the ammonia valve opening angle (see Fig. 2). Each angle is related to a sample (input) represented by levels of concentration of nitrogen oxides, see "NO_x Flow" in Fig. 2. The controller is tested six times: three times for 10 000 samples and three times for 100 000 samples.

The results obtained the implemented type-2 fuzzy controller are now compared to expert's proposals. We take into account two vectors: E – representing expert's proposals of ammonia valve opening angle, and C – representing the angles evaluated by the controller. Both vectors are of the same length $n \in \mathbb{N}$. The vectors are compared using the min-max method.

$$\text{min-max}(E,C) = \frac{\sum\limits_{i=1}^{n} \min\{e_i, c_i\}}{\sum\limits_{i=1}^{n} \max\{e_i, c_i\}}, \tag{19}$$

where $E = \{e_1, e_2, ..., e_n\}$, $C = \{c_1, c_2, ..., c_n\}$. Values of min-max$(E,C)$ represent similarity of vectors E and C. The largest possible value of min-max(E,C) is 1 – it would mean that vectors are identical, so the values proposed by the controller are all equal to the values of ammonia valve opening angle proposed by human experts. As it is mentioned above, two values of n are taken into account: $n_1 = 10\,000$ samples and $n_2 = 100\,000$ samples.

The input data (samples) are fuzzified to interval-valued fuzzy sets presented in Fig. 6. The result of inference, the output interval-valued fuzzy sets representing angles of the ammonia valve opening, are illustrated in Fig. 7.

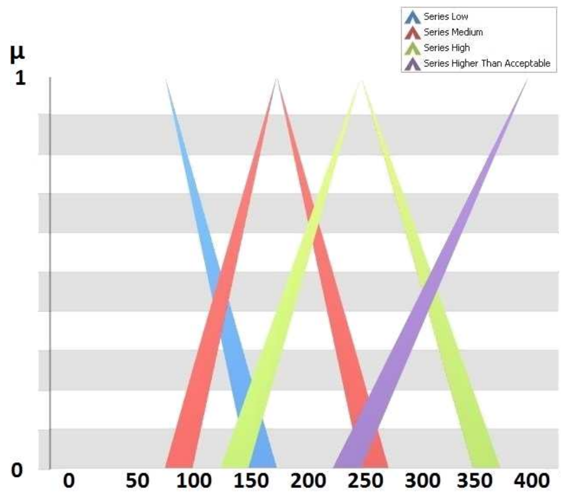

Fig. 6. Interval-valued fuzzy sets for linguistic values of input data on NO and NO_2 (in mg/m^3)

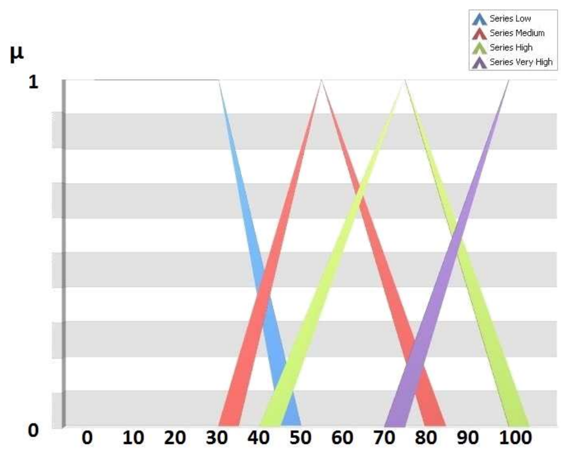

Fig. 7. Interval-valued fuzzy sets for linguistic variable of output data which describe the valve opening angle to determine the amount of ammonia (the y value represents the angle of valve opening in percent)

4.2. Firing rules for interval-valued fuzzy input. There are different methods of selecting rules applied, in particular, to select the most adequate rule from the rules that may be fired for a given interval representing input data. The methods of selecting rules are based on partial order relations for intervals $a = [\overline{a}, \underline{a}], b = [\overline{b}, \underline{b}]$. Three methods of ordering intervals are presented below [19]:

$$a < b \Leftrightarrow \overline{a} \leq \underline{b}, \tag{20}$$

$$a \leq_o b \Leftrightarrow \underline{a} \leq \underline{b} \wedge \overline{a} \leq \overline{b}, \tag{21}$$

$$a \leq_m b \Leftrightarrow m(a) \leq m(b) \wedge w(a) \geq w(b), \tag{22}$$

where $m(a)$ according to [15] is the mid-point of a $\frac{m(a)=\overline{a}+\underline{a}}{2}$ and $\frac{w(a)=\overline{a}-\underline{a}}{2}$.

The use of the methods in evaluation is commented in Table 3, in rows 2, 3, and 4, respectively.

4.3. Results. For each method of selecting rules described by formulae (20), (21), and (22), the interval-valued fuzzy controller is tested. The type-reduction operation is based on (1). The best results of each test are collected in Table 3. The best results – the largest similarity of E and C vectors are bold.

Table 3
Values of $\text{min-max}(E, C)$ similarity of outputs computed by interval-valued fuzzy controllers to expert proposals

	$\text{min-max}(E, C)$	Remarks
1.	0.9298	Type-1 FLS, cf. [1], see Fig. 8
2.	**0.9454**	**IVFLS, firing rules with (20) type-reduction via (1), see Fig. 9**
3.	**0.9454**	**IVFLS, firing rules with (21) type-reduction via (1), see Fig. 10**
4.	0.9445	IVFLS, rules fired with (22), type-red. via (1) type-reduction via (1)

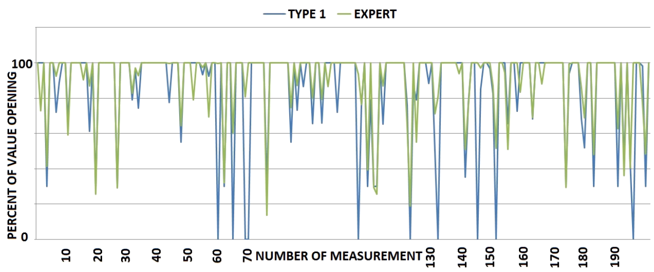

Fig. 8. Angles of the ammonia valve opening evaluated by the type-1 FLS [1] (blue line) and angles proposed by a human expert (green line) Table 3, row 1

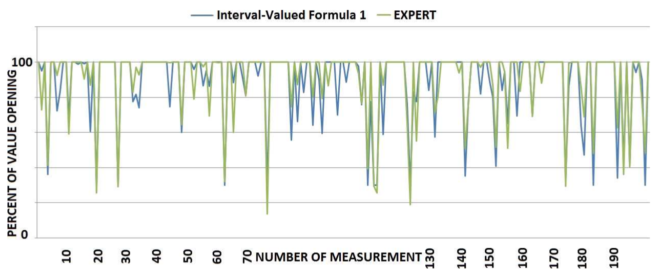

Fig. 9. Angles of the ammonia valve opening evaluated the interval-valued FLS (blue line) and angles proposed by a human expert (green line). IF-THEN rules are fired via (20). Table 3, row 2

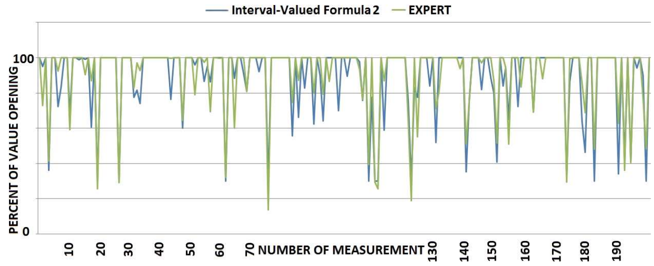

Fig. 10. Angles of the ammonia valve opening evaluated the interval-valued FLS (blue line) and angles proposed by a human expert (green line). IF-THEN rules are fired via (21). Table 3, row 3

As we may conclude from Table 3, the variants of fuzzy logic system based on interval-valued fuzzy sets provides better results than type-1 FLS presented in [1]. The value of $\text{min-max}(E, C)$ for the interval-valued FLS is 0.9454, see Table 3, rows 2 and 3, while the same value for type-1 FLS is smaller. That means the ammonia valve opening angles in the SCR system evaluated by interval-valued fuzzy controller are closer to human expert opinions than those evaluated by the type-1 fuzzy controller described in [1]. It must be underlined, that the difference between $\text{min-max}(E, C)$ values for type-1 FLS and interval-valued FLS is about 0.015, so it may seem to be very small. However, we must be aware that an average company producing nitrogen oxides emits about 8 100 Mg (eight thousand and one hundred tones) of toxic NO_2 to the atmosphere per year [20]. That means that the difference 0.015 implies 121 Mg (one hundred and twenty tones) of toxic gases emitted to the atmosphere each year, that is more than 10 000 kg per month. Hence, we conclude that replacing type-1 FLS with interval-valued FLS is worth taking into account.

The results of fuzzy logic systems designed for controlling angles of the ammonia valve opening are also illustrated in Fig. 8, 9, and 10. The angles evaluated by fuzzy controllers are on the blue lines, and angles proposed by a human expert – on the green lines. The more the green line is covered by the blue line, the more similar are results of FLSs to expert proposals.

5. Experiment II: Triangular Type-2 Fuzzy Controller for SCR system

5.1. Data preparation and evaluation. Similarly to the tests of the interval-valued: fuzzy controller, see Section "Experiment I", the task for the type-2 fuzzy controller is to evaluate ammonia valve opening angles as close to a human expert proposals as possible. Besides, the same datasets are taken into account as input: the type-2 fuzzy controller is tested three times for 10 000 samples and three times for 100 000 samples. Furthermore, the results are compared to expert proposals using the min-max method (19) again.

5.2. Implementation details. Type-2 fuzzy logic controller is based on triangular secondary membership functions of type-2 fuzzy sets representing input in Fig. 11, and output linguistic values in Fig. 12.

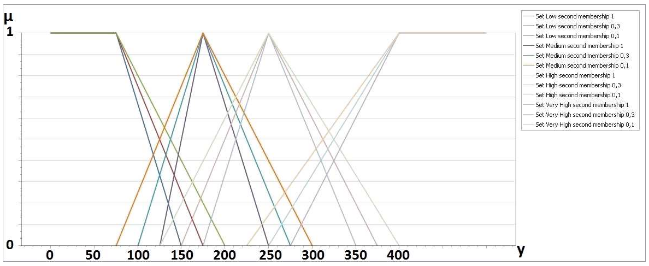

Fig. 11. Type-2 fuzzy sets representing linguistic values of input data on NO and NO_2 (in mg/m^3)

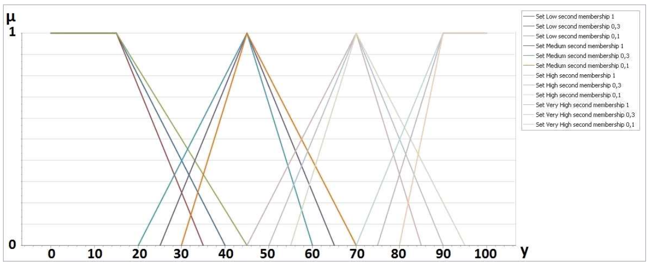

Fig. 12. Type-2 fuzzy sets representing linguistic values of output data: angles of the ammonia valve opening (in percent)

New implication operators (13)–(15) and type-2 fuzzy implications based on them (16)–(18) are applied in IF-THEN rules. Besides, some known implications (e.g. min, Larsen and Łukasiewicz) are also applied and results evaluated with them are compared to those evaluated with newly proposed methods.

5.3. Results. The results evaluated by the presented implementation of a type-2 fuzzy logic system are now commented. They are collected in Table 4. Row 1. of the table presents the $\min\text{-}\max(E,C)$ value (similarity of results obtained by FLS to the expert knowledge) for the type-1 fuzzy logic system presented in [1]. Row 2 presents the value for the best result provided by the interval-valued fuzzy logic system described in Section "Experiment I". Rows 3 and 4 show the $\min\text{-}\max(E,C)$ that describes performance of type-2 FLSs using known implication operators, min in row 3 and Łukasiewicz in row 4. The values are not significantly larger than those provided by the interval-valued FLS. The best results – the largest values of $\min\text{-}\max(E,C)$ are provided by type-2 fuzzy logic systems based on newly proposed implication operators (16) and (18). Especially, the type-2 FLS with implication (16), row 5, provides results significantly higher than the best of interval-valued FLSs presented in Section "Experiment I". Similarly to the results of Experiment I, the difference between $\min\text{-}\max(E,C)$ values shown in rows 5 and 2, i.e. 0.004, can be evaluated as 40 Mg (forty tones) of toxic substances per year. The difference between performance of the type-1 FLS presented in [1] and from the type-2 FLS with implication $\widetilde{I}_{K_1}$ introduced in this paper is $0.9493-0.9298 = 0.0195$, and it is equivalent of 160 Mg (one hundred and sixty tones) of NO_2 emitted to the atmosphere per year.

Table 4
Values of $\min\text{-}\max(E,C)$ similarity of outputs computed by type-2 fuzzy controllers to expert proposals

	$\min\text{-}\max(E,C)$	Remarks
1.	0.9298	Type-1 FLS, cf. [1], see Fig. 8
2.	0.9454	The best result of interval-valued FLS see Table 3 and Fig. 10
3.	0.9477	Type-2 FLS with the min implication (see Fig.13)
4.	0.9445	Type-2 FLS with the Łukasiewicz implication
5.	**0.9493**	**Type-2 FLS with implication** $\widetilde{I}_{K_1}$ (16), see Fig. 14
6.	0.9475	Type-2 FLS with implication $\widetilde{I}_{K_2}$(17)
7.	**0.9484**	**Type-2 FLS with implication** $\widetilde{I}_{K_3}$ (18), see Fig. 15

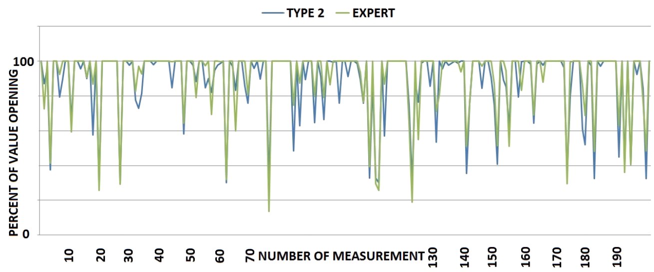

Fig. 13. Angles of the ammonia valve opening evaluated the type-2 FLS (blue) and proposed by a human expert (green). Type-2 inference: the min implication. Compare Table 4, row 2

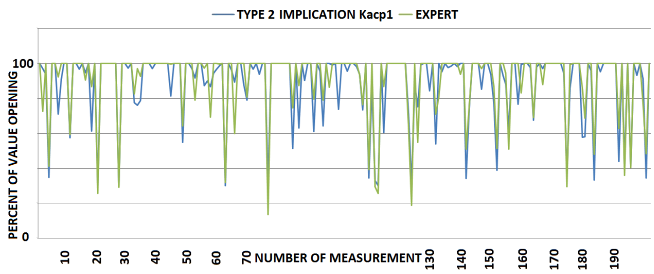

Fig. 14. Angles of the ammonia valve opening evaluated the type-2 FLS (blue) and proposed by a human expert (green). Type-2 inference: implication I_{K_1}. Compare Table 4, row 5

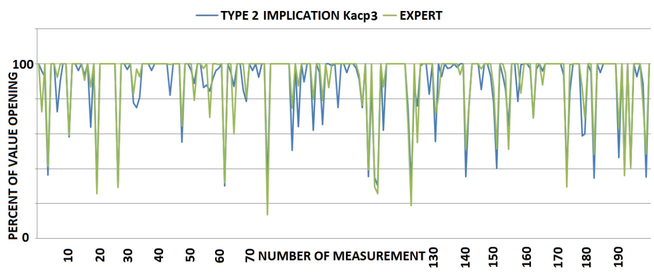

Fig. 15. Angles of the ammonia valve opening evaluated the type-2 FLS (blue) and proposed by a human expert (green). Type-2 inference: implication I_{K_3}. Compare Table 4, row 7

6. Summary and future work

This paper describes new applications of higher order fuzzy logic systems to control Selective Catalytic Reduction Systems in chemical industry. The goal is to support decisions made by a human expert using dedicated fuzzy controllers. This is a continuation and enhancement of our previous work presenting a type-1 fuzzy logic system managing data on air pollution [1]. The idea of applying traditional fuzzy logic systems to control air filters is now extended to an application of higher-order fuzzy logic systems. We test interval-valued fuzzy controllers and type-2 fuzzy controllers. The results of comparison of the system output (proposed parameters of adjusting the filter) to data given by the expert are better than in case of type-1 fuzzy logic system, see Table 4. In the nearest future, two further issues on applying fuzzy logic systems to control the air pollution are undertaken. The first, an application of learning methods to dynamic correction of fuzzy sets. The second issue, possible extensions of functionality of the proposed type-2 FLS is discussed in case of the variability of data on concentration of gases in the air, is enhanced.

All tests and results show that using higher-order fuzzy controllers despite increase allow for significant improvement of the results and in consequently for better modeling of the human perception of reality. These conclusions lead to the final conclusion that further research on applications of higher-order FLSs to manage data on the environment pollution, and additionally, complying with official legal requirements, are reasonable and worth continuing.

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