LOCAL BUCKLING AND LOAD CARRYING CAPACITY OF THIN-WALLED MULTICELL BEAMS SUBJECTED TO BENDING

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The paper deals with local buckling and load carrying capacity of thin-walled multi-cell beam/girder subjected to bending. There are some design hints according the cross-section stiffening method giving significant increase of local buckling stress value what is proved by numerical computations. The results of performed analysis are presented in tables and graphs. The Finite Element software - ANSYS was applied in modelling and calculations.

1. INTRODUCTION

Thin-walled box beams are frequently used in civil engineering and mechanical structures as large load carrying members. Their cross section takes the form of a plate girder, often made of segments which are jointed together on the field of location.

At the beginning we consider the box beam it is one cell girder of rectangular cross-section. Its dimensions are given in Fig. 1. The considered beam/girder is subjected to simple, pure bending with the plane of bending overlapping the vertical plane of beam symmetry. It is assumed that thickness of all beam walls is constant and equal to t_0. Further, it is assumed that:

– the relationship of wall thickness t_0 to the width of beam flange b is: $b/t_0 \geq 200$,
– the relationship of total beam height h to its width is $h/b > 2$,
– beam is made of steel with Young's modulus $E = 2\times10^5$ MPa and Poisson ratio $\nu = 0.3$.

Due to bending one flange is compressed whereas the second is stretched with the stresses of the same absolute magnitude. The webs (side walls) are subjected to in-plane bending with linearly varying stresses. According to the buckling analysis of plates subjected to pure in-plane bending [4] they undergo local buckling. For the given above geometrical and material parameters, the compressed flange of the girder as well as the webs buckle with critical stress value lower than 20 MPa. Therefore, the load carrying capacity of such beam/girder is comparatively low and material properties are not fully exploited. One of methods which lead to increase the capacity of such girders is to reinforce a compressed flange and a compressed part of webs by adding longitudinal ribs of appropriate bending stiffness. The largest number of works dealing with the rib stiffening of thin-walled girders were published in the seventieth of last century - for example [11],[10],[3]. However, a similar effect as by ribbing can be achieved by introducing multi-cell design of a bent girder.

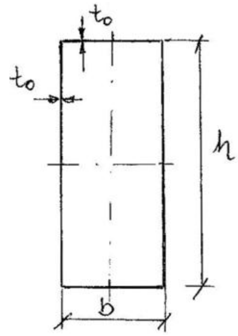

Fig. 1. One cell - box girder dimensions

The research of buckling and post-buckling states of thin-walled multi-cell compressed columns and beams under bending has been began in the first years of current century. The results of theoretical and numerical analyses as well as experimental investigations concerning local buckling and load carrying capacity of multi-cell columns were published in many works (for example [8],[6],[7],[5]) and presented at few conferences. These results were obtained within the framework of the research project No 4T07A02829.

The optimization with respect to local buckling and strength of thin-walled box girder (one cell) subjected to bending was performed in [2]. Świniarski in his PhD Thesis [12], optimized the thin-walled beams of one-, two- and three-cells rectangular cross-section subjected to bending.

The results of the experimental investigations and numerical computations of local buckling and load carrying capacity of thin-walled beam of multi-cell cross section were given in paper [9]. The local buckling analysis of multi-cell beams of opened cross-sections, subjected to pure bending was performed by Barański in his Master Thesis [1].

According to this short review of published works it is noticeable the lack of buckling and load carrying capacity analysis concerned thin-walled beam/girders with the number of cells greater than three, subjected to simple bending.

2. PROBLEM OF INTEREST. BASIC ASSUMPTIONS

Present work deals with the local buckling and load carrying capacity analysis of thin-walled multi-cell beam/girders subjected to pure bending. The cross-section is a one axis symmetrical rectangular profile. The shape of the cross-section beam profile is given in Fig. 2. Used further in the analysis the cell number parameter *n* is equal to the number of cells adjacent to the compressed flange of bent beam.

The numerical analysis will concern beams of total height $h = 2.5 \cdot b$ and the number of cell parameter will be assumed equal to $n = 2, 3, 4\ldots\ 8$. The number of cells having dimensions $b_n \times b_n$, where $b_n = b/n$; adjacent to each web in its upper, compressed part was assumed equal to the number of cells adjacent to the compressed flange of the beam. In the lower part of the web subjected to tension there are no cells (Fig. 2).

The global number N of all $b_n \times b_n$ cells can be calculated from the formula:

$$N = n + 2(n-1) = 3n - 2 \quad \text{for} \qquad n = 2,3,4\ldots$$

Beam with n the cell number parameter equal to 1 ($n = 1$) is not considered. The critical load of such beam is lower than a simple rectangular box girder (it is rectangular cross-section with only one cell as in Fig. 1).

In the case of analysed local buckling and load carrying capacity, for easy comparison of critical stress values for beams of different number of cells (it means different value of n cell number parameter), in subsequent considerations it was assumed that all analysed beams:

- are made of the same isotropic material (equal E Young's modulus and ν Poisson ratio),
- their cross-section area A is equal and identical to the cross-section area of a box girder of dimensions $b \times 2.5b$ and walls thickness t_0, it is $A = 7bt_0$,
- for a beam with a cell number parameter n, a thickness t_n (1) of all beam walls is the same,
- a bending moment acts in the plane of symmetry of beam cross section.

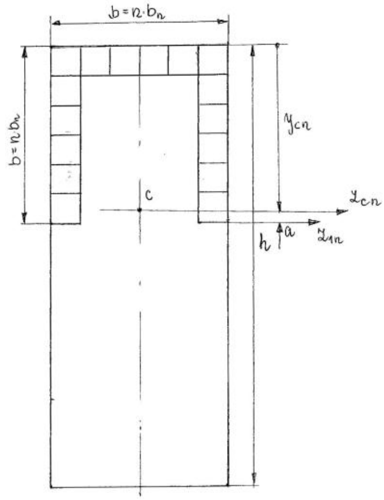

Fig. 2. Multi-cell beam cross section with dimensions description

Equating the cross section area $A = 7bt_0$ of a box girder with the area of multi-cell beam defined by the cell number parameter n, leads to the formula for wall thickness of multi-cell beam:

$$t_n = \frac{7n}{13n-5} t_0 \tag{1}$$

Then, the formula for cross section area of a beam which cell number parameter is *n* can be written as follows:

$$A_n = \frac{13n-5}{n} b t_n \tag{2}$$

From the geometrical relationships the formula for coordinate y_{cn} of the multi-cell beam cross-section centre of gravity can be derived in the form of:

$$y_{cn} = \frac{n}{13n-5}\left[0.75 + \frac{1}{2n^2}\left(3n - 3 + 4\sum_{i=2}^{n} i\right)\right] b \tag{3}$$

what gives the location of beam neutral axis as well. It is worth mentioning that due to location of $b_n \times b_n$ cells only in the upper part of a beam cross-section, it is in its compressed region, the neutral axis position is always above the lowest cell wall ($y_{cn} < b$, see Fig. 2). Knowing a neutral axis location, the formula for beam cross-section second moment of inertia, with respect to this neutral axis is as follows:

$$J_{zcn} = J_{z1n} - A \cdot a^2 \tag{4}$$

where it is introduced:

$$a = b - y_{cn} \tag{5}$$

$$J_{zcn} = \frac{t_n b^3}{3}\left\{ n + 1 - (n-3)\left(\frac{n-1}{n}\right)^3 + 6{,}75 + 3\left[\frac{13}{4} + \left(\frac{n-1}{n}\right)^2\right] + \frac{6}{n}\sum_{i=1}^{n-2}\left(\frac{i}{n}\right)^2 \right\} \tag{6}$$

In Table 1 some exemplary values of $t_n(t_0)$, $A_n(b,t_0)$, $y_{cn}(b)$ and $J_{zcn}(b,t_0)$ for cell number parameter $n = 3,4,..,8$ are collected.

Table 1. Geometrical parameters of multi-cell beams

n	3	4	5	6	7	8
t_n	$0.6176t_0$	$0.5957t_0$	$0.5833t_0$	$0.5753t_0$	$0.5698t_0$	$0.5656t_0$
A_n	$11.333bt_3$	$11.750bt_4$	$12.0bt_5$	$12.166bt_6$	$12.286bt_7$	$12.375bt_8$
Y_{cn}	$0.9877b$	$0.9495b$	$0.9258b$	$0.9098b$	$0.8983b$	$0.8895b$
J_{zcn}	$4.5405t_0b^3$	$4.7066t_0b^3$	$4.6796t_0b^3$	$4.7329t_0b^3$	$4.7763t_0b^3$	$4.8102t_0b^3$

3. NUMERICAL COMPUTATIONS

For greater number of cells the interaction of different modes of buckling may take place what is difficult to predict in theoretical considerations. Among the plate elements (cell walls) there are members subjected to uniform compression and/or in-plane bending which joining edges influence their resistance to local buckling. This interaction causes great analytical difficulties. Thus establishing direct analytical formula for critical load for multi-cell beam/girder subjected to pure bending is not possible. However, it is possible to approximately evaluate the bending stress magnitude equal to local buckling stress for most upper cell walls with the assumption that their longitudinal, common edges are simply supported [1],[3]. In practice only Finite Element Method employment gives the completely solutions for complex structures as considered multi-cell beam/girder is. However, the analytical approach is competitive in terms of 'computation economy'.

The numerical calculations of analysed multi-cell bent beam were performed with the application of FEM and ANSYS software package. It was built a plate parametric model of a multi-cell beam, where for meshing the shell elements were applied. Despite the vertical symmetry of the multi-cell beam a full three dimensional model was created. The boundary conditions were defined in terms of nodes displacements to represent the simple support of a beam and by coupling of rotational degrees of freedom of nodes in outside cross-sections. It was to fulfil the classical beam theory assumption of plane cross-sections shape. According to material properties the previously given isotropic data were applied for computations and additionally its characteristic was defined as bilinear with E_t = 2000 MPa and yield limit equal to $\sigma_{pl} = 235$ MPa [5]. The HMH equivalent stress distribution in the discretised model of multi-cell beam under pure bending in the post-buckling state is presented in Fig. 3.

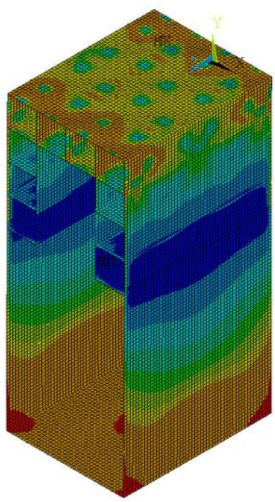

Fig. 3. Equivalent stress distribution in discrete model of multi-cell beam

At this stage of analysis the numerical computations were focused on local buckling loads evaluation for different cell number (parameter *n* value) in the compressed region of beam cross-section. Further, in the next step, the relationships between stress and strains in the post-buckling range were analysed. Their graphical representation - in the case of bending, were presented in the form of a graphs - normalized critical bending moment as a function of angle of rotation (or slope) (as an example Fig. 5).

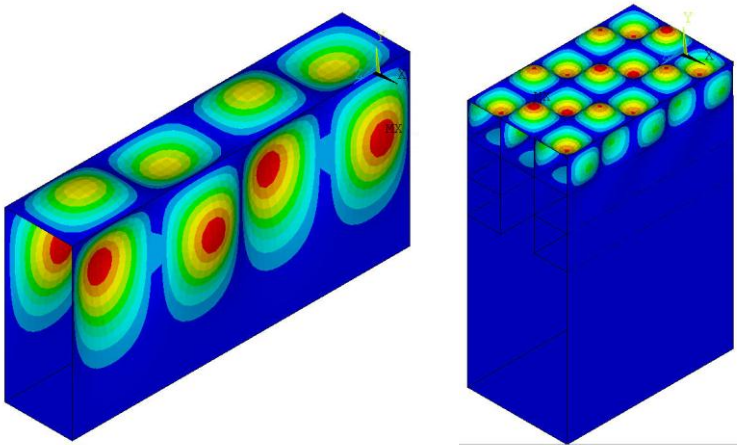

Fig. 4. Buckling modes in box girder and multi-cell beam of *n* = 3

5. RESULTS

The critical loads for bent multi-cell beams under consideration were determined in eigen-buckling linear analysis (Fig. 4). There were multi-cell beam segments of length $5\times b$ modelled to minimize the influence of boundary conditions on the local buckling mode. The reference box girder (one cell beam) was of width $b=300$ mm and wall thickness $t_0=1.5$ mm. The other analyzed multi-cell beams were of dimensions calculated with application formula (1) for walls thickness. The results of these computations are summed up in Table 2.

Table 2. Critical stress value for multi-cell beams

n	1	2	3	4	5	6	7	8
σ_{crn}	15.40	40.79	76.08	122.88	180.85	249.81	329.68	420.31
$\sigma_{crn}/\sigma_{cr0}$	1	2.646	4.937	7.974	11.736	16.211	21.394	27.275

As it’s clearly seen from the bottom row of Table 2, the basic advantage of multi-cell application is a significant increase of critical stress value. For the same amount of material (equal cross-section area) the local buckling phenomena will occur for many times greater load than in a box girder. Similar conclusion is valid for nonlinear buckling and post-buckling analysis when the ultimate load is determined. In practice the number of up to five cells ($n = 5$) is enough to achieve required results - it is increase the local buckling load, since for greater number of cells the local buckling stress is greater than yield limit of construction steel.

For assumed ratio of cross-section dimensions the highest absolute stress value occurs in the lowest flange of the beam. In the case of equal material properties for positive and negative direct stress (steel was assumed in calculations) under bending load the yielding is first single plate wall. In the upper flange and neighbouring cells the yield stress occur later depend on the hardening slope of material characteristic. This phenomenon causes numerical difficulties in nonlinear post-buckling analysis. Excessive distortions of elements which occur above yield limit make the convergence of analysis even impossible. An example of successful nonlinear computations performed for multi-cell beam with cell parameter $n = 3$ is presented in Fig. 5. Its ultimate load is almost 80 % greater than local buckling load. The yield stress in upper part cells is achieved shortly before the ultimate load therefore the beam behaviour is similar to a beam of fully linear material properties.

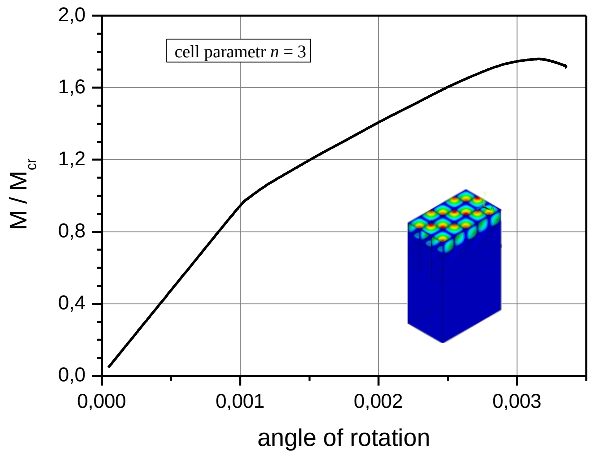

Fig. 5. Nonlinear analysis - n = 3 three cell beam

6. CONCLUSIONS

The results of carried out numerical analysis for multi-cell beams/girders of equal cross-section area, made of the same material and exactly in the same way supported allow one to state the following conclusions:

- local buckling stress of walls increases with the increase of parameter ‘n’ - the number of cells adjacent to beam upper flange,
- the effect of local buckling stress increase is significant - e.g., for cell number parameter $n = 3$ the growth is of five times,
- for some value of parameter n the beam cross-section can undergo yielding before local buckling of it walls occurs,

– for multiple number of adjacent cells the thickness of single wall becomes small and such a design loses practical meaning.

REFERENCES

[1] Barański T., Buckling of multi-cell beams of opened cross section subjected to bending, Master Thesis, supervisor R.J. Mania, Lodz University of Technology, 2010.

[2] Gałkiewicz T., Królak M., Kubiak T., Optimization of thin-walled beams subjected to bending in respect of local stability and strength, Mechanics and Mechanical Engineering, Vol. 11, 2007, pp. 37-48.

[3] Gałkiewicz T., The aproximated stability analysis of ribbed box girder walls subjected to combined bending with tension or compression, Proc. of III Stability of Struc. Symp., Łódź, 1979.

[4] Królak M. (Ed), Post-buckling state and the limit load- carrying capacity of thin-walled girders with flat walls, PWN, Warszawa - Łódź, 1990 (in Polish).

[5] Królak M., Kowal- Michalska K., Mania R.J, Świniarski J., Stability and load carrying capacity of multi- celled thin- walled columns of rectangular cross-section, J. of Theoretical and App. Mechanics, 47, 2, 2009, pp. 435-456.

[6] Królak M., Kowal-Michalska K., Mania R.J, Świniarski J., Experimental tests of stability and load carrying capacity of compressed thin-walled multi-cell columns of triangular cross- section, Elsevier, Thin-Walled Structures, 45, 2007, pp.883-887.

[7] Królak M., Kowal-Michalska K., Mania R.J, Świniarski J., The experimental tests of buckling and load carrying capacity of thin-walled multi-cell column models subjected to uniform compression, Politechnika Warszawska, Research Works, Mechanics, Experimental Investigations in Solid Mechanics, Warsaw, 2007, pp. 73-78.

[8] Królak M., Kowal-Michalska K., Stability and load carrying capacity of multi-cell columns subjected to compression, Proc. of 4-th Int. Confer. on Coupled Instabilities in Metal Structures, Rome-Italy, 2004, pp.213-222.

[9] Królak M., Świniarski J., Mania R.J, Kowal-Michalska K., Experimental and numerical study of stability and ultimate load of thin-walled multi-cell beam, Proc. IV Conf. Thin-walled Vessels and Beams, Poznań-Kołobrzeg, 2007, pp 57-64.

[10] Maquoi R., Massonet Ch., Non linear theory of post-buckling resistance of large stiffened box girders, IABSE Publications, Vol. 31-11, 1971, pp. 91-140.

[11] Massonet Ch., Maquoi R., New theory and tests on the ultimate strength of stiffened box girders, Proc. Int. Confer on Steel Box Girders Bridges, The Inst. of Civil Eng., Londres, 1973.

[12] Świniarski J., Optimization of rectangular multi-cell cross section beam in respect of buckling and load carrying capacity, PhD Thesis, Lodz University of Technology, 2004.