Spectral analysis of granular material reaction to long-term weak dynamic effect

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Abstract. The paper demonstrates the similarity of alternating reaction of rocks to explosions to the response of a granular material to repetitive (up to 330 thousands) light shocks ($3.85 \cdot 10^{-3}$ J, frequency 1.33 Hz). The cause of both reactions is multiple forms of equilibrium of geomaterials. It has been stated that pressure of the sensor in a granular material subject to long-term weak effects doesn't relax to the hydrostatic one, but oscillates about it with the amplitude about its value. The spectral density of this process obeys the power dependence on the frequency; the exponent is typical for black noise.

Introduction

Papers [1, 2] describe the phenomenon of alternating reaction of rock mass to explosion effects. The point of it is that «the process of a natural geophysical material deformation is a heterogeneous one: the deformations localize on the surfaces and in zones with weakening strength, and the structural elements can move independently. Here, the spatial motion of the elements can be alternating» [1]. The change in density of various zones of rock mass is also alternating. For instance, the number of explosions causes a relative rock compaction on particular areas on the rockhole, whereas, the following series of explosions results in relative decompaction of these areas [2].

In generally, the nature of this phenomenon is quite clear. It is caused by multiple forms of equilibrium of geophysical materials. The same edge conditions can be in line with unlimited forms of equilibrium. Therefore, generally speaking, the rock mass has rather good memory for all external effects and all its previous states.

For this reason on can expect that the phenomenon of alternating reaction is also possible in conditions of relatively weak but long-term and systematical effects on rock mass. As the source of these effects one can consider the same explosions made far from the area under consideration, tidal deformations etc.

In [1, 2] explosion effect on the rock mass was considered with the force up to 150 kt (in trinitrotoluene equivalent). Power explosions are unique ones and full-scale research is quite labor intensive even for conventional explosions, and, moreover, not complete. It is interesting to search for models which could be fully investigated in laboratory conditions. It is well known that in most cases the behavior of a rock mass can be modeled with granular materials. This phenomenon can also be used for the issue under consideration in this paper. Actually, mechanical behavior of granular materials is determined by two basic factors: a great quantity of interacting particles and the laws of particles interaction. The major ones are the laws of friction, sliding and rolling of particles. The conditions of dry friction are brought to two-side inequality of forces arising at the contacts. As the consequence, one and the same package of particles can have practically unlimited forms of equilibrium. Here, the quantity of the packages of the same particles is also practically unlimited. It implies that original sample of material has a significant capacity of memory responsible for history of its formation. In particular, on the basis of information on the method the sample was formed and preliminary deformed we can obtain an extremely wide range of various stress states of the sample.

Thus, the first condition – probable and unlimited forms of equilibrium – is met here. The second condition concerns the mosaic structure of the material. It is also met in a granular material to a certain extent. In-depth research demonstrates that the particles of granular material are clustered when deforming (actually, in blocks) [3]. Therefore, principal deformations occur on the borders of clusters. Here, the deformation of clusters is relatively small. The clusters for particles with plane faces can be seen perfectly. To illustrate one can make an experiment involving the flattering of a bundle of hexahedral pencils coupled outside by elastic [4].

Here let us discuss the external effects on the material, which can cause the evolution of its internal self-balancing stresses. In this paper we merely consider weak but long-term effects on the material. The influence of rockholes and other cavities on the rock mass is not studied. We confine ourselves to the simplest situation: the sample of granular material is in a cylindrical container. The container is hit slightly (up to 800 thousands for 7 days of continuous experiment). Inside the sample there is a pressure sensor, which records the evolution of stress state. The problem statement is undoubtedly simple and rather trivial. However, its result is quite unexpected and important – in the course of evolution changing stresses could amount to 20 – 150% of their initial values. Let us describe the results. Here, we pay attention to one typical experiment only, which lasted 68 hours ($3.3 \cdot 10^5$ hits were made).

Methods of experiment

The model of experimental facility is depicted in Fig. 1. A 220 mm high cylindrical polystyrene container *1* 100 mm in diameter and with 1.5 mm thick wall was topfull of bank sand *2*; dimensions of particles were 0.5 – 0.63 mm. To measure the vertical pressure two similar $12 \times 25 \times 3$ mm sensors *3* were placed into sand. The first one was placed at the depth of 155 mm, the second one – 115 mm. The container was fixed on the inflexible base. An impacting facility *4* was placed next to the container to effect on the wall. The head of the striker was made of rubber. The distance from the bottom of the container to the impact point was 60 mm.

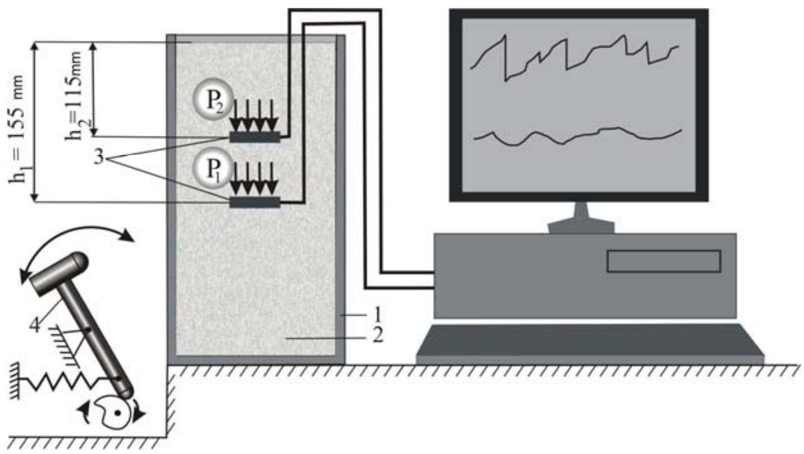

Figure. 1. The model of experimental facility:
1 – cylindrical container; 2 – bank sand; 3 – stress gauges; 4 – impacting facility.

A ready for test experimental facility was placed into the thermostat to exclude the influence of temperature. After the facilities had been warmed up and the indicated values of sensors had been stabilized, an impacting facility was switched on. The energy of impacts E was constant and amounted to $3.85 \cdot 10^{-3}$ J all over the experiment, frequency was - 80 hits a minute. The digitization of signals from the sensors lasted 10 s.

The results of experiment

In the course of experiment the dependencies of pressure on the sensors in time $P_i = P_i(t)$ were recorded on the hard disk of computer, where P_i - the value of the sensor *i* at the moment *t*. According to the obtained data graphs of normalized pressure (Fig. 2) $p_i = P_i / \gamma h_i$, depending on the number of hits *n* were made, where γ, h_i – specific weight and head of the material over the sensor *i*. Horizontal lines depict the pressure γh_i of overlying layers of material.

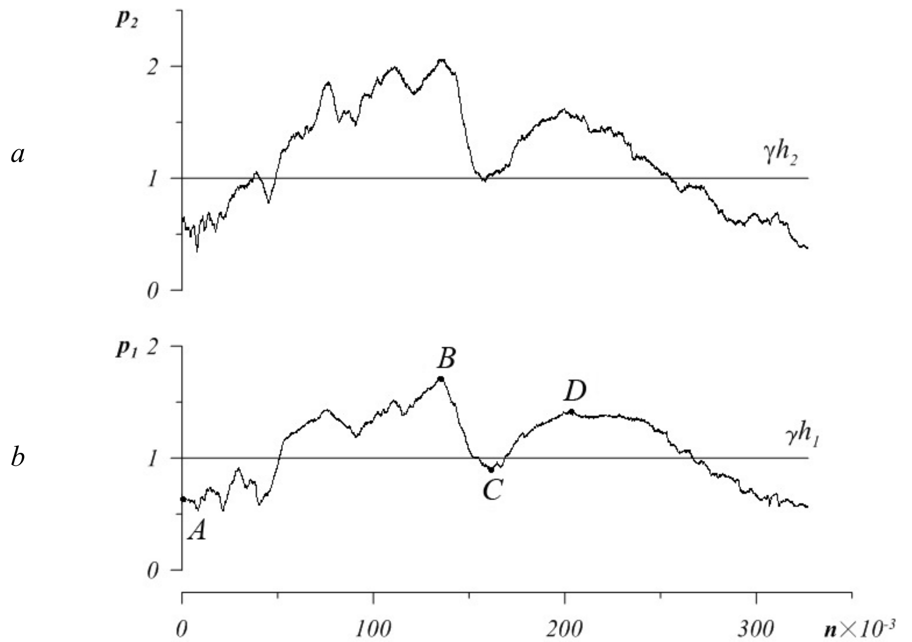

Figure. 2. Graphs of normalized pressure on the sensors: *a* – the second sensor; b – the first one

As we can see, the pressure on the sensors is not constant but varies with the number of hits in a non-monotone way. The pressure increases dropping and surging locally up to 120 thousand of hits (AB segment). It falls sharply on BC segment, then, it grows more smoothly up to the local maximum at D. Then it decreases slightly dropping locally.

For general description of the obtained data series we used the following values: $\overline{p} = \frac{1}{K}\sum_{j=1}^{K} p_j$ - mean pressure on the sensor, *j* – number of reading, p_j - reading *j* on the sensor, K - total readings (series length); $\sigma^2 = \frac{1}{K}\sum_{j=1}^{K}\left(p_j - \overline{p}\right)^2$ - dispersion of the series; $R = p_{\max} - p_{\min}$ - amplitude of the series, $p_{\min}$ и $p_{\max}$ - minimum and maximum pressure recorded by the sensor over the period of experiment. Numerical values of the mentioned parameters are shown in Table 1.

The graphs of pressure demonstrate slow changing elements (trends) and quicker changes against their background. The trends indicate that the series are unsteady at least over the time of the experiment. Let us expand the series into harmonic components by Fourier transform to remove trends. Then, according to Parseval's theorem [5], the mean-square pressure $\frac{1}{K}\sum_{j=1}^{K}\left(p_j\right)^2$ or, in terms of electric engineering, mean power dispersed by the signal p can be expand into the components generated by each harmonic. The mean-square p relative to the mean $\overline{p}$ is the most suitable measure. It simply equals to dispersion or mean power.

$$\sigma^2 = \frac{1}{K}\sum_{j=1}^{K}\left(p_j - \overline{p}\right)^2 = 2\sum_{j=1}^{K/2-1} A_j^2 + A_{K/2}^2 \qquad (1)$$

where A_j - amplitude of *j* harmonic.

Table 1. Major parameters of $P_i = P_i(t)$ series

Series (sensor)	Pressure of the head of material, kPa, γh_i	Mean pressure, $\overline{p}$	Dispersion, σ^2	Minimum pressure, $p_{\min}$	Maximum pressure, $p_{\max}$	Amplitude, R
1	2.26	1.11	0.10	0.58	0.62	1.20
2	1.68	1.21	0.21	0.86	0.86	1.72

Fig. 3 shows calculated according to Fourier transform contributions $2A_j^2$ of low frequencies to the power of signals p and graphs of its accumulation.

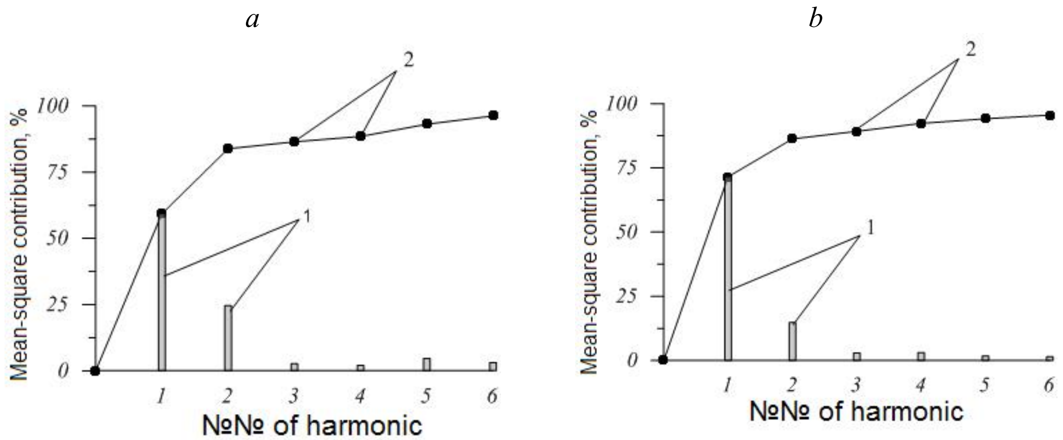

Figure 3. Periodograms (1) and cumulative graphs of mean-square pressure (2):
a – lower sensor; *b* – upper sensor

As we can see the main contribution to the power, which equals to 83.9 and 86.3% for lower and upper sensor, respectively, is made by two first harmonics with periods N_j (*j*=1, 2 – number of harmonic) and 327.1 and 163.6 thousand hits. After deducting mean $\overline{p}_i$ and two most important harmonics from the series $p_i = p_i(n)$ we obtain stationary (or approximately stationary) series $p_i^* = p_i^*(n)$, depicted in Fig. 4.

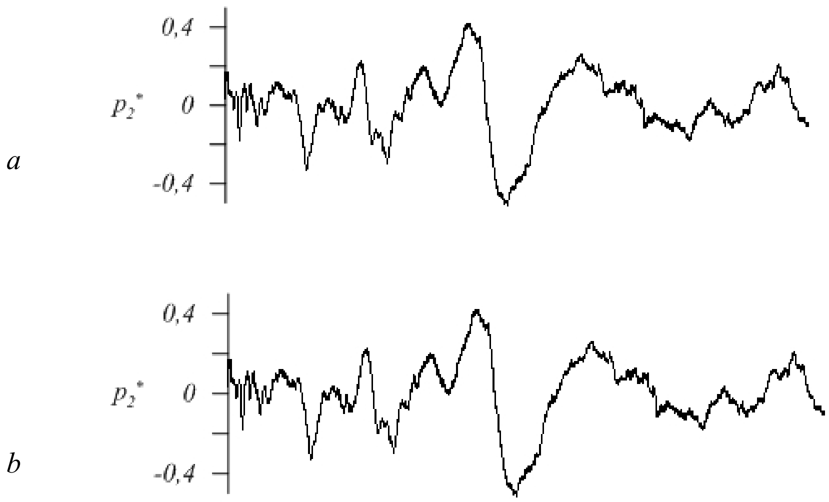

Figure. 4. Graphs of pressure obtained after trend deduction: *a* – the second sensor; b – the first one

Fig. 4 shows that in obtained series p_i^* there are oscillations relatively to zero line. Dispersion and maximum amplitude of deviations p_i^* are provided in Table 2.

Table 2. Characteristics of modified pressure series $p_i^* = p_i^*(n)$

Series (sensor)	Dispersion, σ^2	Minimum pressure, $p^*_{\min}$	Maximum pressure, $p^*_{\max}$	Amplitude, R
1	0.02	-0.33	0.39	0.72
2	0.03	-0.52	0.43	0.95

If we compare Table 1 with Table 2, we can see that dispersion has decreased 5-7 times, and the amplitude – 1.7-2.4 times.

There is the question, whether these series are absolutely random or somehow determined, have inner “memory”? To answer it we make sample autocorrelation functions $r_i = r_i(n)$ of series p_i^* according to equation [6, 7]:

$$r = \frac{1}{K}\sum_{j=1}^{K-u}\frac{(p_j^* - \overline{p})(p_{j+u}^* - \overline{p})}{\sigma^2}, \quad (2)$$

where r - autocorrelation function, u – shift.

The graphs of autocorrelation functions are depicted in Fig. 5, where shifts u measured in thousand of hits are shown on abscissa axis.

Fig. 5 demonstrates that the process under consideration is distinguished by the long-term “memory”. Reducing the correlation function up to zero, i.d. it takes the sample 15.5 thousand hits for "forgetting" of its current state. Furthermore, a periodic component appears when pressure changing. It means that the series are not absolutely random.

Spectral analysis of temporal series is widely applied in research experience [8]. The distribution of spectral density *S* depending on the frequency is rather informative. According to these distributions one can estimate regular characteristics of the processes. In our experiment this analysis was carried out with mathematical system Math Lab. The obtained results are depicted in Fig. 6. A decimal logarithm of spectral density is shown on ordinate axis, and on abscissa axis there is a frequency logarithm *1/N*, where *N* – a period of oscillation measured in number of hits.

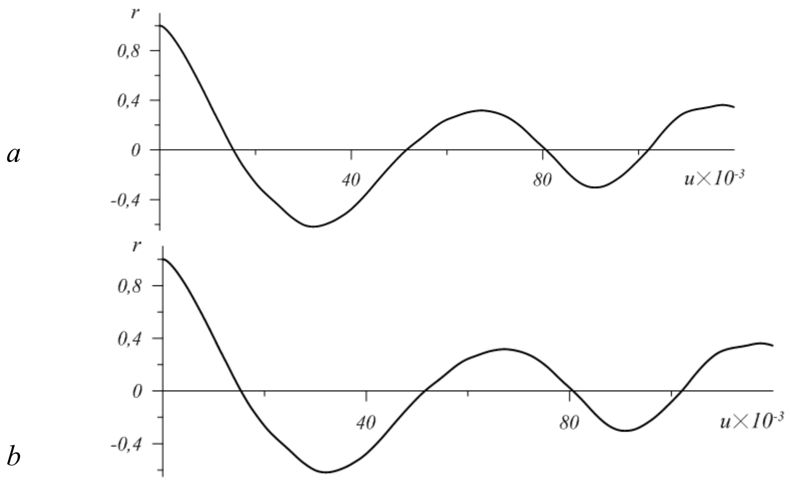

Figure. 5. Autocorrelation functions: *a* – the second sensor; b – the first one

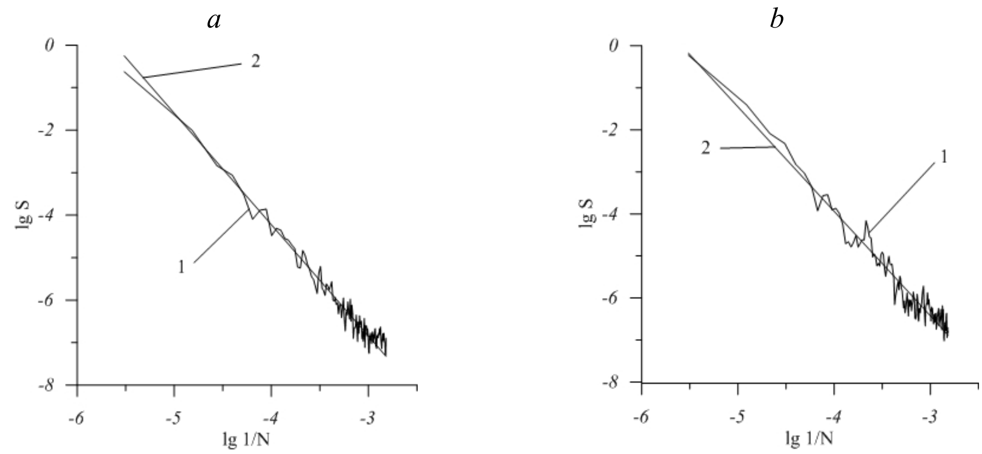

Figure 6. Spectral density of pressure on the lower sensor (*a*) and on the upper one (*b*): 1 – experiment; 2 – approximation

Fig. 6 shows that the logarithmic functions of spectral density in frequency logarithm corresponding with periods $N = 500..320 \cdot 10^3$ hits are well placed on straight lines. That is to say that $S \sim (1/N)^{\alpha}$, i.d. the dependencies are power ones. As far as it is known [9], proportion of spectral density S and $(1/f)^{\alpha}$, where f - frequency, α - exponent, is typical for colored noise. The noises are classified according to α . So, if $\alpha = 2$, the noise is called a brown one and corresponds with the process of Brownian movement. If $\alpha > 2$ the noise is black. In nature we know a lot of processes of black noise. For instance in [10] it is mentioned that «Black noise occurs in long-term cyclical data of observations like river-levels, number of sunspots, thickness of growth rings, and changing prices on stock market. Provided that the system increased over the previous period it will probably grow in the next one». These processes are distinguished by long-term memory; therefore, "long-term correlations of current events and future ones are possible".

In the process under consideration the exponent α is 2.62 for the lower sensor and 2.48 for the upper sensor. Thus, the change in pressure of granular materials caused by long-term weak dynamic effects is the process of black noise.

The sample of granular material consists of a great number of particles, in our case about 10^7. It is quite natural that the state of the system obtained when sample forming is a metastable one. For example, for sand, which had been used in experiments, we obtained initial pressure on the sensor within the range of $(0.4 \div 1.8)\gamma h$ by various formation procedures of packages of particles. Therefore, an initial material isn't in a thermodynamic equilibrium. A somehow energized sample, for example, by light shocks can attain stability in different ways. While a sample is approaching the state of equilibrium dissipative structures can occur [11], including formation and destruction of clusters (zones of compression and decompression). Clusters are unavoidably attend by rearrangement of power chains [12] and repacking of material, the latter is the cause of changing pressure with power range of black noise.

Thus, direct experiments demonstrate alternating reaction of granular material to long-term weak effects. Its comparison with alternating response of rocks to explosion effects [1, 2] brings us to a conclusion that the nature of mentioned phenomena is similar in both cases – unlimited forms of equilibrium are possible in geomaterials.

Conclusions

- The phenomenon of alternating reaction of rocks to explosion effects is similar to the response of granular material to long-term weak effects.
- In both cases the phenomenon is caused by unlimited forms of equilibrium in geomaterials.
- Long-term weak effects result in stress evolution from one state to another. Here, in granular materials stresses can change up to 30 – 150% of initial values.

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