DYNAMIC RESPONSE OF THIN-WALLED GIRDERS SUBJECTED TO COMBINED LOAD

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This paper deals with dynamic response of thin-walled girders subjected to combined load (bending and/or twisting). The load was assumed as stepped dynamic pulse of finite duration. Numerical analysis was conducted with finite element method (FEM). Analysis has concentrated on thin-walled girders of length 200 mm and squared cross-section (100x100 mm) with 1mm wall thickness. Different numerical models were compared and investigation of dynamic response of the structure under combined load was performed for different boundary conditions. The critical values of load were determined according to well-known criteria (Volmir, Budiansky-Hutchinson, Ari-Guru-Simonetta criterion).

1. INTRODUCTION

Wide range of application of thin-walled elements and increasing demand for new enhanced solution affect development of theories and experimental investigations on stability of such structures as well as on the dynamic response of thin-walled structures.

The problem of dynamic response of thin-walled girders subjected to combined load is a complex matter. In order to perform such analysis a well-known finite element method is used to derive solution. Several works concerning the stability, post-buckling in the elastic and elasto-plastic range as well as the load-carrying capacity of thin-walled girders subjected to either simple or combined load is widely discussed in two complete editions by Królak [4, 5]. Particular insight into stability of structures with the debate about theoretical foundations and solutions of such problem is presented in edition by Kołakowski, Kowal-Michalska [3]. Even though that majority of papers deal with thin-walled girders, one can find only a few concerning combined load.

This work shows that the analysis of dynamic response of thin-walled girders subjected to combined load is of huge importance and describes different aspects of analysis of stability with the use of Finite Element Method.

2. SOLUTION METHOD

The problem was solved using finite element method, employing commercial software *ANSYS*. At first step the eigenvalue problem was solved for different bending to torsion ratios. The pulse duration T_p was set based on first period of natural vibration. The critical load - in this case the critical bending moment (M_{bcr}) and torsional moment (M_{tcr}) was used to determine the dynamic load factor *DLF*. Shape of first buckling or modal mode represents the initial imperfection with the amplitude relative to the thickness of the considered girder wall was assumed. The dimensionless amplitude of initial imperfection as $\xi^* = 0.01\ w_{max}/h$ (where w_{max} is the maximal normal to the wall

displacement and *h* is the girder wall thickness) were adopted. The results of these calculations were used as input to the analysis of the dynamic behaviour of the structure, during and immediately after exposure of pulse loads. In the analysis of the dynamic response the equilibrium equation is supplemented by the dynamic blocks, and takes the form:

$$\{P\} = [M] \cdot \{\ddot{u}\} + [C] \cdot \{\dot{u}\} + [K] \cdot \{u\} \,,$$
(1)

where {P} is the vector of generalized nodal forces, [M] is the mass matrix, [C] is a damping matrix and {u} is the vector of generalized nodal displacements.

As it has been shown in many studies (for example [2]) for the short-term load the damping effect can be neglected what leads to the simplification of equation (1) to the form:

$$\{P\} = [M] \cdot \{\ddot{u}\} + [K] \cdot \{u\} \,.$$
(2)

After replacing the time derivatives of displacements {ü} by differences displacement {u} in successive discrete moments of time t, the new static equilibrium equation taking into account the inertia forces [M] {ü} is obtained for the each time step and therefore it is possible to apply the algorithms used in the static analysis. Time integration in the *ANSYS* program is done using the Newmark method and solution of equations in successive time steps is made by Newton-Raphson algorithm.

This approach allows analysing the behaviour of the structure subjected to pulse load. Discretization of thin-walled girders was performed with the quadrilateral, four nodes shell elements (Fig. 1) with six degrees of freedom (three orthogonal displacements and three rotations around the axis in the plane of the element) at each node.
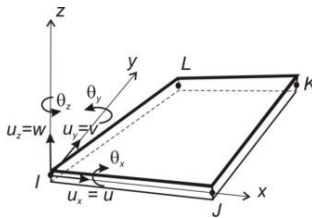

Fig. 1. Shell element [1]

3. NUMERICAL ANALYSIS

Analyzed thin-walled girder of length *l*=200 mm and squared cross-section (*a*=100, *b*=100 mm) with 1mm wall thickness was subjected to combined load. The structure with loading scheme is shown in Fig. 2. The thin-walled girder's material was assumed as isotropic with following material properties: $E=2\cdot10^5$ MPa, ν=0.3.

In order to obtain results comparable to the real structure, two numerical models were taken into consideration for analysis of the stability (eigenvalue method) for different bending to torsional moment ratios. In order to assure the linearity of loaded

edges in first example two plates of relatively high stiffness were added to the ends of the girder (Model 1). In second example the same linearity of the loaded edges were obtained by the use of beam elements (Model 2). The models are presented in Fig. 3.
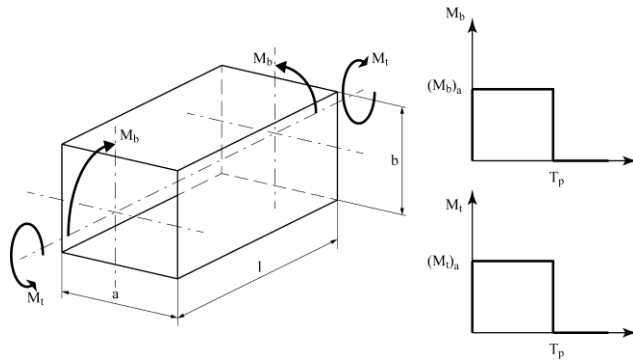
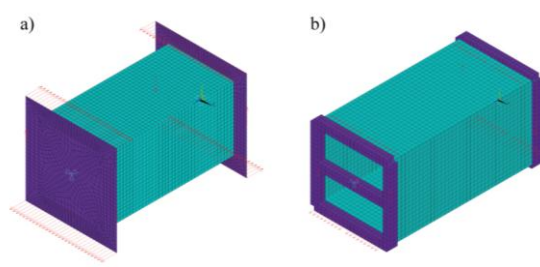

Fig. 2. Dimensions and loading scheme of analyzed girder

Fig. 3. FEM models: a) plate model (Model 1), b) beam model (Model 2)

3. RESULTS OF NUMERICAL CALCULATIONS

The results of eigenvalue analysis for different bending to torsion ratios are depicted in Fig. 4. The figure shows that both models give similar results. Critical values of bending and torsional moments are presented in Table 1. The buckling modes were the same for both models (Fig. 5).

Table 1. Critical values of bending moment and torsion for both considered models

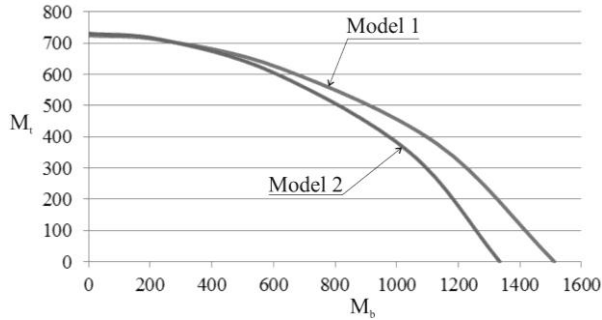|	Model 1		Model 2	
M_b/M_t	M_{bcr} [Nm]	M_{tcr} [Nm]	M_{bcr} [Nm]	M_{tcr} [Nm]
∞	0,725	0	0,732	0
3	0,710	0,237	0,711	0,237
1	0,621	0,621	0,604	0,604
0,333	0,377	1,132	0,348	1,045
0	0	1,512	0	1,335

Fig. 4. Comparison of the results obtained for different models

After preliminary studies the beam model was dismissed because of the difficulties with convergence of the dynamic solution. This was caused by formation of stress concentration at the corners which mainly were due to modelling method of boundary conditions. The pulse duration T_p was obtain on the basis of modal analysis as equal to period of natural vibration and set to $T_p=0.003s$. The deflection of middle node of the upper plate of the given girder in time for different values of DLF is presented in Fig. 6. It can be noted that for *DLF* higher than 2.4 change of buckling mode occurred.
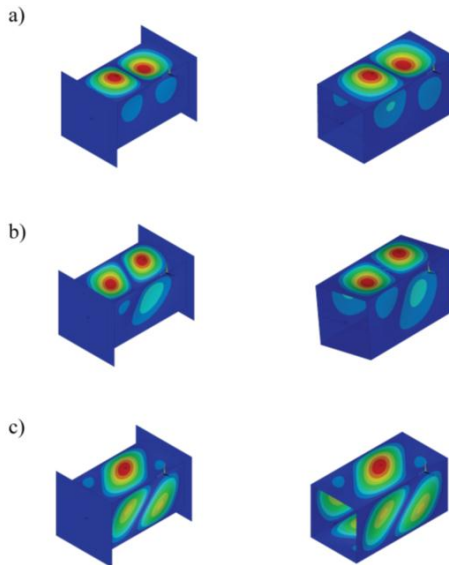

Fig. 5.   Shape of the local buckling modes: a) $M_t/M_b=0$, b) $M_b/M_t=1$, $M_b/M_t=0$

The dynamic response of the structure as the function of ξ - *DLF* is shown in Fig. 7. Two approaches to obtain results of the solution were assumed. One of the curves corresponds to the values of the dimensionless deflection measured in the middle of the upper plate of the girder (*mid)* and the second relates to the maximum values of

dimensionless deflection in whole structure *(max)*. Critical values of dynamic load factor obtained based on different criteria are presented in Table 2.

Table 2.   Critical values of dynamic load factor

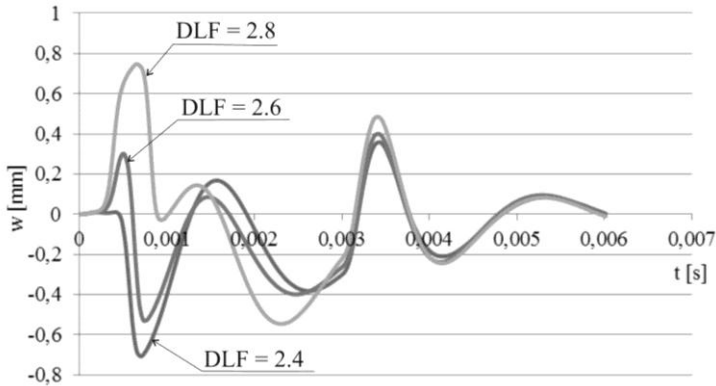|	Middle node	Maximum deflection
Volmir criterion	1.7	1.3
Budiansky-Hutchinson criterion	1.6-1.8	1.4-1.6
Ari-Gur – Simonetta criterion	1.8	2

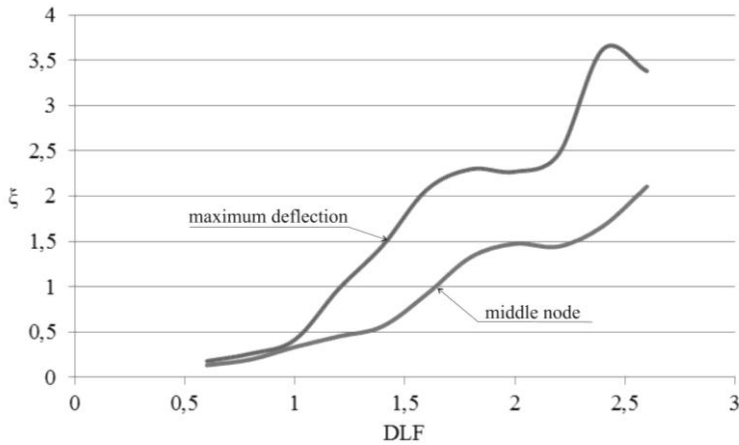Fig. 6. Displacement of middle node of upper plate of the girder in time for different DLF

Fig. 7.   Dimensionless deflections ξ versus DLF for $M_b/M_t=1$

A change of mode shape occurs with the increase of dynamic load factor. This phenomenon is shown in Fig. 8. The results are given for different *DLF* values with respect to the position along the diagonal of the upper wall and the change of mode shape appears between *DLF*=1 and *DLF*=1.2.
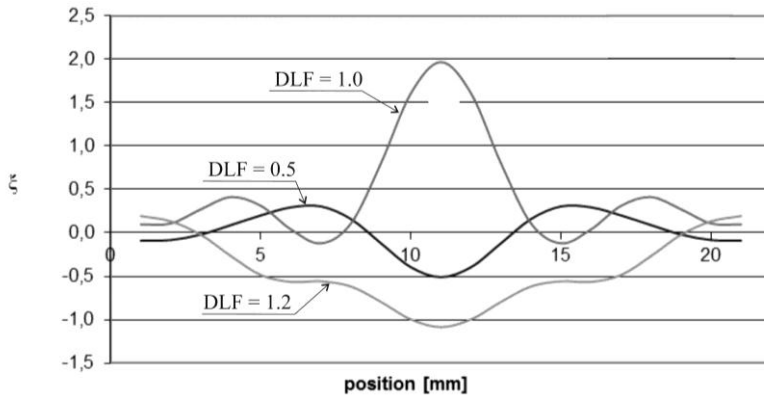
Fig.8.   Dimensionless deflections along the diagonal of the upper wall for different DLF

3. CONCLUSIONS

This paper presents dynamic response of thin walled girders subjected to combined load. According to the analysis it is concluded that:

- suitable choice of elements is crucial in obtaining appropriate results
- value of the deflection for dynamic calculations might be gathered with two different approaches: from middle node of upper plate of the girder or maximum value in a whole structure
- for combined load it is difficult to chose the proper criterion to determining the DLF_{cr}

ACKNOWLEDGEMENTS

REFERENCES

[1]   ANSYS User's Guide rev. 12.1, Ansys, Inc, 2011
[2]   Cui S., Hao H., Cheng H.K., Theoretical study of dynamic elastic buckling of columns subjected to intermediate velocity impact loads, Int. J. of Mechanical Science, 44, 2002, pp. 687-702.
[3]   Kołakowski Z., Kowal-Michalska K., Selected Problems of Instabilities in Composite Structures, Technical University of Lodz, 1999.
[4]   Królak M. , Post-Buckling Behaviour and Load Carrying Capacity of Thin –Walled Plate Girders, PWN, Warsaw-Łódź, 1990 (in Polish).
[5]   Królak M., Stability, Post-Buckling Behaviour and Load Carrying Capacity of Thin, Orthotropic Flat-Walled Structures, Technical University of Lodz, 1995 (in Polish).