

The dependence of cylindrical resonator natural frequencies on the fluid density

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Abstract. The article examines the dependence of cylindrical resonator natural frequencies (sensitive element) on the density (mass) of different fluids flowing through it. The cylindrical resonators are being widely applied in automatic control systems of technological processes as oscillating transducer density meter. The article presents the experimental results that prove the dependence of natural frequencies and vibration amplitude on the fluid density.

1. Introduction

The natural frequency of mechanical resonator under ideal conditions and without considering vibration damping is defined by the following formula [1, 2]:

$$f = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{K}{m}}, \quad (1)$$

where: K – resonator rigidity;
 m – resonator mass.

Under real conditions, resonator frequency depends on the parameters of a given system such as mass, rigidity, and energy loss. As these parameters are of the system itself, this frequency is termed as natural one [2]. The natural frequency of cylindrical resonator with the fluid flowing through is defined by the formula [3, 4]:

$$f = \frac{\lambda^2}{2 \cdot \pi \cdot l^2} \cdot \sqrt{\frac{K}{(m_{st} + m_m)}}, \quad (2)$$

where: λ – root of frequency equation of cylinder bending vibration, which is defined depending on the restraint type [1, 3];

l – equivalent length of resonator branch;

m_{st} – mass per unit length of resonator walls;

m_m – mass per unit length of the medium inside resonator.

The dependence of resonator vibration frequency that has close values to natural frequencies on the fluid density is widely applied, for example, in density sensors [4–6]. Based on the above equations and experimental measurements, it is possible to create a calibration curve for density sensor. For example, this experiment can be carried out using sensor resonance frequency in air and distilled water [4, 5, 7].

Natural frequency of resonator depends on its parameters, precisely, its shape, size, mass, and elasticity of the material it is made of. An illustration for oscillating element is given in figure 1.



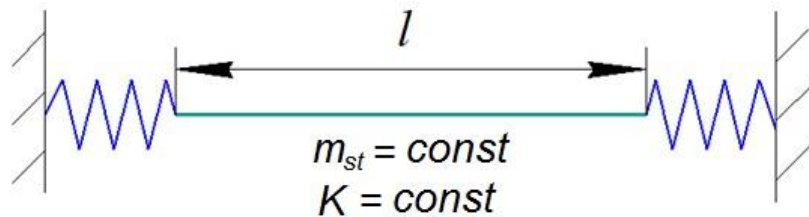


Figure 1. Resonator oscillating element.

To exclude the effect of temperature on resonant frequency calculation, ambient temperature remained constant throughout the course of the experiment. All the experiments were carried out at atmospheric pressure.

Table 1. Oscillating element design parameters.

Tube length, mm	External diameter, mm	Wall thickness, mm	Material	Bellows
1000	32	2.5	precision steel 36NHTYU	38-4-0.21x3 GOST R 55019-2012

The value of resonant vibration frequency of the oscillating element directly depends on the mass of the system itself. It should be emphasized that the design parameters of cylindrical resonator remained constant (table1) during the experiments. Thus, the current value of natural frequency altered in response to fluid density change. The present work is aimed at clarifying this dependence.

Based on the number of degrees of freedom, oscillatory systems are divided into two basic types: system with concentrated or lumped parameters and system with distributed parameters. The former presents a set of elements which is usually reduced to one of the basic parameters, i.e. elasticity and time response. These parameters vary independently from each other. Having a single degree of freedom, these systems are characterized by a finite number of natural frequencies. The latter is one in which all the elements have the same elasticity and time response. These two parameters are distributed within the system so that a change in parameter values causes alteration of the other parameter. It is obvious that the quality of mechanical system with distributed parameters is significantly higher. The quality of the system is referred to the rate of mechanical energy dispersion per oscillation period. Therefore, the oscillatory system with distributed parameters guarantees more precise measurements [4].

The fix-ends of the resonator are conditioned by a high rate of mechanical energy dispersion that passes through the substrates to the resonator body [5, 9]. To decrease energy loss, the resonator is fixed by means of bellows (figure 2).

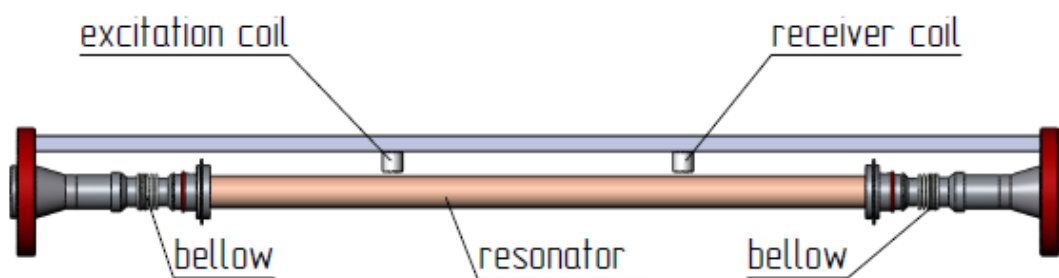


Figure 2. Oscillating system.

2. Materials and methods

Resonator is a cylindrical tube fixed by bellows. It was filled in sequence with fluids (media) of different density (table 2). With an electromagnetic sensor, the resonator oscillated vibrations at a definite frequency. The parameters of the generated signal are listed in table 3.

Table 2. Density of investigated medium.

Medium	Density, kg/m ³
Air in laboratory	1.21
Isopropyl alcohol	774.3
Distilled water	989.7
Solvent 646	831.6
Orange juice	1038.8
Non-freezing solution	957.6

Table 3. Generated signal parameters.

Initial frequency, hertz	End frequency, hertz	Amplitude, V	The number of periods generating the signal at intermediate frequency	The number of points	Signal type
300	2500	10	40	2000	continuous, sinusoidal

The experiment has revealed the difference between the received signals in terms of amplitude and frequency. Figure 3 illustrates the received signals. The presence of several resonance frequencies indicates the fact that the system has more than one degree of freedom.

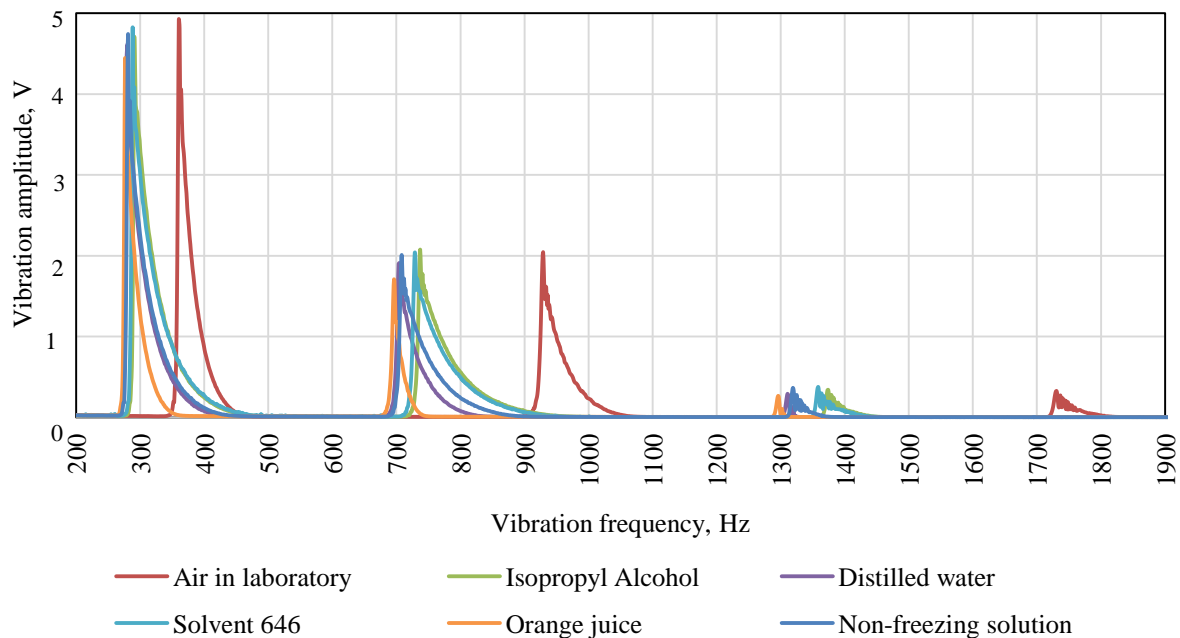


Figure 3. Amplitude-frequency curve of the received signals.

It is possible to specify the value of resonant vibrations for each fluid (medium) and for each harmonic by reducing the frequency range, i.e. generating new signals within narrower frequency range. The parameters of generated signals for determining resonant frequencies for each harmonic are given in table 4.

Table 4. Parameters of the second generated signal.

Generated signal №	Initial frequency, hertz	End frequency, hertz	Amplitude, V	The number of periods generating the signal at intermediate frequency	The number of points	Signal type
1	200	500	10	40	2000	continuous, sinusoidal
2	630	980	10	40	2000	continuous, sinusoidal
3	1200	1750	10	40	2000	continuous, sinusoidal

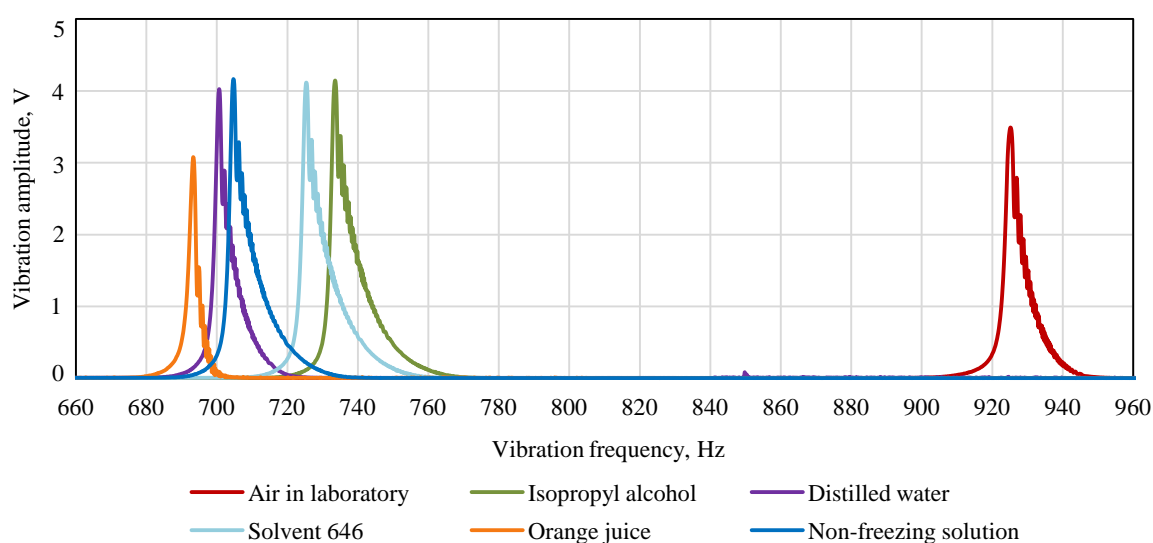
**Figure 4.** Amplitude-frequency curve of the received signal that corresponds to the vibrations within secondary resonant frequency (harmonic).

Figure 4 shows the measurements within the second resonant frequency range. Due to the differences in density and, consequently, in mass, resonant vibration range of each fluid (medium) is different from that of the others. The analysis of the diagrams has revealed that resonant vibration frequency value increases with increasing density. Therefore, air demonstrates the highest value of resonant frequency.

3. Conclusion

To sum up, the conducted experiments enable to define the resonant vibration value of the cylindrical resonator filled with different fluids (media). The resonant frequency values for each investigated fluid (medium) and for each harmonic are listed in table 5.

Table 5. Density of investigated fluids (media).

Investigated fluid (medium)	Resonant frequency for 1 st harmonic	Resonant frequency for 2 nd harmonic	Resonant frequency for 3 rd harmonic
Unit of measurement	Hertz	Hertz	Hertz
Air in laboratory	357.82	923.46	1724.38
Isopropyl alcohol	287.7	732.996	1368.231
Distilled water	275.98	699.157	1306.364
Solvent 646	284.773	723.84	1352.947
Orange juice	273.443	691.95	1291.594
Non-freezing solution	277.389	703.19	1314.228

It is worth noting that the dependence of vibration resonant frequency on the density of fluid inside the resonator is approximately linear (figure 5).

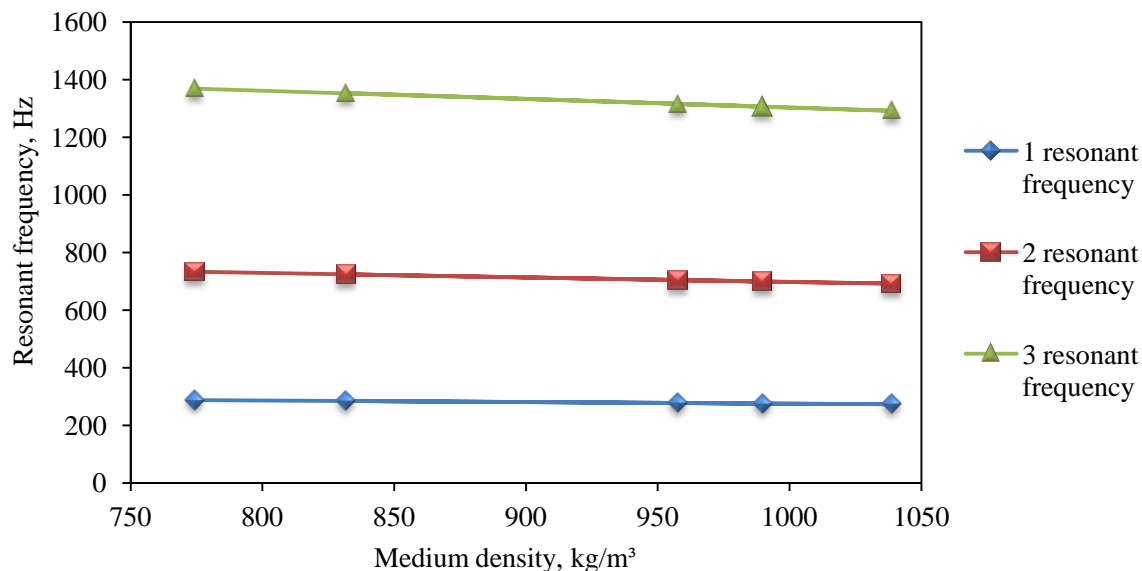


Figure 5. Dependence of vibration resonant frequency on the density of fluid inside the resonator.

Fluid (medium) mass obviously increases with increasing fluid density, which in its turn, contributes to increasing the mass of the whole oscillatory system. The vibration frequency is in inverse proportion to mass. Therefore, density increase results in resonant frequency decrease. Consequently, natural frequencies also decrease.

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