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SYNTHESIS OF LOW-ORDER REGULATORS BY SPECIFIED PLACEMENT OF CLOSED SYSTEM POLES

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Algorithm of regulator synthesis supporting arrangement of closed system dominant poles in specified points and arrangement of non-dominant poles in specified area has been suggested. Two-phase algorithm is based on the method of D-partitioning modified with conditions for system poles arrangement and the method of searching the best solution in parameter region which guarantees the desired pole arrangement. The example is given.

1. Introduction

Urgency of the problem of dynamic low-order regulators synthesis is explained by complexity and practical inappropriateness of full order regulator realization. Manifold examples show that the required quality of control processes may be supported by low-order regulators [1]. This problem has no unambiguous solution and various methods of synthesis are described in scientific literature.

In particular there is a number of papers in which the problem of low-order regulator synthesis is reduced to assignment of closed system dominant poles [2–4]. The number of varied parameters of dynamic regulator is taken equal to a number of assigned dominant poles. Magnitudes of regulator varied parameters are computed explicitly. The suggested methods rely on known fact that dynamic properties of closed system are determined by two-three poles for which dominance conditions are fulfilled. Guidelines on dominant poles assignment may be found, for example, in [5]. Possibilities of these methods are limited as arbitrary placement of nondominant system poles is permitted. Influence of these poles on system properties have to be estimated only at final stage of regulator synthesis.

Algorithm of low-order regulator synthesis is studied in this paper. It is based on the method of *D*-decomposition modified by conditions for system poles placement [6]. A number of regulator varied parameters in comparison with [2–4] increases a number of assigned dominant poles but nondominant poles of closed system are restricted by specified region of their placement. The suggested algorithm of synthesis consists of two stages. On the first stage a part of regulator parameter space is extracted by the method of *D*-decomposition. In this space the conditions imposed to placement of dominant and nondominant poles of the system are fulfilled. To select concrete values of regulator parameters in the obtained region fulfilled on the second stage frequency and integral estimates characterizing quality of control process are suggested to be used.

It should be noted that recently the method of *D*-decomposition used in the given paper attracted attention of researchers. For example, in [1] a two-stage algorithm of regulator synthesis was developed on its base. However, in this case a number of regulator varied parameters is limited by two and stability region is constructed in regulator parameter plane. Optimal regulator parameters are determined by maximal robustness criterion by numerical procedures.

2. Problem statement

Linear stationary system of automatic control the operator-structural scheme of which is shown in Fig. 1 is studied.

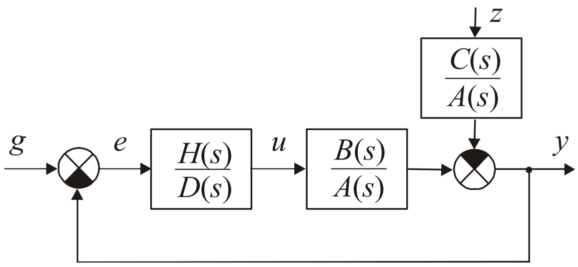

Fig. 1. *Operator-structural scheme of the system*

Control object is described by the equation

$$A(p)y(t) = B(p)u(t) - C(p)z(t),$$

where $p=d/dt$ is the differential operator (formally equivalent to operator s of Laplace transformation); polynomials $A(p)$, $B(p)$ and $C(p)$ are reciprocally simple, their degrees satisfy the conditions: $\deg A(p)=n$, $\deg B(p)\leq n$, $\deg C(p)\leq n$.

The equation of dynamic regulator has the form

$$D(p)u(t) = H(p)[g(t) - y(t)],$$

where $\deg H(p)=\deg D(p)=m<n$.

Regulator synthesis is reduced to selection of its parameters being polynomial coefficients:

$$D(p) = d_m p^m + \ldots + d_1 p + d_0, \qquad (1)$$

$$H(p) = h_m p^m + \ldots + h_1 p + h_0. \qquad (2)$$

A part of regulator parameters may be specified beforehand or satisfy additional conditions. In particular, it is appropriate to accept $d_0=1$ for static regulator. In the case of regulator with first order astaticism $d_0=0$, $d_1=1$, etc. Taking into account the peculiarities of the solved problem the values of other regulator parameters may be assigned. Therefore, a number of regulator parameters which should be determined satisfy the condition $r\leq 2(m+1)$.

The system of automatic control is described by the equation

$$[A(p)D(p) + B(p)H(p)]\cdot y(t) =$$
$$= B(p)H(p)\cdot g(t) - C(p)D(p)\cdot z(t).$$

The task of the given paper is defined in the following way. It is necessary to determine value r of regu-

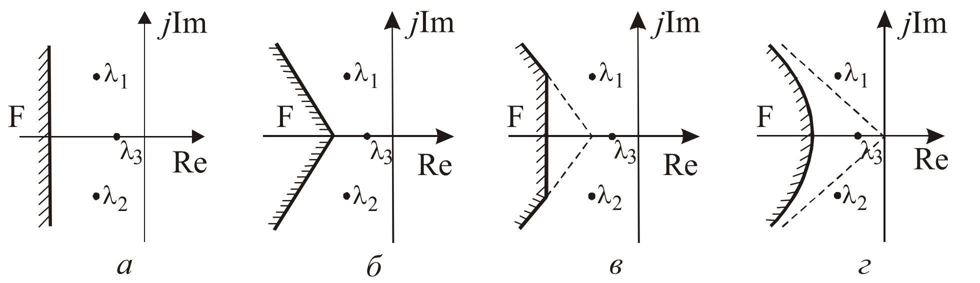

Fig. 2. *Variants of system poles arrangement*

lator varied parameters at which l of dominant poles of closed system take the prescribed values λ_j, j=1,...,l, and the rest $n+r-l$ nondominant poles satisfy some conditions in the form of inequalities limiting region F of their placement on complex plane. The examples of placement of dominant poles $\lambda_1,\lambda_2,\lambda_3$ and possible boundaries of region F of system nondominant pole location are given in Fig. 2.

As a number of assigned dominant poles of the system is less than a number of regulator parameters the stated task has a certain solution set.

3. Derivation of main ratios

Let us denote arbitrarily the coefficients of polynomials (1) and (2), representing variable parameters of regulator by $k_1,k_2,...,k_r$. Regulator parameters are included linearly into characteristic equation of the system. Let us write down the equation in the following form:

$$\sum_{i=1}^{r} k_i\ G_i(p)+G_0(p)=0. \qquad (3)$$

Let us divide the variable parameters $k_1,k_2,...,k_r$ of regulator into two groups. Parameters which are called free ones are added to the first group. Let them form vector $\mathbf{k}_c=(k_1,...,k_q)^T$ the dimension of which equals to $q=r-l$. The dependent variable regulator parameters the values of which are computed after selection of free variable parameters from the condition that l poles of the system take the prescribed values are included into the second group. These parameters are combined into vector $\mathbf{k}_з=(k_{q+1},...,k_r)^T$ with the dimension $l=r-q$.

The main ratios for the case of two free variable parameters (q=2) are obtained. For this purpose let us present characteristic equation (3) in the form

$$\sum_{i=1}^{2} k_i\cdot G_i(p)+\sum_{i=3}^{r} k_i\cdot G_i(p)+G_0(p)=0. \qquad (4)$$

Substitution of $p=\lambda_j$, j=1,...,l into (4) gives l equations:

$$\sum_{i=1}^{2} k_i\cdot G_i(\lambda_j)+\sum_{i=3}^{r} k_i\cdot G_i(\lambda_j)+G_0(\lambda_j)=0,$$
$$j=1,...,l. \qquad (5)$$

These equations connect variable parameters k_i, i=1,...,r with specified dominant poles λ_j, j=1,...,l.

Let us represent (5) in matrix form:

$$\mathbf{Q}_{11}(\lambda)\cdot k_c+\mathbf{Q}_{12}(\lambda)\cdot k_з=\mathbf{R}_1(\lambda), \qquad (6)$$

where

$$\mathbf{Q}_{11}(\lambda)=\begin{bmatrix} G_1(\lambda_1) & G_2(\lambda_1) \\ \cdots & \cdots \\ G_1(\lambda_l) & G_2(\lambda_l)\end{bmatrix};$$

$$\mathbf{Q}_{12}(\lambda)=\begin{bmatrix} G_3(\lambda_1) & \cdots & G_r(\lambda_1) \\ \ldots & \ldots & \ldots \\ G_3(\lambda_l) & \ldots & G_r(\lambda_l)\end{bmatrix};$$

$$\mathbf{R}_1(\lambda)=\begin{bmatrix} -G_0(\lambda_1) \\ \cdots \\ -G_0(\lambda_1)\end{bmatrix}.$$

Let us specify the boundary of plasement region of nondominant poles of the system by the function

$$X(j\omega)=\alpha+\delta(\omega)+j\omega,\ \delta(\omega)=\delta(-\omega),\ \omega\in(-\infty,\infty).$$

To derive the equation of D-decomposition boundary on the plane of two free parameters let us substitute $p=\alpha+\delta(\omega)+j\omega$ into (4) and convert the obtained complex equation into the system of two real equations. It has the form in matrix form

$$\mathbf{Q}_{21}(\omega)\cdot\mathbf{k}_c+\mathbf{Q}_{22}(\omega)\cdot\mathbf{k}_з=\mathbf{R}_2(\omega), \qquad (7)$$

where

$$\mathbf{Q}_{21}(\alpha,\omega)=\begin{bmatrix} \operatorname{Re}G_1(\alpha,\omega) & \operatorname{Re}G_2(\alpha,\omega) \\ \operatorname{Im}G_1(\alpha,\omega) & \operatorname{Im}G_2(\alpha,\omega)\end{bmatrix};$$

$$\mathbf{Q}_{22}(\alpha,\omega)=\begin{bmatrix} \operatorname{Re}G_3(\alpha,\omega) & \cdots & \operatorname{Re}G_r(\alpha,\omega) \\ \operatorname{Im}G_3(\alpha,\omega) & \cdots & \operatorname{Im}G_r(\alpha,\omega)\end{bmatrix};$$

$$\mathbf{R}_2(\alpha,\omega)=\begin{bmatrix} -\operatorname{Re}G_0(\alpha,\omega) \\ -\operatorname{Im}G_0(\alpha,\omega)\end{bmatrix}.$$

As a result of (6) and (7) combination the system of equation is obtained:

$$\mathbf{Q}_{11}(\lambda)\,\mathbf{k}_c+\mathbf{Q}_{12}(\lambda)\,\mathbf{k}_з=\mathbf{R}_1(\lambda);$$
$$\mathbf{Q}_{21}(\alpha,\omega)\,\mathbf{k}_c+\mathbf{Q}_{22}(\alpha,\omega)\,\mathbf{k}_з=\mathbf{R}_2(\alpha,\omega). \qquad (8)$$

The equation system (8) may be considered as parametric equation of D-decomposition boundary of the plane of parameters k_1,k_2, forming vector $\mathbf{k}_c$, when value ω runs the boundary of region F, at additional condition that dominant poles λ_j, j=1,...,l of the closed system take the prescribed values. Let us express vector $\mathbf{k}_з$ of dependant variable parameters from the first equation of the system (8):

$$\mathbf{k}_з = \mathbf{Q}_{12}^{-1}(\lambda)\cdot\mathbf{R}_1(\lambda) - \mathbf{Q}_{12}^{-1}(\lambda)\cdot\mathbf{Q}_{11}(\lambda)\cdot\mathbf{k}_c. \quad (9)$$

From the second equation of the system (8) after substitution of (9) we obtain:

$$\mathbf{k}_c(\alpha,\omega) = \begin{bmatrix} k_1(\alpha,\omega) \\ k_2(\alpha,\omega) \end{bmatrix} =$$

$$= [\mathbf{Q}_{21}(\alpha,\omega) - \mathbf{Q}_{22}(\alpha,\omega)\cdot\mathbf{Q}_{12}^{-1}(\lambda)\cdot\mathbf{Q}_{11}(\lambda)]^{-1}\times$$

$$\times[\mathbf{R}_2(\alpha,\omega) - \mathbf{Q}_{22}(\alpha,\omega)\cdot\mathbf{Q}_{12}^{-1}(\lambda)\cdot\mathbf{R}_1(\lambda)]\ . \quad (10)$$

Numerical value of vector $\mathbf{k}_c$ determining one point of *D*-decomposition boundary on the plane of free parameters k_1 and k_2 of regulator is computed for each concrete value on the basis of (10). Changing in (10) ω from $-\infty$ to ∞ the whole curve of *D*-decomposition on this plane may be plotted.

Some values frequencies give at uncertainty determination. Not separate points but so-called singular straight lines correspond to these values ω. The first singular straight line corresponds to ω=0. To obtain its equation let us substitute $p=\alpha+\delta(\omega)+j\omega|_{\omega=0}=\alpha$ into (4). We have

$$k_1\cdot G_1(\alpha) + k_2\cdot G_2(\alpha) + \sum_{i=3}^{r} k_i\cdot G_i(\alpha) + G_0(\alpha) = 0.$$

Let us write down this equation in vector-matrix form:

$$\mathbf{Q}_{21}(\alpha)\cdot\mathbf{k}_c + \mathbf{Q}_{22}(\alpha)\mathbf{k}_з = G_0(\alpha), \quad (11)$$

where

$$\mathbf{Q}_{21}(\alpha) = [G_1(\alpha)\ G_2(\alpha)];\ \ \mathbf{Q}_{22}(\alpha) = [G_3(\alpha) \dots G_r(\alpha)].$$

And, finally, substituting (9) into (11) the equation of the first singular straight line in vector-matrix form is obtained:

$$[\mathbf{Q}_{21}(\alpha) - \mathbf{Q}_{22}(\alpha)\cdot\mathbf{Q}_{12}^{-1}(\lambda)\cdot\mathbf{Q}_{11}(\lambda)]\cdot\mathbf{k}_c =$$

$$= G_0(\alpha) - \mathbf{Q}_{22}(\alpha)\cdot\mathbf{Q}_{12}^{-1}(\lambda)\cdot\mathbf{R}_1(\lambda)\,.$$

The second singular straight line corresponding to $\omega=\infty$ is found by equating coefficient a_n to zero at a summand with high degree of characteristic equation (3) if this coefficient depends on regulator variable parameters.

D-decomposition boundary is hatched according to the sign of determinant

$$\Delta(\omega) = \mathbf{Q}_{21}(\omega) - \mathbf{Q}_{22}(\omega)\cdot\mathbf{Q}_{12}^{-1}(\lambda)\cdot\mathbf{Q}_{11}(\lambda)$$

of equation system (9) on the basis of known rules. Singular straight lines are hatched as usual according to hatching *D*-decomposition boundary about junction points.

4. Algorithm of low-order regulator synthesis

On the basis of the obtained ratios the following order of low-order regulator parametric synthesis is suggested.

Stage 1. Initial data formation for calculation by the suggested synthesis method. On the basis of prior information about control object properties the type (static, astatic) of regulator and its order are determined. A set of calculated parameters of regulator consisting of transfer function coefficients is divided into free and dependent ones. Coefficients of transfer function influencing greatly the system properties of interest are selected as two free parameters in the plane of which the region of *D*-decomposition is supposed to be constructed. Values λ_j, j=1,...,l of system dominant poles are specified. The amount of dominant poles of the system should be equal to a number of dependent parameters of regulator.

Stage 2. Construction of *D*-decomposition region in the plane of regulator free parameters. Matrices $\mathbf{Q}_{11}(\lambda)$, $\mathbf{Q}_{12}(\lambda)$, $\mathbf{Q}_{21}(\omega)$, $\mathbf{Q}_{22}(\omega)$ and vectors $\mathbf{R}_1(\lambda)$, $\mathbf{R}_2(\omega)$ are formed at this stage. With the help of formulas obtained in part 3 the boundary of *D*-decomposition and singular straight lines are constructed in a space of free parameters. For this purpose modern specialized programming systems having facilities of solving the systems of linear algebraic equations in their composition are suggested to be used. MathCAD referred, for example, to such systems. Then the region where the specified plasement of dominant and other poles of the system is supported is marked out by hatching rules.

Stage 3. Этап 3. Searching for numerical values of free variable parameters from *D*-decomposition region. The problem of optimization by two free parameters of regulator is solved at this stage. Various quality indices characterizing system operation in steady state and transient regimes may be selected as an optimality criterion. The constructed region of *D*-decomposition is considered as a region of feasible problem solutions. To search for optimal solution it is appropriate to use numerical methods or simulation with directed enumeration of acceptable possibilities.

5. The example of regulator synthesis

Let us consider the stabilizing system the operator-structural scheme of which is shown in Fig. 3.

Control object is described by the equation

$$(10p+1)(p+1)^2\,y(t) = 1{,}2\cdot u(t) - 0{,}1\cdot z(t).$$

PID regulator in which derivative action is formed with real differentiator directly by system output coordinate is used as regulator. In comparison with usually accepted assumption that differentiator time constant $T_д$ is small and may be neglected let us consider this time constant the forth parameter which should be determined at synthesizing. Regulator transfer function by control value has the form

$$W_р(s) = \frac{U(s)}{Y(s)} = \frac{(k_д + k_п T_д)s^2 + (k_п + k_и T_д)s + k_и}{T_д s^2 + s}.$$

Let us find values of parameters $k_и$, $k_п$, $k_д$, $T_д$ of PID regulator which:

- ensure the placement of two dominant poles of closed system in points $\lambda_1=-0{,}2+j0{,}3$ and $\lambda_2=-0{,}2-j0{,}3$ provided that the rest poles of the system satisfy the inequality

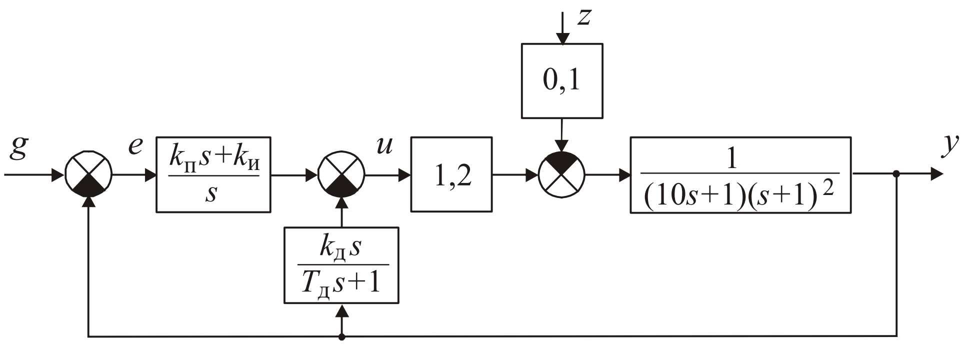

Fig. 3. *Operator-structural scheme of synthesized system*

$$\operatorname{Re} s_i < \begin{cases} -0,2+0,5\omega \text{ при } \omega \in (-\infty, 0], \\ -0,2-0,5\omega \text{ при } \omega \in [0, \infty); \end{cases}$$

- maximal disturbance suppression z at specified conditions fulfillment on plasement of closed system poles. Quality of disturbance suppression is estimated with integral

$$J = \int_0^\infty y_z^2(t)\,dt,$$

where $y_z(t)$ is the reaction of the system for step change of disturbing action.

Let us introduce the following notations: $k_1=T_д$, $k_2=k_и$, $k_3=k_п+k_и T_д$, $k_4=k_д+k_п T_д$ and write down characteristic equation of the system in the form satisfying the conditions of the described method application:

$$k_1 \cdot G_1(p) + k_2 \cdot G_2(p) + k_3 \cdot G_3(p) +$$
$$+ k_4 \cdot G_4(p) + k_5 \cdot G_5(p) + G_0(p) = 0,$$

where

$$G_1(p) = 10p^5 + 21p^4 + 12p^3 + p^2;$$
$$G_2(p) = 1,21;$$
$$G_3(p) = 1,21p;$$
$$G_4(p) = 1,21p^2;$$
$$G_0(p) = 10p^4 + 21p^3 + 12p^2 + p.$$

Regulator parameters $T_д=k_1$ and $k_и=k_2$ are considered to be free and k_3 and k_4 are the dependant ones. On the basis of the ratios obtained before and initial data the required vectors and matrices are formed for the given system, D-decomposition is carried out by two free parameters. The region in plane of free parameters $T_д$ and $k_и$ corresponding to specified placement of closed system poles is plotted Fig. 4.

The lines of equal meaning of disturbing action suppression criterion selected for determining optimal values of regulator parameters are drawn in the region of D-decomposition (Fig. 4). Minimal value of the criterion is achieved at value $T_д=0$ that is for idealized PID regulator. Let us chose $T_д=0,05$ s. In this case maximum possible value is $k_и=2,2$.

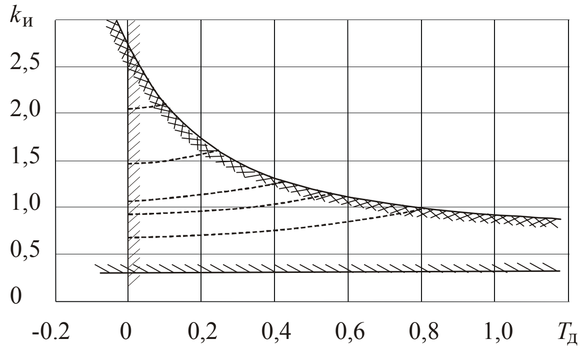

Fig. 4. *Region of D-decomposition in the plane of regulator free parameters*

Having substituted these values into expression connecting free and dependent regulator parameters the following values of dependent parameters are obtained: $k_п=7,68$; $k_д=13,43$. At found values of regulator parameters the closed system has the poles:

$$\lambda_1 = -0,2 + j0,3; \quad \lambda_2 = -0,2 - j0,3;$$
$$\lambda_3 = -0,805 + j1,179; \quad \lambda_4 = -0,805 - j1,179;$$
$$\lambda_5 = -20,09.$$

Conclusion

The algorithm of synthesis of low-order regulator parameters supporting specified plasement of dominant and nondominant poles of closed system was suggested. It is based on the method of constructing D-decomposition boundaries subject to limiting for arranging system dominant poles in specified points of complex plane.

Substantially the suggested algorithm of synthesis is on the basis of multicriterion approaches to design of automated control systems being rather widespread at present. It may serve as a base for development of human-computer (interactive) procedures of designing automated control systems using, in particular, the universal programming environment MathCAD including methods of solving systems of linear algebraic equations. The given example confirms the efficiency of the suggested algorithm.

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STRUCTURALLY PARAMETRIC IDENTIFICATION OF OBJECT DESCRETE MODELS WITH DELAY FOR TUNING SMITH CONTROLLERS

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Construction of Smith digital controller on the basis of equivalence principle of dynamic object models with delay has been suggested.

Synthesis of control systems starts, as a rule, with object model determination. In practice, information about object is obtained in the form of inputs and outputs available for direct measurement. Actually, impact reaction time delay τ typical for heat-and-power engineering, chemical, metallurgical processes is always observed in objects. It is known that delay adversely affects stability, accuracy and quality of closed system [1]. There are many ways for solving this problem. The most widespread method is the use of delay compensation methods (for example, Smith, Reswick controllers etc.) [2]. These controllers had significant disadvantage from the point of view of delay element practical realization at analog engineering [3]. Realization of such element the delay time of which could be changed in wide range is rather difficult. Digital technique development allowed solving the given problem. However, in both cases, it is necessary to know rather exactly the mathematical model of object inertial part which does not contain delay and it is necessary to know exactly delay value as well [3].

The methods of parametric identification based on the fact that the accepted model should approximate well the experimental data became the most widespread. Accuracy of reclaimed delay value is determined to a large extent by correspondence of mathematical model to the object and namely its inertial part which does not contain delay. Delay time differs significantly from actual one at model structural deviations. Using such models for Smith type controllers does not result in desired results.

Mathematical models in which input action equality involves response reaction equality are called equivalent ones in paper [4]. It is stated in paper [5] that it is possible to say about strict equivalence at coincidence of object and model dynamic properties. The modified method of V. Viskovatov of structurally parametric identification suggested in [6] and based on continued fraction theory allows constructing discrete model strictly equivalent to continuous object.

The matter of the method is in the fact that calculated identifying matrix is formed on the basis of discrete in-output data. The first two lines of this matrix form consecutive measurements of input and output variables and the rest elements are calculated by recurrence relation until a line with null elements appears. The first column till a null line determines structure and values of parameters of discrete transfer function (DTF). And if in the second line in object reaction measurements (in deviations) the first k elements are null ones then the line shifts by k elements to the left. This shift determines delay in discrete representation with accuracy up to sampling increment parameter. Object DTF is obtained in the form:

$$G(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_n z^{-n}}{1 + a_1 z^{-1} + \ldots + a_n z^{-n}} z^{-d} = \frac{P_n(z)}{Q_n(z)} z^{-d}, \quad (1)$$

where n is the order of the model determined by matrix dimension and d determines delay by time $d\Delta t$. The obtained model (1) possesses the same dynamic properties as continuous object as one-to-one correspondence is determined between nulls and poles, continuous object $G(s)$ and discrete model with consistent Z-conversion $z=e^{s\Delta t}$. The carried out numerous model researches for objects: aperiodic, stable and unstable, nonminimum-phase etc. confirmed entirely the validity of redu-