Control, computer engineering and information science

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ARRANGEMENT OF DOMINANT POLES LOCALIZATION AREAS OF AUTOMATIC CONTROL INTERVAL SYSTEM IN SPECIFIED TRUNCATED SECTOR

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Characteristic polynomial of automatic control system has been considered. Its coefficients contain linearly entering interval parameters of control object and regulator adjustable parameters. The technique of determining regulator tunings supporting specified root quality indices of interval system was developed. The numerical illustration is given.

Sufficient quantity of papers [1–4] devoted to the problem of regulator synthesis for the systems with interval-specified parameters are known. The majority of methods for robust regulator synthesis suggested recently is based on optimization by different criteria and has a number of disadvantages [3, 4]:

- require computational intensity;
- result in obtaining high order regulators;
- allow synthesizing interval system by not more than two parameters of regulator.

It is known that dynamics of any linear system with constant coefficients depends essentially on arrangement of its dominant poles [5, 6]. Therefore, to support guaranteed dynamic properties of interval systems (IS) the principle of pole dominant arrangement is suggested to be used. According to the given principle the dominant poles should be arranged in the required way and the rest (free) poles should be arranged considerably to the left of the dominant ones for obtaining IS required quality.

Solving the problem of arrangement of stationary system dominant poles in specified points of complex plane is considered in a number of papers and solved by different methods: by nominal synthesis equations, interpolation method of dominant pole assignment [6] and on the basis of the method of *D*-partitioning [5].

As the coefficients of IS characteristic polynomial have fixed ranges then the system poles are allocated in certain closed regions which should be arranged in the required way. The required arrangement of the dominant and free poles assumes that their localization regions should not overrun at any magnitudes of interval parameters.

In paper [7] the location mode of IS pole localization region was suggested; it guarantees its acceptable oscillation and degree of stability. The given mode allows arranging localization regions of IS dominant poles in specified truncated sector and free ones – in the required region. However, application of the given mode is limited by the condition

$$\Theta_0(i-1) \notin (\frac{\pi}{2};-\Theta_0] \cap (-\frac{\pi}{2};\pi-\Theta_0], \qquad (1)$$

where Θ_0 is the angle determining IS acceptable oscillation, i is the index of interval coefficient, $i=\overline{1,n}$, n is the order of polynomial.

The method allowing arranging localization regions of IS dominant poles in truncated sector at any magnitude of angle is suggested in the paper.

Problem statement

Let IS characteristic polynomial is presented in the form:

$$R(p)=\sum_{i=0}^{n} a_i(\bar{k})p^i, \qquad (2)$$

$$a_{i\min}(\bar{k}) \le a_i(\bar{k}) \le a_{i\max}(\bar{k}), \qquad (3)$$

where $\bar{k}$ is the vector of regulator adjustable parameters linearly entering in the polynomial coefficients (2), $a_i(\bar{k})$ are the interval coefficients, p is the Laplacian.

The task is set: to search such magnitudes of parameters k_i, $i=1,2,...,r$, that at possible variations of polynomial interval coefficients (2) in the range (3) the regions of dominant poles were arranged in specified truncated sector and free poles were localized in the specified region (Fig. 1).

As interval coefficients enter the expression (2) then it corresponds to polynomial family. Oscillation and degree of stability of corresponding stationary system may be determined by roots of each of these polynomials. To estimate IS root quality indices it is desirable not to consider all polynomial family but to select only those of them which determine maximal oscillation and minimal degree of stability of IS at specified regulator parameters. In this case the required polynomials should be vertex ones that is to be specified by a set of limit values of interval coefficients. In the paper [7] the assigned task is solved by using only one vertex polynomial. However, the disadvantage of such solution is impossibility of applying the developed approach for all values of , specifying acceptable oscillation. Therefore, the method of robust regulator parametric synthesis free of the above mentioned limitation is proposed in the given paper.

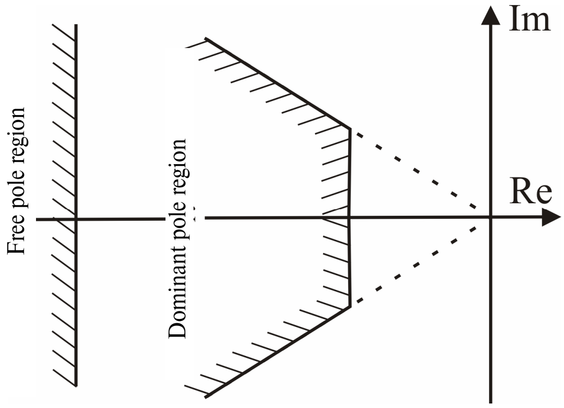

Fig. 1. *IS pole location regions*

Vertex polynomial selection

To extend the range of application of the method two vertex polynomials are suggested to be used at regulator synthesis. One of them – $R^v_{b1}(p)$ determines minimal degree of stability of IS and the second one – $R^v_{b2}(p)$ specifies maximal oscillation. In this case two conditions should be fulfilled simultaneously (Fig. 2): $\Theta_0<\Theta_i^q<\Theta_0+\pi$ for root p_0 and $\pi/2<\Theta_i^w<2\pi/2$ for root p_0'.

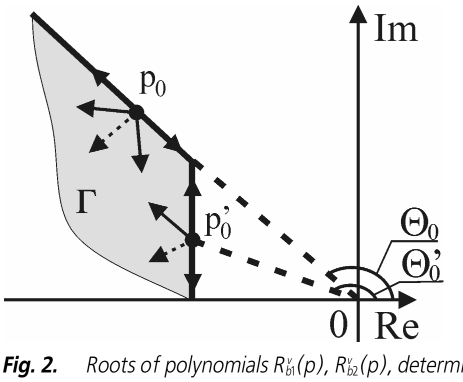

Fig. 2. *Roots of polynomials $R^v_{b1}(p)$, $R^v_{b2}(p)$, determining IS quality*

On the basis of the results of the paper [7] conditions of $R^v_{b1}(p)$ and $R^v_{b2}(p)$formation are obtained.

For polynomial $R^v_{b1}(p)$:

If $\Theta_0(i-1)\in(\pi/2;3\pi/2]$, then $a_i=a_{i\max}=\overline{a_i}$ and max$C_i=\Theta_0(i-1)-\pi/2$.

If $\Theta_0(i-1)\in(-\pi/2;\pi/2]$, then $a_i=a_{i\max}=\underline{a_i}$ and max$C_i=\Theta_0(i-1)+\pi/2$.

Magnitude maxC_i is necessary for determining the boundary of free pole arrangement region: $d_0=\dfrac{\beta_2}{\mathrm{tg}((\max C_i)/m)}+\alpha_2$, where m is the number of free poles, α_2, β_2 are the coordinates of dominant root of the obtained polynomial $R^v_{b1}(p)$, d_0 is the straight line to the left of which there is free pole region. If there are no free polynomials in this region then the obtained polynomial will not determine minimal degree of stability of IS.

For polynomial $R^v_{b2}(p)$:

If $i\Theta'_0\in(\pi;2\pi]$, then $a_i=a_{i\max}=\overline{a_i}$ and max$C_i=i\Theta'_0-\pi$.

If $i\Theta'_0\in(0;\pi]$, then $a_i=a_{i\max}=\underline{a_i}$ and max$C_i=i\Theta'_0$.

As value Θ'_0 is unknown then the conditions for polynomial $R^v_{b2}(p)$ formation could not be used. Let polynomials $R^v_{b1}(p)$ and $R^v_{b2}(p)$ are vertex ones of one edge branch and therefore they differ in the range of only one coefficient. Then for determining $R^v_{b2}(p)$ it is necessary to find such coefficient of polynomial $R^v_{b1}(p)$ the range of which is changed to the opposite one at sector angle changing from Θ_0 to Θ'_0 and changing condition of vector direction of edge branch output. For this purpose, after formation of polynomial $R^v_{b1}(p)$, Θ'_0 are determined according to the ranges of coefficients a_i, obtained at each i from conditions: $i\Theta'_0\in(\pi;2\pi]$ at $\overline{a_i}$ or $i\Theta'_0\in(0;\pi]$ at $\underline{a_i}$. If the condition $\Theta_0<\Theta'_0<\pi$ is not fulfilled at coefficient with index i then $R^v_{b2}(p)$ differs from $R^v_{b1}(p)$ in the range of this coefficient.

Let us consider the method of regulator parameter selection at two determinative vertex polynomials. Let characteristic equation of linear continuous control system is reduced to the form

$$\sum_{i=1}^{r} k_i\, A_i(p)+B(p)=0, \qquad (4)$$

where k_i, $i=1,2,...,r$ are the parameters the values of which should be chosen so that the required control quality is supported, $A_i(p)$, $i=1,2,...,r$, $B(p)$ are the polynomials.

Let root p_0 is on the straight line $\Lambda(\alpha)=-\alpha+j\beta(\alpha)$ determining maximal IS oscillation. If $\beta(\alpha)=\alpha\cdot\mathrm{tg}(\pi-\Theta_0)$, then $\Lambda(\alpha)=-\alpha+j\alpha\cdot\mathrm{tg}(\pi-\Theta_0)$.

Substituting

$$p(\alpha)=\Lambda(\alpha)=-\alpha+j\beta=-\alpha+j\alpha\cdot\mathrm{tg}(\pi-\Theta_0)$$

in (4) we obtain

$$k_1\cdot A_1(\Lambda(\alpha))+\sum_{i=2}^{r}k_i\cdot A_i(\Lambda(\alpha))+B(\Lambda(\alpha))=0,$$
$$i=1,\ldots,l. \qquad (5)$$

Equations (5) connect the varied parameters k_i, $i=1,...,r$ with roots $\lambda_j(\alpha)$, $j=1,...,l$. Let us presenting the equation system (5) in a matrix form

$$\mathbf{Q}_{11}(\alpha)\cdot k_1+\mathbf{Q}_{12}(\alpha)\cdot\mathbf{g}_2=\mathbf{R}_1(\alpha), \qquad (6)$$

where

$$\mathbf{Q}_{11}(\lambda)=\begin{bmatrix}A_1(\lambda_1(\alpha))\\ \dots\\ A_1(\lambda_l(\alpha))\end{bmatrix};$$

$$\mathbf{Q}_{12}(\lambda)=\begin{bmatrix}A_2(\lambda_1(\alpha)) & \cdots & A_r(\lambda_1(\alpha))\\ \dots & \dots & \dots\\ A_2(\lambda_l(\alpha)) & \dots & A_r(\lambda_l(\alpha))\end{bmatrix};$$

$$\mathbf{g}_2=\begin{bmatrix}k_2\\ \cdots\\ k_r\end{bmatrix};\quad \mathbf{R}_1(\lambda)=\begin{bmatrix}-B(\lambda_1(\alpha))\\ \cdots\\ -B(\lambda_l(\alpha))\end{bmatrix}.$$

To arrange free poles in specified region let us substitute $p=-\delta(\omega)+j\omega$ in (4). And obtain:

$$k_1\cdot A_1(-\delta(\omega)+j\omega)+\sum_{i=2}^{r}k_i\cdot A_i(-\delta(\omega)+j\omega)+$$
$$+B(-\delta(\omega)+j\omega)=0, \qquad (7)$$

In matrix form (7) has the form

$$\mathbf{Q}_{21}(\omega)\cdot k_1+\mathbf{Q}_{22}(\omega)\mathbf{g}_2=\mathbf{R}_2(\omega), \qquad (8)$$

where

$$\mathbf{Q}_{21}(\omega)=A_1(-\delta(\omega)+j\omega);$$
$$\mathbf{Q}_{22}(\omega)=[A_2(-\delta(\omega)+j\omega)\ \dots\ A_r(-\delta(\omega)+j\omega)];$$
$$\mathbf{R}_2(\omega)=-B(-\delta(\omega)+j\omega).$$

To determine the equation of D-partitioning boundary let us combining (6) and (8) in one equation system

$$\begin{cases}\mathbf{Q}_{11}(\alpha)\cdot k_1+\mathbf{Q}_{12}(\alpha)\cdot\mathbf{g}_2=\mathbf{R}_1(\alpha),\\ \mathbf{Q}_{21}(\omega)\cdot k_1+\mathbf{Q}_{22}(\omega)\mathbf{g}_2=\mathbf{R}_2(\omega).\end{cases} \qquad (9)$$

From the first equation of the system (9) we have

$$\mathbf{g}_2=\mathbf{Q}_{12}^{-1}(\alpha)\cdot\mathbf{R}_1(\alpha)-\mathbf{Q}_{12}^{-1}(\alpha)\cdot\mathbf{Q}_{11}(\alpha)\cdot k_1. \qquad (10)$$

After substitution of (9) into the second equation of the system (8) we obtained the required equation of D-partitioning boundary

$$k_1(\omega)=\frac{\mathbf{R}_2(\omega)-\mathbf{Q}_{22}(\omega)\cdot\mathbf{Q}_{12}^{-1}(\alpha)\cdot\mathbf{R}_1(\alpha)}{\mathbf{Q}_{21}(\omega)-\mathbf{Q}_{22}(\omega)\cdot\mathbf{Q}_{12}^{-1}(\alpha)\cdot\mathbf{Q}_{11}(\alpha)}. \qquad (11)$$

Then, specifying values ω in the range from $-\infty$ to ∞ and probable values α we construct the boundaries of D-partitioning on complex plane on the basis of (11). They divide plane of the parameter k_1 into a number of regions among which the region corresponding to the required arrangement of system free poles (if there is) should be segregated.

After choosing the value k_1 the functions of dependent parameter value $k_2(\alpha)$,..., $k_r(\alpha)$ are computed on the basis of the expression (10).

Let the second root p_0' is on the straight line $\Lambda_j'(\beta')=-\alpha'+j\beta'$ parallel to the imaginary axis and determining minimal degree of IS stability.

Substitution of $p_j(\beta')=\Lambda_j'(\beta')=-\alpha'+j\beta'$, $j=1,...,l$, into (4) gives equations

$$k_1\cdot A_1(\Lambda'_j(\beta'))+\sum_{i=2}^{r}k_i\cdot A_i(\Lambda'_j(\beta'))+$$
$$+B(\Lambda'_j(\beta'))=0,\quad j=1,\dots,l. \qquad (12)$$

These equations connect the varied parameters k_i, $i=1,...,r$ with roots being on the straight line $\Lambda_j'(\beta')$, $j=1,...,l$.

Let us presenting the equation system (12) in matrix form

$$\mathbf{Q}_{11}(\beta')\cdot k_1+\mathbf{Q}_{12}(\beta')\cdot\mathbf{g}_2=\mathbf{R}_1(\beta'), \qquad (13)$$

where

$$\mathbf{Q}_{11}(\lambda)=\begin{bmatrix}A_1(\lambda'_1(\beta'))\\ \dots\\ A_1(\lambda'_l(\beta'))\end{bmatrix};$$

$$\mathbf{Q}_{12}(\lambda)=\begin{bmatrix}A_2(\lambda'_1(\beta')) & \cdots & A_r(\lambda'_1(\beta'))\\ \dots & \dots & \dots\\ A_2(\lambda'_l(\beta')) & \dots & A_r(\lambda'_l(\beta'))\end{bmatrix};$$

$$\mathbf{g}_2=\begin{bmatrix}k_2\\ \cdots\\ k_r\end{bmatrix};\quad \mathbf{R}_1(\lambda)=\begin{bmatrix}-B(\lambda'_1(\beta'))\\ \cdots\\ -B(\lambda'_l(\beta'))\end{bmatrix}.$$

To determine the equation of D-partitioning boundary let us combining (8) and (13) into one equation system

$$\begin{cases}\mathbf{Q}_{11}(\beta')\cdot k_1+\mathbf{Q}_{12}(\beta')\cdot\mathbf{g}_2=\mathbf{R}_1(\beta'),\\ \mathbf{Q}_{21}(\omega)\cdot k_1+\mathbf{Q}_{22}(\omega)\mathbf{g}_2=\mathbf{R}_2(\omega).\end{cases} \qquad (14)$$

From the first equation of the system (14) we have:

$$\mathbf{g}_2=\mathbf{Q}_{12}^{-1}(\beta')\cdot\mathbf{R}_1(\beta')-\mathbf{Q}_{12}^{-1}(\beta')\cdot\mathbf{Q}_{11}(\beta')\cdot k_1. \qquad (15)$$

After substitution of the obtained expression (15) for the vector $\mathbf{g}_2$ of dependent varied parameters into the second equation of the system (14), we obtain the required equation of D-partitioning boundary:

$$k_1(\omega)=\frac{\mathbf{R}_2(\omega)-\mathbf{Q}_{22}(\omega)\cdot\mathbf{Q}_{12}^{-1}(\beta')\cdot\mathbf{R}_1(\beta')}{\mathbf{Q}_{21}(\omega)-\mathbf{Q}_{22}(\omega)\cdot\mathbf{Q}_{12}^{-1}(\beta')\cdot\mathbf{Q}_{11}(\beta')}. \qquad (16)$$

After selection of value k_1 from the found region of D-partitioning on the basis of (15) functions of values of dependent parameters $k_2(\beta')$,...,$k_r(\beta')$ are computed.

Solving the system of equations, $k_i(\alpha)$,...,$k_i(\beta')$, $i=\overline{2,r}$, we find α and β'. Substituting the searched values α and β' into (15), we obtain the value of regulator adjustments.

The technique of pole arrangement by two vertex polynomials

On the basis of carried out investigations the technique of IS poles arrangement in specified truncated sector by two vertex polynomials including the following stages was developed.

1. The required quality indices of IS: maximal oscillation and minimal degree of stability are specified.
2. Minimal value maxC_i and corresponding set of ranges of polynomial coefficients $R_{b1}^{v}(p)$ are determined from the conditions of polynomial $R_{b1}^{v}(p)$ formation.
3. According to the ranges of coefficients of polynomial $R_{b1}^{v}(p)$ from conditions $i\Theta_0'\in(\pi;2\pi]$ at $a_i=\overline{a_i}$ and $i\Theta_0'\in(0;\pi]$ at $a_i=\underline{a_i}$ are determined Θ_0'. If the condition $\Theta_0<\Theta_0'<\pi$ is not fulfilled at a certain coefficient then $R_{b2}^{v}(p)$ differs from $R_{b1}^{v}(p)$ in this coefficient range.
4. The equation of straight line d_0 is determined.

5. The straight line $\Lambda_j(\alpha)=-\alpha+j\alpha\cdot\mathrm{tg}(\pi-\Theta_0+0)$ determining maximal oscillation of IS is specified and boundaries of D-partitioning in the space of parameter k_1 at different values α are constructed.
6. The straight line $\Lambda_j'(\beta')=-\alpha'+j\beta'$ determining minimal degree of stability of IS is specified and boundaries of D-partitioning in space of parameter k_1 at different values α are constructed.
7. Value k_1 is selected from the regions of D-partitioning obtained in 5, 6 of the technique and functions of magnitudes of dependent parameters $k_2(\alpha)$,..., $k_r(\alpha)$ and $k_2(\beta')$,..., $k_r(\beta')$ are determined.
8. Solving the system of r–1 equations $k_i(\alpha)$,..., $k_i(\beta')$, $i=\overline{2,r}$, values α and β' are found and regulator adjustments are determined.
9. Arrangement of localization regions of dominating and free poles of IS with the obtained adjustments is checked. In case of region overrunning a number of adjusted parameters should be increased or system requirements should be changed.

Example

Let us consider the example of searching the IS regulator adjustments for arrangement of two dominant poles supporting minimal degree of stability equal 1 in truncated sector with angle $\Theta_0=\pm7\pi/9$ specifying maximal oscillation.

Let the control object and regulator are described respectively by transfer functions:

$$W_0(p)=\frac{1}{a_4p^3+a_3p^2+b_2p+b_1}; \qquad (17)$$

$$W_\text{p}(p)=\frac{k_3p^2+k_2p+k_1}{p}. \qquad (18)$$

On the basis of (17) and (18) we obtain the characteristic equation of the system:

$$a_4p^4+a_3p^3+(b_2+k_3)p^2+(b_1+k_2)p+k_1=0,$$

where $a_2=b_2+k_3$, $a_1=b_1+k_2$, $a_0=k_1$, a_4=[0,7;0,8], a_3=[17;19], b_2=[198;200], b_1=[1025;1026].

The ranges of polynomial coefficients corresponding to sector $\Theta_0=\pm7\pi/9$ are determined from conditions of $R_b^v(p)$ formation. In this case the least value maxC_i=maxC_2=$5\pi/18$. The obtained polynomial has the ranges: $\overline{a_0}\underline{a_1}\overline{a_2}\underline{a_3}\underline{a_4}$, where

$$\underline{a_4}=0,7,\ \underline{a_3}=17,\ \overline{a_2}=200+k_3,\ \underline{a_1}=1025+k_2.$$

Let us determining the ranges of coefficients of the second vertex polynomial defining the degree of stability: $\overline{a_0}\underline{a_1}\overline{a_2}\underline{a_3}\overline{a_4}$. Let us specifying the boundaries of positioning roots determining maximal oscillation and minimal degree of stability:

$$\Lambda_j(\alpha)=-\alpha+j\alpha\cdot\mathrm{tg}(\frac{2}{9}\pi),\ \ \Lambda'_j(\beta')=-1+j\beta'.$$

To determine straight line d_0 let us specifying certain assumed maximal values α_2 and β_2

$$d_0=\frac{b_2}{\mathrm{tg}((\max C_i)/m)}+a_2=\frac{1.1}{\mathrm{tg}((5\pi/18)/2)}+1,2=3,5.$$

To support dominance principle let us specifying the boundary of IS free poles $X(j\beta)=-8$, $-\infty<\beta<\infty$. Then, specifying by values ω, in the range from $-\infty$ to ∞, probable values $\alpha\in(1;1,15)$ and $\beta'\in(0;0,84)$ the boundaries of two regions of D-partitioning are constructed on complex plane (Fig. 3, 4).

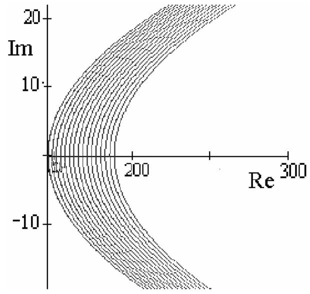

Fig. 3. *Regions of D-partitioning by $k_1(\alpha)$*

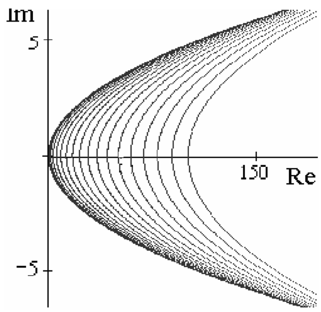

Fig. 4. *Regions of D-partitioning by $k_1(\beta')$*

At common value k_1=200 chosen among them we obtain k_2=–762,6 and k_3=–49,7. Regions of IS pole localization given in Fig. 5, 6 are constructed for the given regulator adjustment.

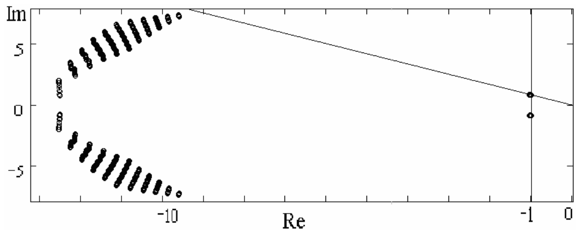

Fig. 5. *Arrangement of IS pole localization regions*

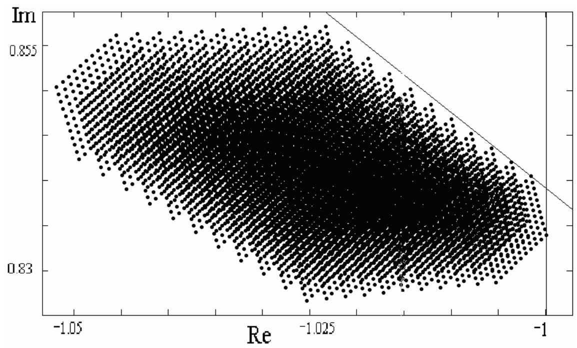

Fig. 6. *Arrangement of IS dominant pole localization region*

It can be concluded from Fig. 5, 6 that the obtained regulator adjustments support the specified quality in IS.

Conclusion

The suggested approach allows arranging localization regions of system dominant poles with interval coefficients of characteristic polynomial in specified truncated sector with any angle Θ_0, that is supporting guaranteed oscillation and degree of stability of IS.

The suggested technique is not highly computational and allows carrying out parametric synthesis of low order regulator.

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PARAMETRIC SYNTHESIS OF LINEAR REGULATOR IN INTERVAL SYSTEM WITH GUARANTEED ROOT QUALITY INDICES

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Automatic control system containing proportional-plus-integral action regulator and control object which has interval specified parameters has been considered. Using robust expansion of root-locus method the technique of synthesis of proportional-plus-integral action regulator parameters guaranteeing minimal degree of stability and maximal degree of system oscillation was developed. The technique is based on vertex analysis of root quality indices applying the equation of Theodorchik-Evans. The numeric illustration is given.

Introduction

In real systems of automatic control there are cases when some their parameters are not known exactly or change in the system maintenance process by laws unknown beforehand and their values can not be available for measuring. If the ranges of possible values of constant parameters or unstable parameters are known then it is said about parametric interval uncertainty. Systems having interval-indefinite parameters were called interval ones.

Designing interval system the main task is in supporting the desired quality of its functioning at any possible values of interval-indefinite parameters. Let us introduce root indices of system quality: degree of stability α and oscillation φ. It is obvious that at system parameter instability these quality indices may be changed. Therefore, the task of supporting the guaranteed minimal degree of stability and maximal oscillation in interval system is of interest.

To specify the desired quality of the system corresponding to these root indices the sector ABCD (Fig. 1) specifying the boundary of localization region of roots Γ may be used.

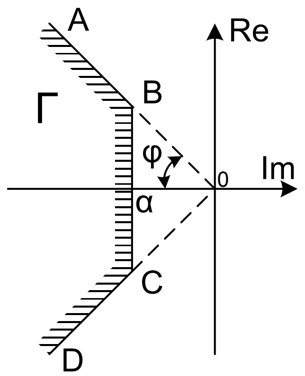

Fig. 1. *The region of desired root arrangement*

1. Problem statement

Let us consider the system of automatic control, Fig. 2.