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APPLICATION OF PROBABILITY METHODS TO RESEARCH OF ONE TYPE OF EXOTIC OPTIONS IN DIFFUSION MODEL (*B*,*S*)- OF THE FINANCIAL MARKET

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The decision of optimum hedging problem for the European options of purchase and sale of the exotic type when possible payments on options are limited by the set value is resulted. The formulas defining costs of options and also evolution in time of portfolios and capitals, i. e. hedging strategy and corresponding to them are obtained. Some properties of decisions are investigated.

Introduction

In case of standard options of purchase and sale the payment on option can be big enough which represents essential risk for the investor [1–4]. The first way of this risk restriction consists in option consideration when payment obligation is carried out with probability of smaller unit [5, 6]. The second way consists in problem decision concerning payment functions which provide payments not exceeding the set value and which can be appropriated to the class of so-called exotic options [7]. In the survey work [8], written on the basis of foreign scientific periodical press, wide enough circulation in the financial markets of exotic options and at the same time absence of the developed theory for them is marked. In the given work for the diffusion model [3,4] (B,S)-market of securities and payment functions of the specified type the problem-solving of optimum hedging in case of purchase and sale options with fixed time of execution (options of the European type) is resulted.

1. Problem stating

Consideration of the problem is carried out in standard probability space $(\Omega, F_t, \mathbb{F}=(F_t)_{t\geq0}, \mathbb{P})$ [3]. Risk and non-risk assets are circulating in financial market, current prices of which S_t and B_t during the time interval $t\in[0,T]$ are defined by the equations [1–4]

$$dS_t = S_t(\mu dt + \sigma dW_t), \quad dB_t = rB_t dt, \qquad (1.1)$$

where W_t is the Vinerovskiy process, $\sigma>0$, $r>0$, $S_0>0$, $B_0>0$, their solution looks like

$$S_t(\mu) = S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right\},$$
$$B_t = B_0 \exp\{rt\}. \qquad (1.2)$$

Lets consider that current value of the investor capital X_t is defined in the form of [1–4]

$$X_t = \beta_t B_t + \gamma_t S_t,\ t\in[0,T], \qquad (1.3)$$

where $\pi_t=(\beta_t,\gamma_t)$ is a pair of F_t – measurable processes composing portfolio of the investor securities. The purpose of portfolio management is achievement of $X_T=f_T(S_T)$ equality, where X_T is capital, S_T is price of risk asset during the moment of time T, when the option is presented to execution, $f(\cdot)$ is payment function. In the given work for options of the purchase and sale payment functions accordingly look like [3,4,7,8]

$$f(S_T) = \min\{(S_T - K_1)^+, K_2\}, \qquad (1.4)$$
$$f(S_T) = \min\{(K_1 - S_T)^+, K_2\}, \qquad (1.5)$$

where $K_1>0$ is stipulated during the moment of contract conclusion price of risk asset realization during the moment of execution T, and $K_2>0$ is value limiting payment by option, for sale option $K_2<K_1$. The essence of payment functions (1.4), (1.5) consists in the following. The option of purchase is presented to execution, if $S_T>K_1$. Thus, the option owner receives income equal to S_T-K_1, if $S_T-K_1<K_2$, and equal to K_2 otherwise. Sale option is presented to execution, if $S_T<K_1$. Thus, the option owner receives income equal to K_1-S_T, if $K_1-S_T<K_2$, and equal to K_2 otherwise.

2. Purchase option

Further everywhere E is average of distribution, $N\{a,\sigma^2\}$ is the Gauss' distribution with parameters a and σ^2, $\Phi(y)$ is the Laplas' function, i.e.

$$\Phi(y) = \int_{-\infty}^{y} \phi(x)dx, \quad \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}.$$

Theorem 1. Let $d_1^C(t)$, $d_2^C(t)$, $b_1^C(t)$ and $b_2^C(t)$ be defined by the formulas

$$d_1^C(t)=\frac{\ln(K_1/S_t)-\left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \qquad (2.1)$$

$$d_2^C(t)=\frac{\ln((K_1+K_2)/S_t)-\left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}. \qquad (2.2)$$

$$b_1^C(t)=\frac{\ln(K_1/S_t)-\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \qquad (2.3)$$

$$b_2^C(t)=\frac{\ln((K_1+K_2)/S_t)-\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}. \qquad (2.4)$$

Then rational cost of the purchase option is defined by the formula

$$C_T=S_0[\Phi(b_2^C)-\Phi(b_1^C)]-$$
$$-K_1e^{-rT}[\Phi(b_2^C)-\Phi(b_1^C)]+K_2e^{-rT}\Phi(-d_2^C), \quad (2.5)$$

and portfolio $\pi_t^C=\{\gamma_t^C,\beta_t^C\}$ and capital X_t^C respectively by the formulas

$$\gamma_t^C=\Phi(b_2^C(t))-\Phi(b_1^C(t)), \qquad (2.6)$$

$$\beta_t^C=-\frac{K_1}{B_t}e^{-r(T-t)}[\Phi(d_2^C(t))-\Phi(d_1^C(t))]+$$
$$+\frac{K_2}{B_t}e^{-r(T-t)}\Phi(d_2^C(t)), \qquad (2.7)$$

$$X_t^C=S_t[\Phi(d_2^C(t))-\Phi(d_1^C(t))]-$$
$$-K_1e^{-r(T-t)}[\Phi(d_2^C(t))-\Phi(d_1^C(t))]+$$
$$+K_2e^{-r(T-t)}\Phi(d_2^C(t)), \qquad (2.8)$$

where $b_1^C=b_1^C(t)$, $b_2^C=b_2^C(t)$, $d_1^C=d_1^C(t)$, $d_2^C=d_2^C(t)$ at $t=0$.

Proof. According to [4] $F_{T-t}(S_t)=E\{f_T(S_T(r))|S_t\}$, where the process $S_t(r)$ is defined by the equation (1.1) and the formula (1.2) with replacement μ by r. As $f^C(S_T)=\min\{(S_T-K_1)^+,K_2\}$, and $W_t\sim N\{0;t\}$, doing obvious replacements of variables consistently in view of (2.1) – (2.4) we receive that

$$F_{T-t}^C(S_t)=\frac{1}{\sqrt{2\pi}}\times$$
$$\times\int_{-\infty}^{\infty}\min\left\{\left(S_te^{\left(r-\frac{\sigma^2}{2}\right)(T-t)+\sigma z\sqrt{T-t}}-K_1\right)^+,K_2\right\}\times$$
$$\times e^{-\frac{z^2}{2}}dz=$$
$$=\frac{1}{\sqrt{2\pi}}\int_{d_1^C(t)}^{d_2^C(t)}\left(S_te^{\left(r-\frac{\sigma^2}{2}\right)(T-t)+\sigma z\sqrt{T-t}}-K_1\right)e^{-\frac{z^2}{2}}dz+$$
$$+\frac{1}{\sqrt{2\pi}}\int_{d_2^C(t)}^{\infty}K_2e^{-\frac{z^2}{2}}dz=$$
$$=\frac{1}{\sqrt{2\pi}}S_te^{r(T-t)}\int_{d_1^C(t)}^{d_2^C(t)}e^{-\frac{z^2}{2}+\sigma z\sqrt{T-t}-\frac{\sigma^2}{2}(T-t)}dz-$$
$$-\frac{1}{\sqrt{2\pi}}K_1\int_{d_1^C(t)}^{d_2^C(t)}e^{-\frac{z^2}{2}}dz+\frac{1}{\sqrt{2\pi}}K_2\int_{d_2^C(t)}^{\infty}e^{-\frac{z^2}{2}}dz=$$
$$=\frac{1}{\sqrt{2\pi}}S_te^{r(T-t)}\int_{d_1^C(t)}^{d_2^C(t)}e^{-\frac{(z-\sigma\sqrt{T-t})^2}{2}}dz-$$
$$-K_1(\Phi(d_2^C(t))-\Phi(d_1^C(t)))+K_2\Phi(-d_2^C(t))=$$
$$=\frac{1}{\sqrt{2\pi}}S_te^{r(T-t)}\int_{b_1^C(t)}^{b_2^C(t)}e^{-\frac{y^2}{2}}dy-$$
$$-K_1(\Phi(d_2^C(t))-\Phi(d_1^C(t)))+K_2\Phi(-d_2^C(t))=$$
$$=S_te^{r(T-t)}(\Phi(d_2^C(t))-\Phi(d_1^C(t)))-$$
$$-K_1(\Phi(d_2^C(t))-\Phi(d_1^C(t)))+K_2\Phi(-d_2^C(t)). \qquad (2.9)$$

Согласно общей теории [3, 4]

$$X_t=e^{-r(T-t)}F_{T-t}(S_t), \qquad (2.10)$$

$$C_T=e^{-rT}F_T(S_0), \qquad (2.11)$$

$$\gamma_t=e^{-r(T-t)}\frac{\partial F_{T-t}(s)}{\partial s}(S_t), \qquad (2.12)$$

$$\beta_t=\frac{1}{B_T}\left[F_{T-t}(S_t)-S_t\frac{\partial F_{T-t}(s)}{\partial s}(S_t)\right]. \qquad (2.13)$$

Thus (2.5) – (2.8) follow from (2.9) – (2.13). The theorem is proved.

3. Sale option

Theorem 2. Let $d_1^P(t)$, $d_2^P(t)$, $b_1^P(t)$ and $b_2^P(t)$ be defined by the formulas

$$d_1^P(t)=\frac{\ln(K_1/S_t)-\left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \qquad (3.1)$$

$$d_2^P(t)=\frac{\ln((K_1-K_2)/S_t)-\left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \qquad (3.2)$$

$$b_1^P(t)=\frac{\ln(K_1/S_t)-\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \qquad (3.3)$$

$$b_2^P(t)=\frac{\ln((K_1-K_2)/S_t)-\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}. \qquad (3.4)$$

Then rational cost of sale option of P_T is defined by the formula

$$P_T = -S_0[\Phi(b_1^P) - \Phi(b_2^P)] +$$
$$+K_1 e^{-rT}[\Phi(b_1^P) - \Phi(b_2^P)] + K_2 e^{-rT}\Phi(b_2^P), \quad (3.5)$$

and portfolio $\pi_t^P=\{\gamma_t^P,\beta_t^P\}$ and capital X_t^P respectively by the formulas

$$\gamma_t^P = -[\Phi(b_1^P(t)) - \Phi(b_2^P(t))], \quad (3.6)$$

$$\beta_t^P = \frac{K_1}{B_t} e^{-r(T-t)}[\Phi(b_1^P(t)) - \Phi(b_2^P(t))] +$$
$$+\frac{K_2}{B_t} e^{-r(T-t)}\Phi(b_2^P(t)), \quad (3.7)$$

$$X_t^P = -S_t[\Phi(b_1^P(t)) - \Phi(b_2^P(t))] +$$
$$+K_1 e^{-r(T-t)}[\Phi(b_1^P(t)) - \Phi(b_2^P(t))] +$$
$$+K_2 e^{-r(T-t)}\Phi(b_2^P(t)), \quad (3.8)$$

where $b_1^P=b_1^P(t)$, $b_2^P=b_2^P(t)$, $d_1^P=d_1^P(t)$, $d_2^P=d_2^P(t)$ at t=0.

Proof. As, then similar to the conclusion (2.9)

$$F_{T-t}^P(S_t) = \frac{1}{\sqrt{2\pi}} \times$$
$$\times \int_{-\infty}^{\infty} \min\left\{\left(K_1 - S_t e^{\left(r-\frac{\sigma^2}{2}\right)(T-t)+\sigma z\sqrt{T-t}}\right)^+, K_2\right\} \times$$
$$\times e^{-\frac{z^2}{2}} dz =$$
$$= \frac{1}{\sqrt{2\pi}} \int_{d_2^P(t)}^{d_1^P(t)} \left(K_1 - S_t e^{\left(r-\frac{\sigma^2}{2}\right)(T-t)+\sigma z\sqrt{T-t}}\right) e^{-\frac{z^2}{2}} dz +$$
$$+\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d_2^P(t)} K_2 e^{-\frac{z^2}{2}} dz =$$
$$= -\frac{1}{\sqrt{2\pi}} S_t e^{r(T-t)} \int_{d_2^P(t)}^{d_1^P(t)} e^{-\frac{z^2}{2}+\sigma z\sqrt{T-t}-\frac{\sigma^2}{2}(T-t)} dz +$$
$$+\frac{1}{\sqrt{2\pi}} K_1 \int_{d_2^P(t)}^{d_1^P(t)} e^{-\frac{z^2}{2}} dz + \frac{1}{\sqrt{2\pi}} K_2 \int_{-\infty}^{d_2^P(t)} e^{-\frac{z^2}{2}} dz =$$
$$= -\frac{1}{\sqrt{2\pi}} S_t e^{r(T-t)} \int_{d_2^P(t)}^{d_1^P(t)} e^{-\frac{(z-\sigma\sqrt{T-t})^2}{2}} dz +$$
$$+K_1(\Phi(d_1^P(t)) - \Phi(d_2^P(t))) + K_2\Phi(d_2^P(t)) =$$
$$= -\frac{1}{\sqrt{2\pi}} S_t e^{r(T-t)} \int_{b_2^P(t)}^{b_1^P(t)} e^{-\frac{y^2}{2}} dy +$$
$$+K_1(\Phi(d_1^P(t)) - \Phi(d_2^P(t))) + K_2\Phi(d_2^P(t)) =$$
$$= -S_t e^{r(T-t)}(\Phi(d_1^P(t)) - \Phi(d_2^P(t))) +$$
$$+K_1(\Phi(d_1^P(t)) - \Phi(d_2^P(t))) + K_2\Phi(d_2^P(t)). \quad (3.9)$$

Substitution of (3.9) in (2.10) – (2.13) leads to (3.5) – (3.8). The theorem is proved.

4. Solution analysis

Theorem 3. Parity correlations take place:

$$P_T = C_T + S_0[\Phi(b_2^P) - \Phi(b_2^C)] -$$
$$-K_1 e^{-rT}\Phi(b_2^P) - \Phi(b_2^C)] + K_2 e^{-rT}\Phi(b_2^P) - \Phi(b_2^C)], \quad (4.1)$$

$$\gamma_t^P = \gamma_t^C + \Phi(b_2^P(t)) - \Phi(b_2^C(t)), \quad (4.2)$$

$$\beta_t^P = \beta_t^C + \frac{K_1}{B_t} e^{-r(T-t)}[\Phi(d_2^C(t)) - \Phi(d_2^P(t))] +$$
$$+\frac{K_2}{B_t} e^{-r(T-t)}[\Phi(d_2^C(t)) - \Phi(-d_2^C(t))], \quad (4.3)$$

$$X_t^P = X_t^C + S_t[\Phi(d_2^P(t)) - \Phi(d_2^C(t))] -$$
$$-K_1 e^{-r(T-t)}[\Phi(d_2^P(t)) - \Phi(d_2^C(t))] +$$
$$+K_2 e^{-r(T-t)}[\Phi(d_2^P(t)) - \Phi(-d_2^C(t))]. \quad (4.4)$$

The proof follows directly from correlations of the Theorems 1 and 2 in view of the Laplas' function property $\Phi(y)+\Phi(-y)=1$.

Remark. As in the case of sale option $K_2<K_1$, then parity correlations are fair at this condition.

Theorem 4. Let $\overline{C_T}$, $\overline{\gamma_t^C}$, $\overline{\beta_t^C}$, $\overline{X_t^C}$ are limits C_T, γ_t^C, β_t^C, X_t^C at $K_2\uparrow\infty$. Let $\overline{P_T}$, $\overline{\gamma_t^P}$, $\overline{\beta_t^P}$, $\overline{X_t^P}$ are limits P_T, γ_t^P, β_t^P, X_t^P at $K_2\uparrow K_1$. Then

$$\overline{C_T} = S_0\Phi(-b_1^C) - K_1 e^{-rT}\Phi(-d_1^C), \quad (4.5)$$

$$\overline{\gamma_t^C} = \Phi(-b_1^C(t)), \quad (4.6)$$

$$\overline{\beta_t^C} = -\frac{K_1}{B_t} e^{-r(T-t)}\Phi(-d_1^C(t)), \quad (4.7)$$

$$\overline{X_t^C} = S_t\Phi(-b_1^C(t)) - K_1 e^{-r(T-t)}\Phi(-d_1^C(t)), \quad (4.8)$$

$$\overline{P_T} = -S_0\Phi(b_1^P) + K_1 e^{-rT}\Phi(d_1^P), \quad (4.9)$$

$$\overline{\gamma_t^P} = -\Phi(b_1^P(t)), \quad (4.10)$$

$$\overline{\beta_t^P} = \frac{K_1}{B_t} e^{-r(T-t)}\Phi(d_1^P(t)), \quad (4.11)$$

$$\overline{X_t^P} = -S_t\Phi(b_1^P(t)) + K_1 e^{-r(T-t)}\Phi(d_1^P(t)), \quad (4.12)$$

$$\overline{P_T} = \overline{C_T} - S_0 + K_1 e^{-rT}, \quad (4.13)$$

$$\overline{\gamma_t^P} = \overline{\gamma_t^C} - 1, \quad (4.14)$$

$$\overline{\beta_t^P} = \overline{\beta_t^C} + \frac{K_1}{B_t} e^{-r(T-t)}, \quad (4.15)$$

$$\overline{X_t^P} = \overline{X_t^C} - S_t + K_1 e^{-r(T-t)}. \quad (4.16)$$

Yielded results follow directly from correlations of the Theorems 1–3 as a result of the specified limiting transitions and represent full solution of hedging problems of standard European options of purchase and sale [4, 9]. The formula (4.5) is known as the Black-Shouls formula [1, 3, 4].

Let's compare costs of exotic options of purchase and sale to standard European options.

Theorem 5. Let $\Delta C_T = \overline{C_T} - C_T$, $\Delta P_T = \overline{P_T} - P_T$. Then

$$\Delta C_T = S_0\Phi(-b_2^C) - (K_1 + K_2)e^{-rT}\Phi(-d_2^C), \quad (4.17)$$

$$\Delta P_T = -S_0\Phi(b_2^P) + (K_1 - K_2)e^{-rT}\Phi(d_2^P). \quad (4.18)$$

Formulas (4.17), (4.18) follow directly from (2.5), (3.5), (4.5), (4.9).

Theorem 6. Sensitivity coefficients $\kappa^C = dC_T/dK_2$, $\kappa^P = dP_T/dK_2$, $\zeta^C = dC_T/dK_1$, $\zeta^P = dP_T/dK_1$, $\xi^C = dC_T/dS_0$, $\xi^P = dP_T/dS_0$, characterizing fluctuation of option costs respectively by the parameters K_2, K_1, S_0, are defined by the formulas

$$k^C = e^{-rT}\Phi(-d_2^C) + \frac{1}{\sigma\sqrt{T}}\left(\frac{S_0}{K_1 + K_2}\phi(b_2^C) - e^{-rT}\phi(d_2^C)\right), \quad (4.19)$$

$$k^P = e^{-rT}\Phi(d_2^P) + \frac{1}{\sigma\sqrt{T}}\left(e^{-rT}\phi(d_2^P) - \frac{S_0}{K_1 - K_2}\phi(b_2^P)\right), \quad (4.20)$$

$$\varsigma^C = \frac{S_0}{\sigma\sqrt{T}}\left[\frac{\phi(b_2^C)}{K_1 + K_2} - \frac{\phi(b_1^C)}{K_1}\right] - e^{-rT}[\Phi(d_2^C) - \Phi(d_1^C)] - \frac{e^{-rT}}{\sigma\sqrt{T}}[\phi(d_2^C) - \phi(d_1^C)], \quad (4.21)$$

$$\varsigma^P = \frac{S_0}{\sigma\sqrt{T}}\left[\frac{\phi(b_2^P)}{K_1 - K_2} - \frac{\phi(b_1^P)}{K_1}\right] - e^{-rT}[\Phi(d_2^P) - \Phi(d_1^P)] - \frac{e^{-rT}}{\sigma\sqrt{T}}[\phi(d_2^P) - \phi(d_1^P)], \quad (4.22)$$

$$\xi^C = \Phi(b_2^C) - \Phi(b_1^C) - \frac{1}{\sigma\sqrt{T}}[\phi(b_2^C) - \phi(b_1^C)] + \frac{e^{-rT}}{S_0\sigma\sqrt{T}}[(K_1 + K_2)\phi(d_2^C) - K_1\phi(d_1^C)], \quad (4.23)$$

$$\xi^P = \Phi(b_2^P) - \Phi(b_1^P) - \frac{1}{\sigma\sqrt{T}}[\phi(b_2^P) - \phi(b_1^P)] + \frac{e^{-rT}}{S_0\sigma\sqrt{T}}[(K_1 - K_2)\phi(d_2^P) - K_1\phi(d_1^P)]. \quad (4.24)$$

The proof is made by differentiation of (2.5), (3.5) by corresponding parameters.

Theorem 7. Asymptotical properties of rational cost of exotic options, and also portfolios and capitals which correspond to them, consist in the following:

1. $\lim_{K_1\to 0} C_T = S_0\Phi(b_2^C) + K_2e^{-rT}\Phi(-d_2^C)$;
   $\lim_{K_1\to\infty} C_T = 0$; $\lim_{K_2\to 0} C_T = 0$; $\lim_{K_2\to\infty} C_T = \overline{C_T}$;
   $\lim_{S_0\to 0} C_T = 0$; $\lim_{S_0\to\infty} C_T = K_2e^{-rT}$.
2. $\lim_{K_2\to K_1} P_T = -S_0\Phi(b_1^C) + K_1e^{-rT}\Phi(d_1^C) = P_T$;
   $\lim_{K_2\to 0} P_T = 0$; $\lim_{K_1\to\infty} P_T = K_2e^{-rT}$; $\lim_{S_0\to 0} P_T = K_2e^{-rT}$;
   $\lim_{S_0\to\infty} P_T = 0$.

The proof of the formulated results is conducted directly with use of the Laplas' function properties: $\lim_{x\to\infty}\Phi(x)=1$; $\lim_{x\to-\infty}\Phi(x)=0$; $\Phi(x)=1-\Phi(-x)$; $\Phi(x)$ is continuous on the right on x.

2. Conclusion

1. The researches show that $\Delta C_T>0$, $\Delta P_T>0$, i. e. costs of standard options are above cost of exotic options. The given property is justified on the basis that for the option owner reception of higher income is possible in case of standard rather than exotic option, payments by which are limited, and in order to get higher income it is necessary to pay more.
2. The researches show that $\kappa^C>0$, if $S_0e^{rT}>K_1+K_2$, and $\kappa^P>0$, if $S_0e^{rT}>K_1-K_2$. Thus, option costs of purchase and sale are the increasing functions of the parameter K_2 at execution of the specified conditions. The economic sense of this property consists in the following. With growth of the parameter K_2 the option owner's opportunity of greater profit increases. In order to get such opportunity it is necessary to pay more, therefore the price of the option increases.
3. The economic sense of limiting properties of option costs (Theorem 7) is obvious.

REFERENCES

1. Black F., Scholes M. The pricing of options and corporate liabilities // J. of Political Economy. – 1973. – May/Jun. – P. 637–657.
2. Merton R. Theory of rational option pricing // Bell J. Economics and Management Science. – 1973. – V. 4. – P. 141–183.
3. Shiryaev A.N. Fundamentals of stochastic financial mathematics. – Moscow: FAZZIS, 1998. – 1010 p.
4. Shiryaev A.N., Kabanov Yu.M., Kramkov D.O., Melnikov A.V. To the theory of option calculations of the European and American types. Continuous time // Probability Theory and its Applications. – 1994. – V. 39. – № 1. – P. 80–129.
5. Novikov A.A. Option hedging with set probability // Probability Theory and its Applications. – 1998. – V. 43. – № 1. – P. 152–161.
6. Follmer H., Len K.P. Quantile hedging // Finance and Stochastic. – 1999. – V. 3. – № 3. – P. 251–273.
7. Rubinstein M. Exotic options // Finance working paper. – 1991. – № 220. – Inst. of Business and Economic research. University of California. Berkley.
8. Kozhin K. Everything about exotic options // Securities Market. – 2002. – № 15. – P. 53–57; № 16. – P. 60–64; № 17. – P. 68–73.
9. Demin N.S., Lazatnikova A.V. Research of portfolio, capital and option cost in case of standard European option of sale with continuous time // Bulletin of TGU. Appendix. – 2002. – № 1 (I) – P. 147–149.

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