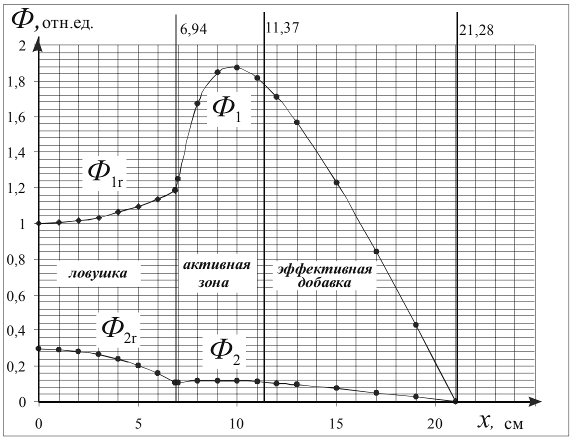


***Fig. 4.*** *Distribution of neutron flux*

In the case of using central reflector (fig. 2) the value of this relation in one-group approximation is equal to 0,82 according to the data [6]. This result is not unexpected one and can be explained, first of all, by the fact that one-group model does not include the slow neutron storage in the reflector due to slowing down fast neutrons.

Solutions in two-group approximation (fig. 4) show visually the role of central reflector. The efficiency of neutron trap permits us to increase the relation of slow neutron average flux to its maximum value up to 0,966.

It should be noted that at transition to the effective boundary the neutron field distribution near the «core-external reflector» boundary is deformed. A more detailed consideration requires increase in system order and significant complication of computations.

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*Arrived on 13.06.2006*

UDC 621.391

# REGULARISING ALGORITHM OF PARAMETER IDENTIFICATION OF ELECTRIC CHARGE EQUIVALENT CIRCUIT. P. I.

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*A new algorithm of parameter identification of equivalent circuit for electrical charge replacement is suggested. The approach is based on the solution of integral equation of the I type with respect to the function of indicial admittance, by which then determination of replacement circuit parameters is carried out. Application of smoothing splines and original regulating algorithm including kernel setting error of integration equation permits to obtain a stable algorithm of parameter identification. The investigation of algorithm shows high calculating efficiency and sufficient accuracy of parameter identification.*

## 1. Introduction

One of the most interesting from the physical point of view and practically important trends in different fields of engineering is barrier discharge. In particular, barrier discharge is used in water purification, plasma technology, etching, etc. However, a strong spatial irregularity and short durability of physical processes taking place in barrier discharge make it difficult to study this phenomenon.

In characteristic description of electric discharge (barrier discharge, in particular) their description as objects of electric circuit is widely used [1]. The bases of such approaches are the replacement of electrophysical phenomena by the phenomena taking place in electric circuit consisting of definite electric elements (resistance, capacity, and inductance). Such electric circuit will be called an equivalent circuit of electric discharge replacement.

Investigating the discharge physics voltage $U(t)$ and current $I(t)$ in circuit with discharge gap are available for measuring. Therefore, there appears the problem of determination of electric discharge replacement circuit parameters with registered function values $U(t)$, $I(t)$. In fact, we have the problem of identification of replacement equivalent circuit.

In the works [2, 3] the parameter values are along indicial admittance $g(t)$. In its turn, the function $g(t)$ is defined as a solution of the Volterra integral equation-convolution of the I genus, which is an ill-defined specified problem [4, 5]. However, in these works a number of significant aspects are not taken into account, they are connected with the solution of this ill-defined specified problem, which affected the accuracy of parameter identification negatively. One can refer to such aspects: insufficient stability of the involved algorithm differentiated by noise function $U(t)$ when calculating integral equation kernel; neglecting random error of specified equation kernel both at the stage of regularising solution construction and at choosing the regularization parameters.

Therefore, in the present work a stable algorithm of identification of replacement circuit parameters, based on regularizing algorithm of solving integral equation-convolution with inaccurately specified kernel [6] and involving the aspects mentioned above fully has been suggested.

#### Setting up the problem

If the voltage in the circuit is impulse, the transition in discharge gap in terms of $U(t)$ and $I(t)$ is convenient to describe by means of the Duhamel integral [7]. If the voltage $U(t)$ is a finite function, i.e. out of the interval $[0,T_U]$ it is turned out to zero, then the following expression is true for the current in the circuit

$$I(t) = U(0)g(t) + \int_0^t \frac{dU(\tau)}{d\tau} g(t-\tau)d\tau, \qquad (1)$$

where $g(t)$ is the indicial admittance. As a rule, the value $U(0)=0$ hence, we obtain the Volterra integral equation-convolution of the I genus:

$$I(t) = \int_0^t g(t-\tau)\frac{dU(\tau)}{d\tau}d\tau. \qquad (2)$$

The function $g(t-\tau)$ is called the integral equation kernel, $I(t)$ is the right part of the equation.

The integral equation (2) is necessary to be solved relative to the function $g(t)$, which is an ill-defined specified problem, and then by the function $g(t)$ define the parameters of the replacement equivalent circuit of electric discharge.

Thus, the problem of parameter determination of the replacement equivalent circuit includes the following stages:

Stage 1. Calculation of derivative $\frac{dU(\tau)}{d\tau}$ on the changed (with errors) magnitudes of the function $U(t)$.

Stage 2. Solution of the integral equation (2) relative to the function $g(t)$.

Stage 3. Determination (probably by the form of the function $g(t)$) of the replacement equivalent circuit structure and function parameterization $g(t)$.

Stage 4. Estimation of the function parameters $g(t)$ and calculation of values of resistance, capacity and inductances forming the replacement equivalent circuit.

The solution of the formulated determination is considered with the following assumptions:

1. The function $U(t)$ is different from zero in the interval $(0,T_U]$ (i.e. it is finite) and is measured in this interval at the moments $t_j=\Delta\cdot j$, $j=0,1,...,N_U-1$, where $N_U=\text{ent}[T_U/\Delta]+1$, $\Delta$ is the sampling increment, is the integer part of the real number $z$. The changed values $\tilde{U}_j$ admit the statement

$$\tilde{U}_j = U(j\Delta)+\zeta_j, \quad j=0,1,...,N_U-1, \qquad (3)$$

where $\zeta_j$ are random quantities with the expectation function $M(\zeta_j)=0$, dispersion $D(\zeta_j)=\delta_\zeta^2$ and showing the voltage measurement errors.

2. The function $g(t)$ is different from zero in the interval $[0,T_g]$.

Supposing these, the function $I(t)$ is finite with the interval $[0,T_I]$, of the determination interval, where $T_I=T_U+T_g$.

3. The function $I(t)$ is measured in the interval $[0,T_I]$ at the moments $t_j=\Delta\cdot j$, $j=0,1,...,N_I-1$, where $N_I=\text{ent}[T_I/\Delta]+1$. The measured magnitudes assume the statement

$$\tilde{I}_j = I(t_j)+\eta_j, \quad j=0,1,...,N_I-1, \qquad (4)$$

where $\eta_j$ are random magnitudes with numerical characteristics $M(\eta_j)=0$, $D(\eta_j)=\delta_\eta^2$.

In short, let us dwell on the solution algorithms of each stage of the involved problem of the parameter identification of the replacement equivalent circuit.

#### 2. Calculation of derivative by the changes values of voltage

The kernel of the integral equation (2) is a derivative $\frac{dU(t)}{d\tau}$ of voltage $U(t)$. It is known that differentiation procedure is an ill-defined specified problem (in particular, minute errors can cause any great mistake in derivative).

For stable differentiation of the function $U(t)$, specified by the changed at the moments $t_j$ values $\tilde{U}(t_j)$ as an approximation for $U(t)$ let us take *smoothing cubic spline* (SCS) $S_\lambda(t)$. Recall that cubic polynomial is called smoothing cubic spline, meeting the conditions:

1. In each interval $[t_j,t_{j+1}]$ $S_\lambda(t)$ has the following form

$$S_\lambda(t) = a_j + b_j(t-t_j) + c_j(t-t_j)^2 + d_j(t-t_j)^3, \quad (5)$$

where $t_j<t<t_{j+1}$.

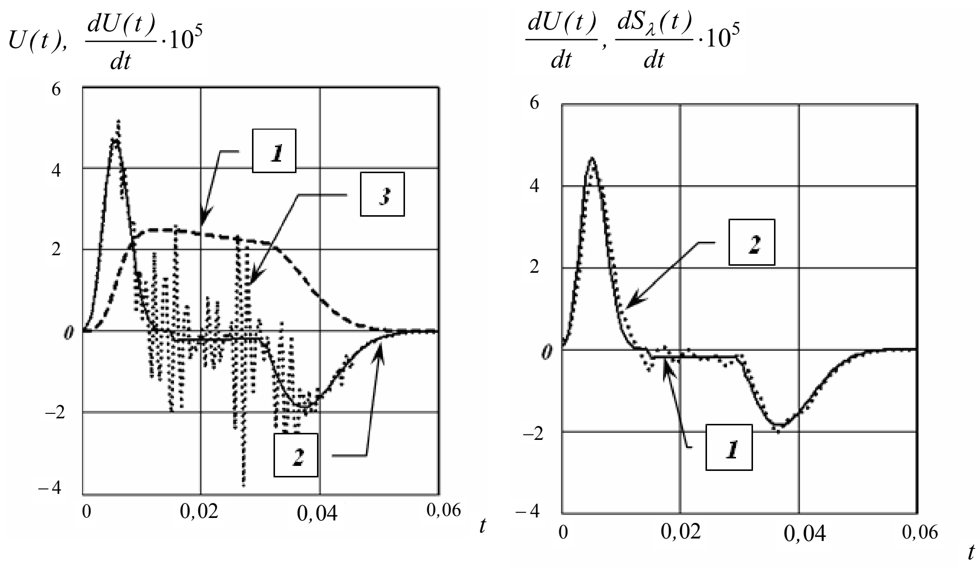
2. The function $S_\lambda(t)$ has the continuous second derivative on the whole segment $[0,T_U]$.

Calculations of the coefficients $a_j$, $b_j$, $c_j$, $d_j$ of SCS (which depend on *the smoothing parameters* $\lambda$) are described in the works in details [5, 8] and for their unique calculation let us adopt *the edge conditions* of the form:

$$S_\lambda{}'(0)=0; \ S_\lambda{}'(T_U)=0. \qquad (6)$$

These conditions correspond to typical form of the voltage impulse $U(t)$ (see fig. 1). One can show that SCS with the conditions (6) returns minimum to the functional

$$\int_0^{T_U} (f''(t))^2 dt + \lambda \cdot \sum_{j=0}^{N_u-1} p_j (f(t_j)-\tilde{U}_j)^2 \qquad (7)$$




a b

***Figure.*** *Calculation of the function U(t) derivatives*

among all the functions $f(t)$ with integrable by the square of the second derivative and meeting the condition (6).

After calculation of the SCS coefficients the first derivative $S'_\lambda(t)$ (which is the estimation for the derivative $\frac{dU(t)}{d\tau}$) can be found by the formula

$$S_\lambda{}'(t) = b_j + 2c_j(t-t_j) + 3d_j(t-t_j)^2, \quad (8)$$

where $t_j<t<t_{j+1}$.

The main problem in plotting the SCS is the choice of the smoothing parameter $\lambda$, which can change in the range from 0 (smoothing spline becomes interpolated, crossing the values $\tilde{U}_j$, i. e. $S_0(t_j)=\tilde{U}_j$) to $\infty$ (SCS becomes a straight line). If $\lambda$ turns out to be small, the high-frequency constituents are present in the spline, which is explained by the errors $\zeta_j$, particularly developing in the spline derivatives in the form of high-frequency oscillations. If this parameter is too large, the plotted spline appears to be «oversmoothing» and leading and descending edges of the impulse $U(t)$ are strongly smoothed in it, which affects the accuracy of the first derivative calculation negatively.

One can point out the two approaches to the choice of parameter $\lambda$: estimation of $\lambda$ from minimum condition by smoothing root-mean-square error [5] and the choice $\lambda$ by the spline accuracy characteristics [9]. Let us dwell on the second approach, more appropriate for the solved differentiation problem $U(t)$.

In this approach the smoothing spline is referred to as an output signal of some filter (spline filter), to the input of which the digital string consisting of changes values $\tilde{U}_j$ of the function $U(t)$ comes. In such interpretation the smoothing properties of spline can be defined by its *hardware function* $h_\lambda(t)$, that characterises a hard error of smoothing and differentiating: the less the function «width» is, the less the hard error is. As a numerical characteristics of hardware function its width is taken $\Delta_h(\lambda)$:

$$\Delta_h(\lambda) = \frac{\int_0^\infty |h_\lambda(t)|\,dt}{h_\lambda(0)}.$$

Physical interpretation of this characteristic is rather simple for the differentiation problem: in smoothing spline and its derivative the function constituents $U(t)$ and the derivative $U'(t)$ are kept with small amplitude distortion, if their width is more than that of the hardware function $h_\lambda(t)$. Specifying «limit» size $\Delta_{lim}$ of constituents, those are to be kept in spline, the value $\lambda$ of can be determined from the solution of non-linear equation:

$$\Delta_h(\lambda) = \Delta_{lim}. \quad (9)$$

The hardware function $h_\lambda(t)$ is calculated with the formula

$$h_\lambda(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_\lambda(\omega)e^{i\omega t}\,d\omega.$$

Frequency characteristic of spline is defined by the expression [9]

$$H_\lambda(\omega) =$$

$$= \frac{2}{\Delta\omega^2}\left(\frac{\sin(\frac{\omega T}{2})}{\frac{\omega T}{2}}\right)^2 \left[\frac{1-\cos(\omega\Delta)}{q_0 + 2q_1\cos(\omega\Delta) + 2q_2\cos(2\omega\Delta)}\right],$$

where $q_0 = \frac{2\Delta}{3} + \frac{6\lambda p}{\Delta^2}$; $q_1 = \frac{\Delta}{6} - \frac{4\lambda p}{\Delta^2}$; $q_2 = \frac{\lambda p}{\Delta^2}$; $p_j = p$ are the weighting coefficients of the functional, $\Delta$ is the sampling increment.

Let us demonstrate the presented approach to the choice of smoothing parameter $\lambda$ by the results of the following calculation experiment. In figure, *a*, the graph of function is shown (curve 1) and its «exact» derivative $\frac{d}{dt}U(t)$ (curve 2). The function values are multiplied by

100, so that the function would differ from zero in the figure scale. Curve 3 corresponds to derivative of integral spline $S_0(t)$, plotted by the changed (with error) values $\tilde{U}_j$, $j$=1,2,...,$N_U$=240. Relative level of errors $\zeta_j$ is specified to be equal to 0,05. In figure, *b*, the graph of «exact» derivative $\frac{dU(t)}{d\tau}$ and the derivative values calculated by interpolating cubic spline are shown. One can see significant oscillations of this derivative, characteristic for differentiating inexact specified function. In figure, *b*, the graph of derivative $\frac{dU(t)}{d\tau}$ and the derivative values $S_\lambda'(t)$, calculated by smoothing cubic spline are presented $S_\lambda(t)$, see (8). A rather good (in comparison with the derivative of interpolating spline) agreement of these two derivatives is obvious.

Smoothing parameter is chosen from the solution of the equation (9) at $\Delta_{np}$=5·10$^{-3}$ s (sampling interval $\Delta_{np}$=2,5·10$^{-4}$ s). The magnitude is specified to be equal to the half width of the leading edge of the voltage impulse $U(t)$, that permits the derivative $S_\lambda'(t)$ to keep «fine» details of derivative $U'(t)$ (in particular in the range [0, 0.01]). Small oscillations $S_\lambda'(t)$ of the derivative in the range [0.01, 0.03] are at the second stage of interpretation as specifying errors of integral equation kernel and are taken into account in construction of regularising solution of this integral equation. For this purpose let us present the values of spline $S_\lambda(t)$ derivative in the junction $t_j$ in the form:

$$S_\lambda'(t_j) = \frac{d}{dt}U(t)\Big|_{t=t_j} + \xi_j\,,\ j = 0,1,...,N_U - 1. \quad (10)$$

The random magnitudes $\xi_j$ reflect the errors in calculation of derivative on smoothing spline $S_\lambda(t)$. If the errors $\zeta_j$ of measurement $U(t_j)$ have the same dispersion, the random errors of differentiating have also dispersion $D(\xi_j)=\sigma_\xi^2$.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*Arrived on 18.07.2006*