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SENSITIVITY ANALYSIS OF STEEL BOX-SECTION GIRDERS

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The paper deals with the load-carrying capacity stochastic variance based sensitivity analysis of thin-walled box-section girder subjected to pure bending. The lower- and upper-bound load-capacity estimation is performed. The methodology is based on the Monte-Carlo method. The exemplary results are presented in diagrams and pie charts showing the sensitivity of load-capacity to different random input variables. The analysis is focused on the variance of the yield stress of the girder material and girder's wall thickness. Some final conclusions, concerning an efficiency of the applied models and the sensitivity analysis are derived.

1. INTRODUCTION

In recent years the deterministic approach to the design of TWS has been often replaced by the probabilistic one [1, 2, 3, 9, 13]. It concerns especially thin-walled girders [1, 14]. Also some new codes, particularly concerning TWS in civil engineering, treat the structural reliability and load-carrying capacity of TWS as a probabilistic problem [4]. However, since using any probabilistic method one has to perform a great number of calculations, the main limitation becomes the time of computation, which depends on the method applied.

The strength of thin-walled structures is usually calculated on the basis of "effective width" model and their ultimate capacity is evaluated using a reduced or effective cross-section and, additionally, the elastic limit for maximum stress. This approach is currently used in almost all design codes and leads to the lower-bound estimation of the load-carrying capacity. The elastic post-buckling behaviour of the thin-walled beam was analysed by Kolakowski et al [5] who solved the problem using the asymptotic method in the range of the second order approximation. The algorithm based on the asymptotic method is relatively simple and delivers the lower-bound estimation of the load-carrying capacity (LBELC) in the short time of computation.

However, TWS members display a significant post-elastic capacity. It means that the actual load-carrying capacity of any thin-walled member is higher than the ultimate load calculated using the method mentioned above.

Thus, the alternative approach is the upper-bound estimation of the load-carrying capacity, consisting in the determination of the intersection –point of a post-buckling path (evaluated using either analytical method or numerical one, e.g. Finite Element Method) and a rigid-plastic *failure curve* obtained from the plastic mechanism analysis – Kotełko et al. [6, 7].

Compilation of post-buckling analysis with the yield-line analysis (plastic mechanism approach) leads to a relatively simple and quick solution of the upper-bound estimation of load-carrying capacity (UBELC). Thus, both the asymptotic method

(LBELC) and yield-line analysis (UBELC) have advantages over numerical methods, particularly FEM.

The paper deals with the sensitivity analysis of the load-carrying capacity (LBELC and UBELC) of thin-walled, box-section girder subjected to pure bending (Fig.1).

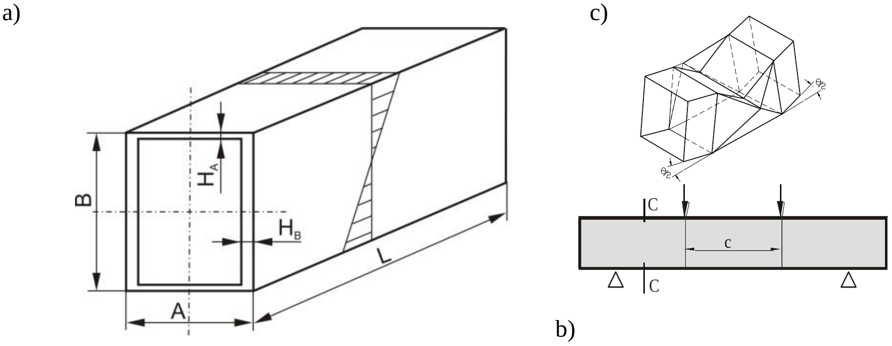


Fig. 1. Box-section girder under pure bending: a) dimensions, b) load and support lay-out, c) theoretical model of the plastic mechanism of failure

2. LOAD-CAPACITY COMPUTATION MODEL

The load-carrying capacity of the girder was calculated using the software code 'NOSNOSC' elaborated by Kołakowski, Kotełko and Kubiak [8]. The code provides information about the girder's structural behaviour in the whole range of loading (up to and beyond the ultimate load) and calculates the lower bound and upper-bound load-carrying capacity estimations (denoted below as LBELC and UBELC, respectively). LBELC corresponds to the first yield in the beam's compressed flange, while UBELC is calculated as an ordinate of inter-section of the post-buckling elastic path with the failure path. The post-buckling path is calculated using the asymptotic method. The study is based on the numerical method of the transition matrix using Godunov's orthogonalization [5, 12]. In order to determine maximum stresses in girder's plate members under compression, the width of a compressed flange is reduced to the effective width to obtain the real decrease in a flexural stiffness of the cross-section after local buckling. The first yield threshold criterion is used in order to estimate load-capacity of the girder (lower-bound estimation) - (Kołakowski & Kotełko, 2004).

The failure path is derived from the yield-line analysis, based on the theoretical model of plastic mechanism shown in Fig. 2. The energy method is applied in order to calculate an actual bending moment at the global plastic hinge [7].

The out-put quantities obtained from the code "NOSNOSC" are the lower-bound (LBELC) and upper-bound (UBELC) maximum bending moments of the girder.

1. LOAD-CAPACITY SENSITIVITY ANALYSIS

The sensitivity analysis was performed to determine the sensitivity of LBELC and UBELC with respect to the variance of several random input quantities i.e. dimensions of the girder and material parameters. The initial geometrical imperfections were not taken into account. The input random quantities are indicated in Table 1. The material parameters and their standard deviations are taken from publication by Kala et al. [9]. The methodology based on the Monte Carlo method [2, 1] is applied in the analysis.

The analysis consists in the polynomial decomposition, carried out using the multi-dimensional linear regression. The calculations were performed using the program Minitab 15 [10]. Knowing the distribution of input variables one is able, using the Monte-Carlo method, to generate adequate data files (Mikulski [2]). After generating the data files the values of out-put variables have to be determined. Then, after generating in-out files one can derive equations of regression. Afterwards, performing the analysis of variance of particular variables multiplied by direction coefficients of regression one can determine the significance of each variable and its contribution in the final value of a predicted quantity. Within the framework of each run of the Monte Carlo method, the LBELC and UBELC were found, using the code 'NOSNOSC'. For each calculation case 100 iterations were conducted. After performing iterations, the procedure of multi-dimensional linear regression was carried out.

Table 1. Input random quantities

Random quantity	Unit	Mean value	Standard deviation	Type of distribution
Width *A*	m	0.1	0.0005	Normal (Gauss)
Hight *B*	m	0.1	0.0005	Normal (Gauss)
Length *L*	m	0.1	0.0005	Normal (Gauss)
Wall thickness *H* ($H_A = H_B$)	m	0.001	1, 2, 7, 8, 9, 10, 15 [%]*	Normal (Gauss)
Young's modulus E	GPa	210	12.6	Normal (Gauss)
Poission's ratio ν	-	0.27	0.03	Normal (Gauss)
Yield stress R_e	MPa	284.5	21.5, (real), 22.5, 23.5**	Normal (Gauss)

(*) wall thickness sensitivity analysis (variance of wall thickness), (**) yield stress sensitivity analysis (variance of yield stress)

3.1. WALL THICKNESS SENSITIVITY

The sensitivity analysis was performed in two steps: in the first one the analysis was carried out in terms of the variance of wall thickness H, with the standard deviations shown in Table 1. The results of the regression analysis and sensitivity analysis in terms of wall thickness variance are discussed in details in [11, 12]. Fig. 2 shows the results of the sensitivity analysis of UBELC in terms of the variance of wall thickness *H* represented by pie charts. The corresponding diagram is shown in Fig. 8.

On the basis of the results of wall thickness standard deviation change (thickness tolerance) one can conclude, that the UBELC induction is generated mainly by the yield stress (60%), when the tolerance of thickness is restrictive (here 1mm ±0.01). Increment of the thickness tolerance changes this structure [11]. In the next steps of the analysis the magnitude of thickness standard deviation was checked using the test ANOVA. It allowed one to conclude, that the deviation of thickness does not generate any distinction of samples (based on means of difference between UBELC and LBELC) as a different materials on requested (as a standard 95%) confidence level (Fig.3).

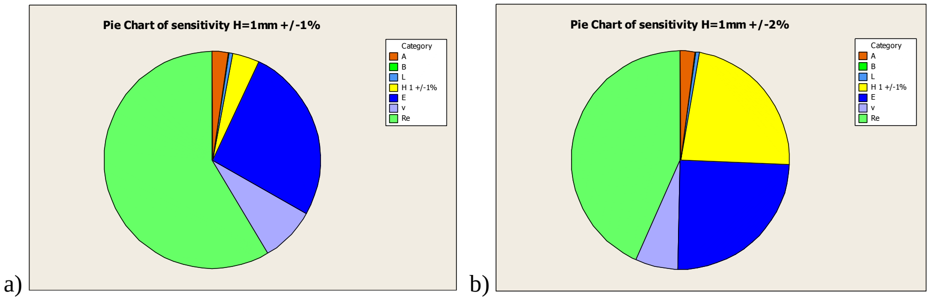
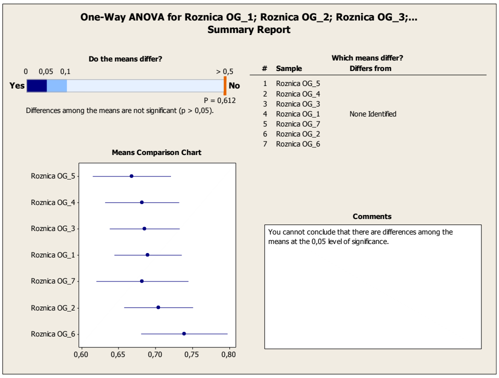


Fig. 2. Exemplary pie charts (UBELC sensitivity analysis – wall thickness variance): a) 1 % standard deviation, b) 2 % standard deviation


One-Way ANOVA for Roznica OG_1; Roznica OG_2; Roznica OG_3;...
Summary Report

Do the means differ?

Differences among the means are not significant (p > 0,05).

Which means differ?

#	Sample	Differs from
1	Roznica OG_5	
2	Roznica OG_4	
3	Roznica OG_3	
4	Roznica OG_1	None Identified
5	Roznica OG_7	
6	Roznica OG_2	
7	Roznica OG_6	

Comments

You cannot conclude that there are differences among the means at the 0,05 level of significance.

Fig. 3. ANOVA test results for of UBELC (OG) and LBELC (OD) difference analysis for wall thickness variance

3.2. YIELD-STRESS SENSITIVITY

In the second step the analysis was carried out in terms of the variance of the yield stress Re, with the standard deviations shown in Table 1. The results of the regression analysis and sensitivity analysis in terms of the yield stress variance are shown in pie charts in Fig. 4. The corresponding diagram is shown in Fig.7. The analysis indicates that the larger is the standard deviation of Re, the higher is an influence of this quantity on UBELC. It varies linearly from 47.5% up to about 70%. The increase of the Re influence is associated with a decrease of the influence of other material out-put quantities: Young modulus and Poisson ratio (both of linear character). Influence of geometrical parameters (dimensions) is approximately constant.

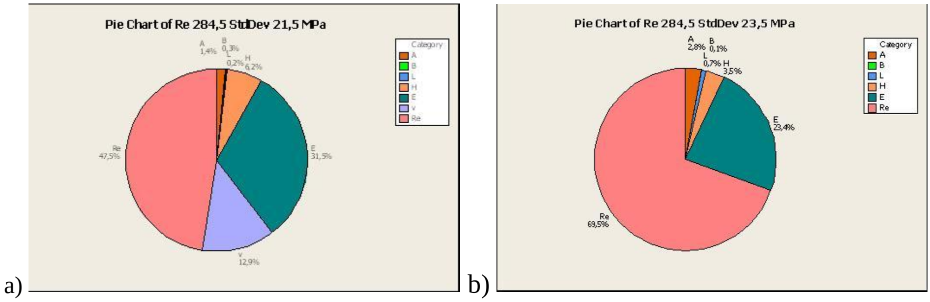


Fig. 4. Exemplary pie charts (UBELC sensitivity analysis – yield stress variance): a) 21.5 MPa standard deviation, b) 23.5 MPa standard deviation

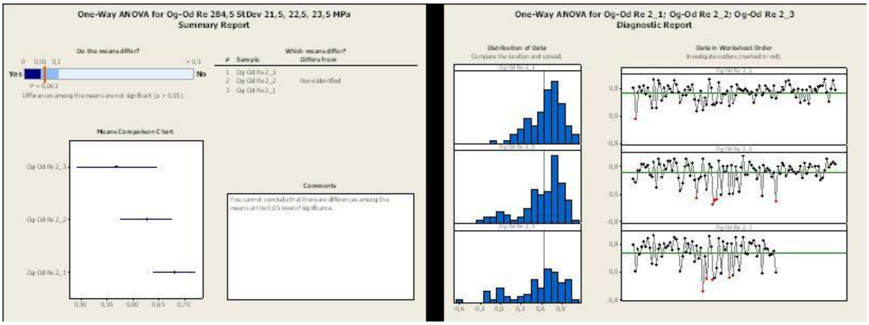


Fig. 5. ANOVA test results of UBELC (OG) and LBELC (OD) difference analysis for Re variance: Re = 284.5 MPa, standard deviations 21.5, 22.5 i 23.5 MPa

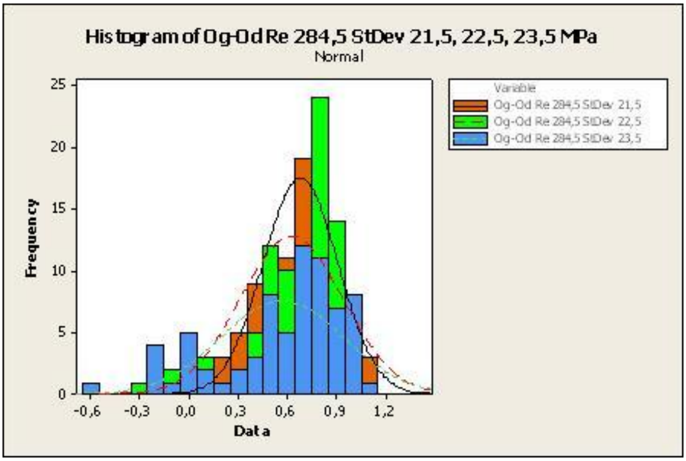


Fig. 6. Exemplary histograms of UBELC (og) and LBELC (od) differences for the yield stress variance

The ANOVA test shows that at the standard confidence level (95%) mean values of LBELC and UBELC for each tested class (for subsequent standard deviations) are not the same (Fig.5). The similar tests for the wall thickness variance show, that the samples are the same [11] – as it was mentioned above.

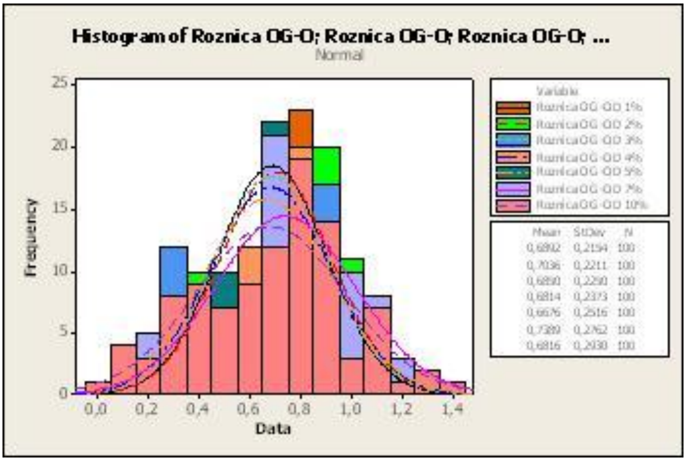


Fig. 7. Exemplary histograms UBELC (og) and LBELC (od) differences for wall thickness variance

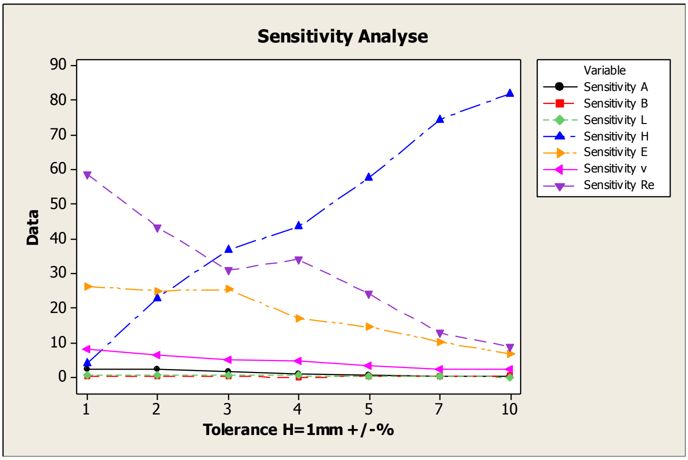


Fig. 8. Results of UBELC sensitivity analysis – wall thickness variance

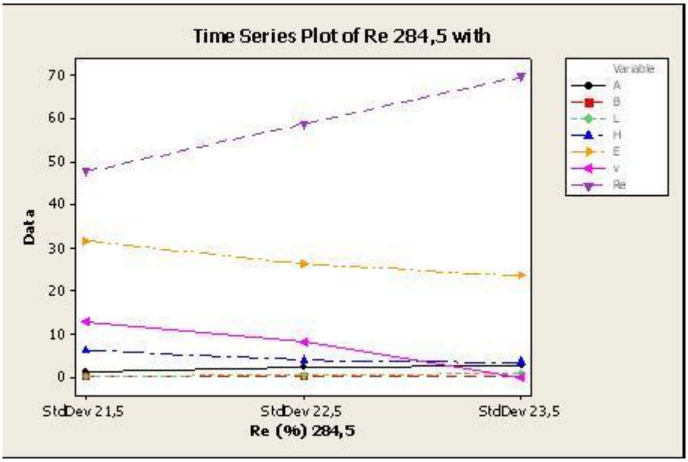


Fig. 9. Results of UBELC sensitivity analysis – yield stress variance

4. FINAL REMARKS

The regression analysis confirms that a statistically significant empirical multi-dimensional model exists for the lower-bound estimation (LBELC) in terms of considered input random quantities. However, its efficiency is weak. Accuracy of the model based on the least squares method was connected with 25% error.

On the contrary, the efficiency of analogous empirical model for the upper-bound estimation - (UBELC) is high (above 98%). It concerns both the yield stress and wall thickness variance.

The increase of the yield stress standard deviation induces an increase of the differences of UBELC and LBELC (see the “shift” of the histogram in Fig. 6). Also a “shift” of means of those differences is noticed. It is not observed for the wall thickness variance (Fig. 7). The distribution of UBELC-LBELC differences is not normal for the 95 % confidence level (Fig.6) in the case of the yield stress variance, while for the wall thickness variance at the same confidence level this distribution is normal (Fig.7).

Results of the performed analysis show, how a quality of structural steel affects the load-carrying capacity of the girder. The upper-bound estimation (UBELC) induction is generated mainly by the yield stress. Activity of the yield stress is reduced with the tolerance change of wall thickness, but is elevated by the increase of the yield stress standard deviation itself.

The results presented in the paper are based on linear models of analysis, without interactions. The relations between indicators of UBELC and LBELC were checked with use of non-linear models. However, the improvement of the estimation efficiency of those models was about 4%.

Results based on the algorithm, which applies the yield-line approach (plastic mechanism approach) for the approximate determination of the upper-bound load-carrying capacity of TWS, indicate that this approach is useful for the sensitivity analysis. The empirical multi-dimensional model used in the presented sensitivity analysis based on this approach is more efficient than the model based on the lower-bound estimation.

The analysis performed allows one to conclude, that a “redundancy” of the load capacity of the girder (the post-elastic capacity) is more sensitive to the yield stress deviation than to the wall thickness deviation.

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