

Thus, the derived formulas may be used for calculating parameters of pulsed current sources with inductive energy storages. On the basis of analysis of formulas and carried out calculations the following conclusions may be stated.

1. The oscillating charge is the most efficient when the equivalent inductance is rather lower than the critical one ( $\lambda \ll 1$ ) and more than 50 % of generator energy may be transferred to the load.
2. To support high efficiency of the source at low average power of generator it is necessary to have maximal constants of the charge  $\tau_c$  and storage  $\tau$ .
3. Increasing initial stress of generator  $U_g$  at constant parameters  $W, j_m, \gamma, K_z$  and  $r_g$  the efficiencies of the charge  $\eta_c$  and generator energy transfer to the load  $\eta$

grow as the current achieves the required maximal value  $I_m$  at low magnitudes of  $\lambda$ .

4. Application of accumulator batteries, unipolar and synchronous generators as generators for aperiodic charge of inductive storage is inefficient owing to low magnitudes  $\eta_c$  and  $\eta$ ; use of these generators at oscillating charge requires implementation of special constructive solutions directed to increase of rate of current rise, i.e. to increase of stress  $U_g$  and decrease of inductance  $L_g$  and resistance  $r_g$ .
5. Oscillating charge of inductive storage from a capacitor bank is the most efficient and appropriate especially if the battery has high density of accumulated energy  $W_g$ , which can exceed 3 J/g [6].

#### REFERENCES

1. Explosive generators of electric current power pulses / E.I. Asinovskiy, E.F. Lebedev, A.A. Leontiev et al; ed. by V.E. Fortov. – Moscow: Nauka, 2002. – 398 p.
2. High power pulsed systems / Ed. by E.I. Asinovskiy. – Moscow: Mir, 1981. – 248 p.
3. Glebov I.A., Kasharskiy E.G., Rutberg F.G. Synchronous generators of short-term and impact action. – Leningrad: Nauka, 1985. – 224 p.
4. Electrotechnical reference book: 3 volumes, V. 1. General questions. Electrotechnical materials / Ed. by professor of MEI V.I. Gerasimov et al. – Moscow: Energoatomizdat, 1985. – 488 p.
5. Physics and engineering of power pulsed systems / Ed. by E.P. Velikhov. – Moscow: Energoatomizdat, 1987. – 352 p.
6. Aviation week and space technology. – 1990. – V. 132. – № 19. – P. 88–89.

Received on 25.09.2007

UDC 621.311.001

## UNIVERSAL MATHEMATICAL MODEL OF POWER THREE-PHASE TRANSFORMERS AND AUTOTRANSFORMERS

A.S. Gusev, S.V. Svechkarov, I.L. Plodistiy

Tomsk Polytechnic University  
E-mail: Svech@tpu.ru

*The substantiation of necessity in essential increase of completeness and reliability of modeling processes in energy systems has been shown. The results of synthesis of universal mathematical model of one of the main elements of energy systems – power transformers and autotransformers are given. The demanded quality of reproduction of processes is confirmed by experience of using the developed model in structure of all-mode multiprocessing modeling complexes of real time of the hybrid type. The examples illustrating quality of process modeling are shown.*

According to statistics [1, 2] about 50 % of severe failures in electric systems (ES), including EES, occurs due to incorrect actions of dispatcher staff, relaying, manufacturing and emergency automation, the main reason of which is the use of under-complete and under-reliable information on possible processes, especially emergency ones, in EES, at design, commissioning and maintenance.

EES specific character excludes practically a possibility of obtaining this information in a full-scale way and extraordinary complication of up-to-date EES restricts considerably applicability of their physical modeling. As a result, the main method of obtaining information on various normal and emergency processes in EES is mathematical modeling, the possibilities of which depend on presence:

- and use of rather accurate mathematical models for all kinds and types of equipment;
- means capable of solving reliably and efficiently the EES equation systems formed by these models.

The stated constantly high emergency component in EES indicates objectively the fact that the existing implementations of these factors do not support the completeness and reliability of mathematical modeling required for its considerable decrease and in particular, for efficient dispatcher control of its operability.

The detailed analysis of these facts and their interaction is given in [3–8]; the urgent need and topicality of further development of these factors follows from this. It is obvious also that the first of them becomes logically first-priority.

The level of mathematical description of the processes in various elements of energy equipment achieved by the present allows setting and solving the problems of substantiated synthesis of mathematical models for all kinds and types of the equipment applied in EES, describing the whole spectrum of the processes rather completely and reliably without decomposition.

Power transformers and autotransformers, being integral and significant elements of transformation and distribution of electric energy, influence greatly the processes in EES in whole. Therefore, to achieve the required completeness and reliability of process reproduction in EES, their mathematical model should be rather high-precision and take into account the technological and structurally required variety of the transformers and autotransformers applied in EES: with general and separate magnetic cores for windings of each phase; with different quantity of windings (2–4) in each phase, including electrically independent circuits split into (2–4) as well as three applied connecting circuits of phase windings ( $Y_0, Y, \Delta$ ) at each voltage stage and their possible combinations. The listed types of transformers and autotransformers may be combined by a virtual adaptable five-winding transformer and take it as a prototype for synthesis of mathematical model universal for all given types.

Forming the equation system, describing the electromagnetic processes in this transformer, the embodiments of windings and magnetic cores are considered; they include the armoured-bar ones which allow, in particular, neglecting the electromagnetic interference of various phase windings without substantial damage for accuracy of process reproduction and taking into account the interaction of each phase winding only with its own stray fluxes and the main magnetic flux of its phase, as well as the possibility of magnetic core saturation for this flux [9]. Such presentation allows imaging the three-phase group and the three-phase transformers and autotransformers by the given universal mathematical model. Besides, phase unbalance can be introduced if necessary.

According to the denoted approach in modeling transformers and autotransformers, their universal mathematical model should combine the equation systems for three phases of each of five windings. Applying the proper indexing of phases and numbers of windings for model compact presentation, this model may be described by the equation system of:

- 1) magneto-connected by phase flux of winding circuits of the form

$$\omega_{i\xi} \frac{d\Phi_{\xi}}{dt} + L_{i\xi} \frac{di_{i\xi}}{dt} + r_{i\xi} i_{i\xi} - u_{i\xi} = 0,$$

where  $i=1,2,\dots,5$  is the number of winding, and  $\xi=A,B,C$  is the phase index subject to which:  $\omega_{i\xi}$  is the number of turns;  $\Phi_{\xi}$  is the instantaneous value of the main magnetic flux;  $L_{i\xi}$  is the leakage inductance;  $i_{i\xi}$  is the instantaneous current;  $r_{i\xi}$  is the active resistance;  $u_{i\xi}$  is the instantaneous voltage,

- 2) balance of magnetomotive force for each phase

$$\sum_{i=1}^{i=5} i_{i\xi} \omega_{i\xi} = F_{\mu\xi},$$

where  $F_{\mu\xi}$  is the excitation of phase electromagnetic system of five-winding transformer, determined subject to the possible steel saturation for  $\Phi_{\xi}$  by approximating expression [10]  $F_{\mu\xi} = K_{\mu} \Phi_{\xi}^{\rho}$ , in which  $K_{\mu}$  is the coefficient of dimension implemented at transition to relative units. Approximation of magnetization curve is specified by power dependence with odd or fractional magnitudes of the index in the range of 3...5, and fractional values are used for extra accurate approximation,

- 3)  $u_{i\xi}$  formation depending on winding connecting circuit  $\omega_{iA}, \omega_{iB}, \omega_{iC}$ :

$$u_{iA} = (u_{Ai} - u_{Bi}) / \sqrt{3}; \quad u_{iB} = (u_{Bi} - u_{Ci}) / \sqrt{3};$$

$$u_{iC} = (u_{Ci} - u_{Ai}) / \sqrt{3} \quad \text{for } \Delta;$$

$$u_{iA} = u_{Ai}; \quad u_{iB} = u_{Bi}; \quad u_{iC} = u_{Ci} \quad \text{for } Y \text{ and } Y_0,$$

where  $u_{Ai}, u_{Bi}, u_{Ci}$  are the instantaneous values of proper phase voltages.

Additionally, for excluding residual currents in respect to winding connecting circuit  $Y$  and for supporting various a priori unregulated functional possibilities connected with zero string the equations

$$i_{Ai} = i_{Ai} - K_{0i} \cdot i_{i0}; \quad i_{Bi} = i_{Bi} - K_{0i} \cdot i_{i0};$$

$$i_{Ci} = i_{Ci} - K_{0i} \cdot i_{i0}; \quad i_{i0} = \frac{1}{3}(i_{iA} + i_{iB} + i_{iC}),$$

are included into this system, where:  $i_{Ai}, i_{Bi}, i_{Ci}$  are the instantaneous values of proper phase currents;  $K_{0i}$  is the adjusted coefficient, the intermediate (between the end) values of which allow simulating, if necessary, the specific conditions of residual current flow.

For reproduction of the magnetization curve  $F_{\mu\xi} = f(\Phi_{\xi})$  the implicit equation [11]

$$F_{\mu\xi} = \Phi_{\xi}^2 - \alpha \Phi_{\xi} (\Phi_{\xi} - F_{\mu\xi}) \approx \Phi_{\xi}^{\rho},$$

supporting more flexible and efficient approximation due to variation of the coefficient  $\alpha$  may be used by exponential approximation expression.

It should be noted that, as at least one of windings, for example  $\omega_1$ , of any transformer is obligatory exciting (feeding) and another one, for example  $\omega_5$ , turns out to be passive (receiving), then the signs in the equations for circuits and magnetomotive forces for these windings become definite while the signs in the equations of the rest of windings depend on concrete assignment of the latter.

It is seen from the given equation system that any type of power transformer and autotransformer may be simulated by specifying and variation of mathematical model parameters (coefficients in the equations).

The examined mathematical model is reliably and thoroughly tested in laboratory environment at classical no-load, impedance-drop and nominal load tests; as well as in practice, that is the most important and co-

gent, composed of fully-variable multiprocessor modeling complexes of real time of hybrid type, developed and made for training and research aims of Electro-technical institute departments at Tomsk polytechnic university and for «Tyumenenergo». Hybrid modeling complex of Tyumen power system (GMK TE) was long and successfully tested in central dispatcher service of «Tyumenenergo». On the basis of the examined model, all block and network transformers and autotransformers are reproduced in GMK TE. The results of maintenance confirm completely and visually the considerable increase of quality and reliability of modeling various normal and emergency processes in TE obtained owing to application of much more accurate mathematical models for all elements of EES and, in particular, synthesized for transformers and autotransformers, in comparison with the usually used ones. Separate fragments from the file of numerous results of GMK TE pilot operation illustrating the quality of reproduction of emergency processes in EES are given in Fig. 1–4. They are greatly influenced by completeness and reliability of modeling the processes directly in transformers and autotransformers.

The current and voltage oscillograms introduced in Fig. 1–4 image the most often, securely recognizable and estimated by specialists symmetric and asymmetric emergency processes in EES.

It may be additionally noted that on the basis of the results of operational testing of the earlier developed GMK TE and on the basis of up-to-date achievements of integral microelectronics, microprocessor technology and hardware-information technologies the project of modernized fully-variable modeling complex of real time of hybrid type for «FSK EES» is developed and delivered; the examined universal mathematical model is used in it for modeling transformers and autotransformers.

### Conclusion

1. The universal mathematical model of one of the main power system elements – power transformer and autotransformer is synthesized. The required quality of process reproduction is confirmed by the experience of using the developed model in composition of fully-variable multiprocessor modeling complexes of real time of hybrid type.
2. The examined universal mathematical model allows reducing more accurately the whole spectrum of processes in any power transformers and autotransformers used in EES without decomposition. Its application at modeling of processes in EES allows increasing significantly the reliability of calculations.

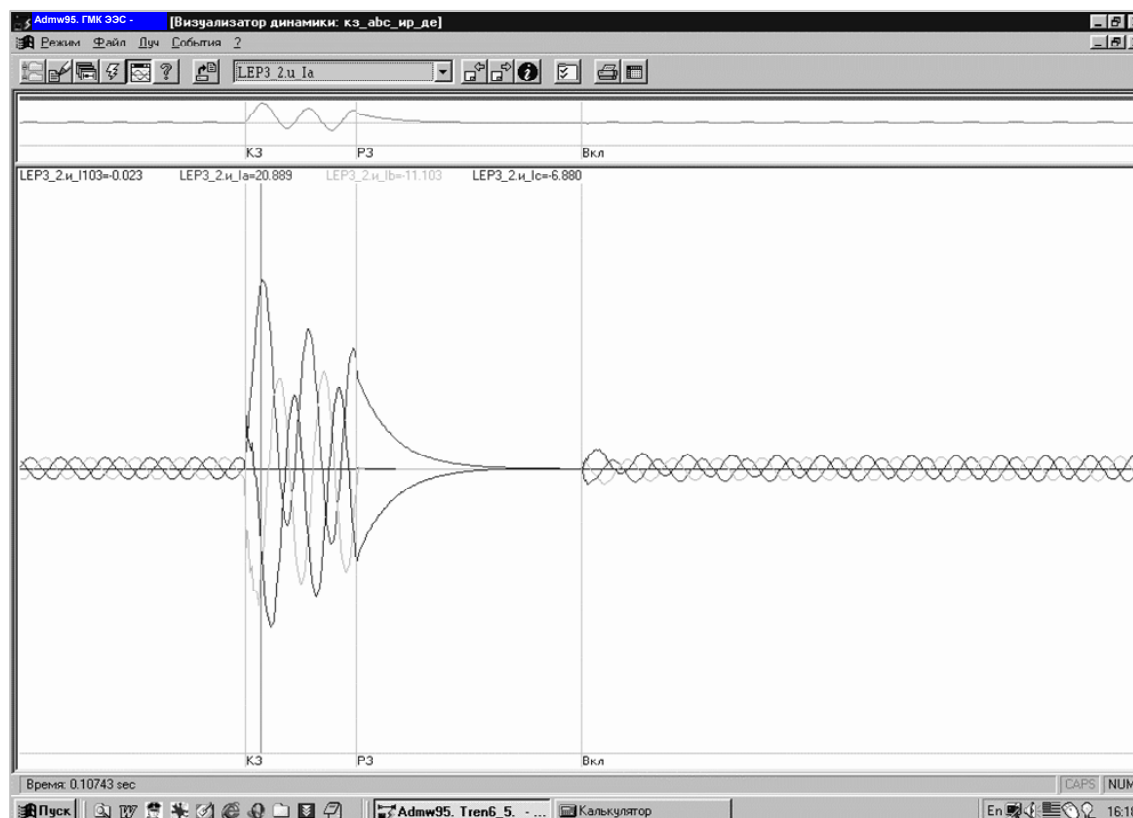


Fig. 1. Phase currents at modeling three-phase short circuit

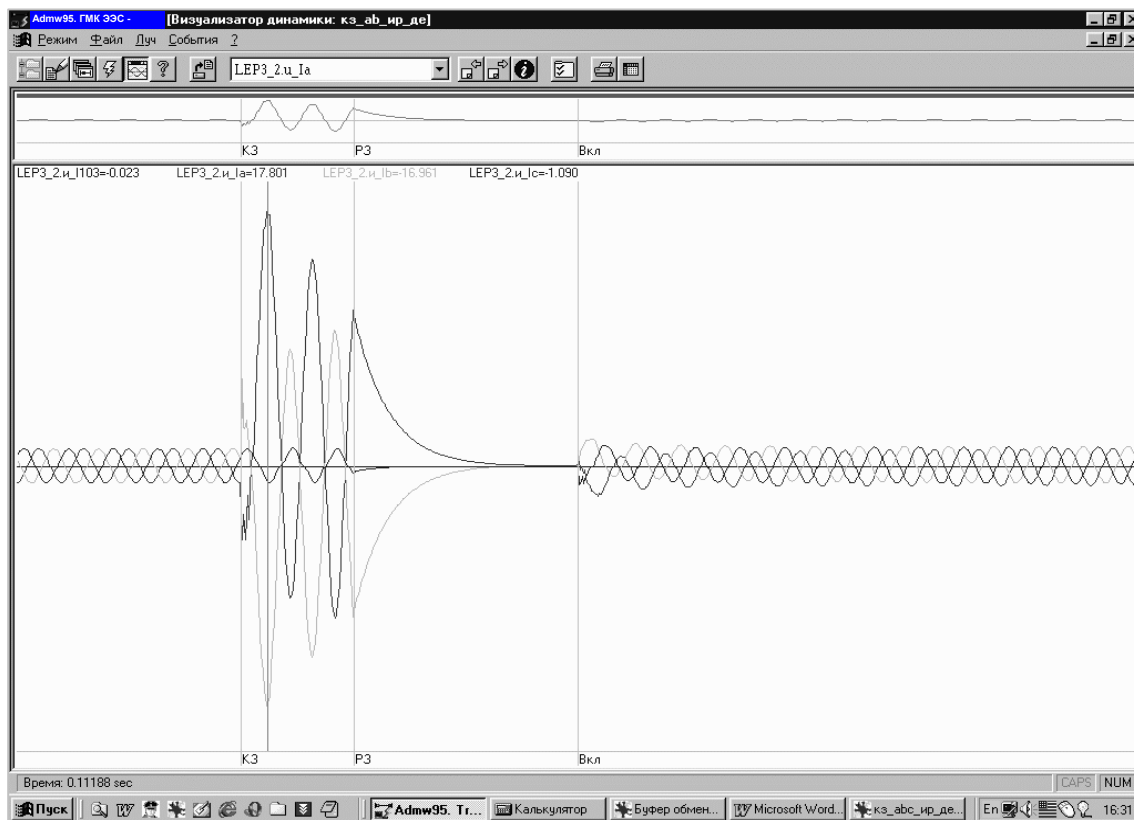


Fig. 2. Phase currents and residual current at interphase short circuit of two phases

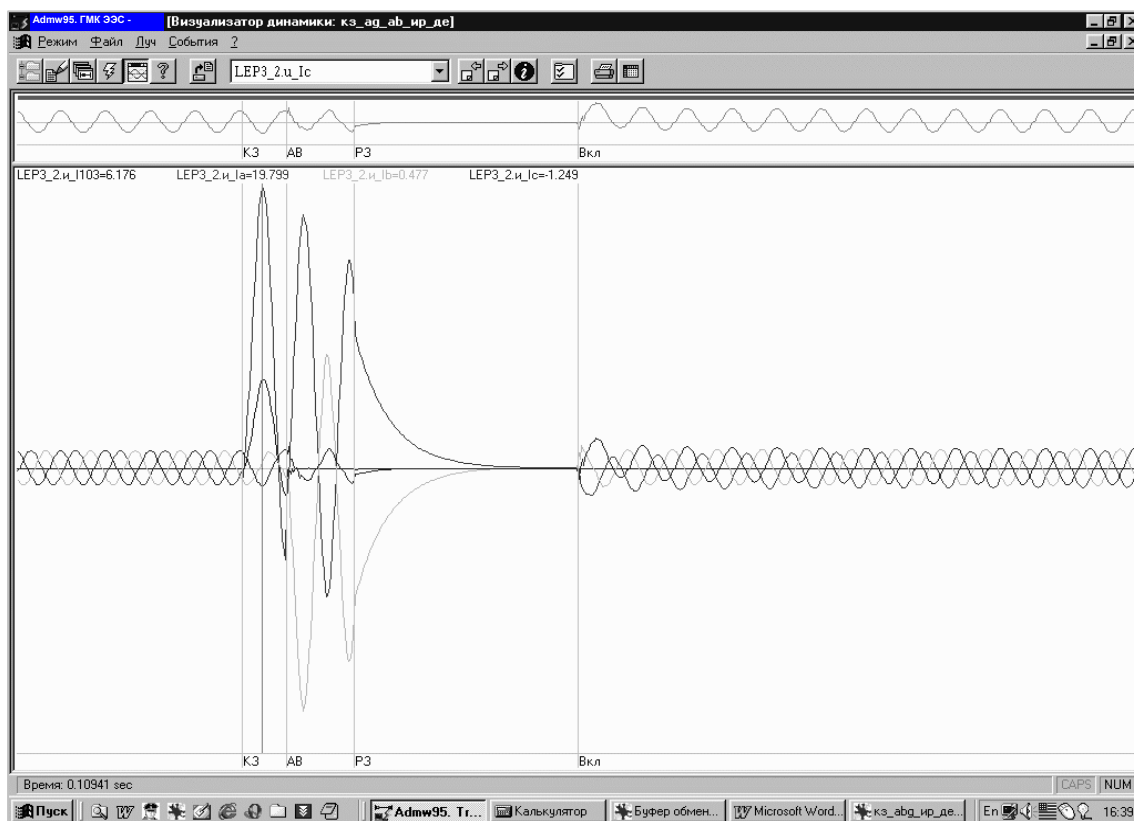


Fig. 3. Phase current and residual current at single-phase and consequent two-phase short circuits

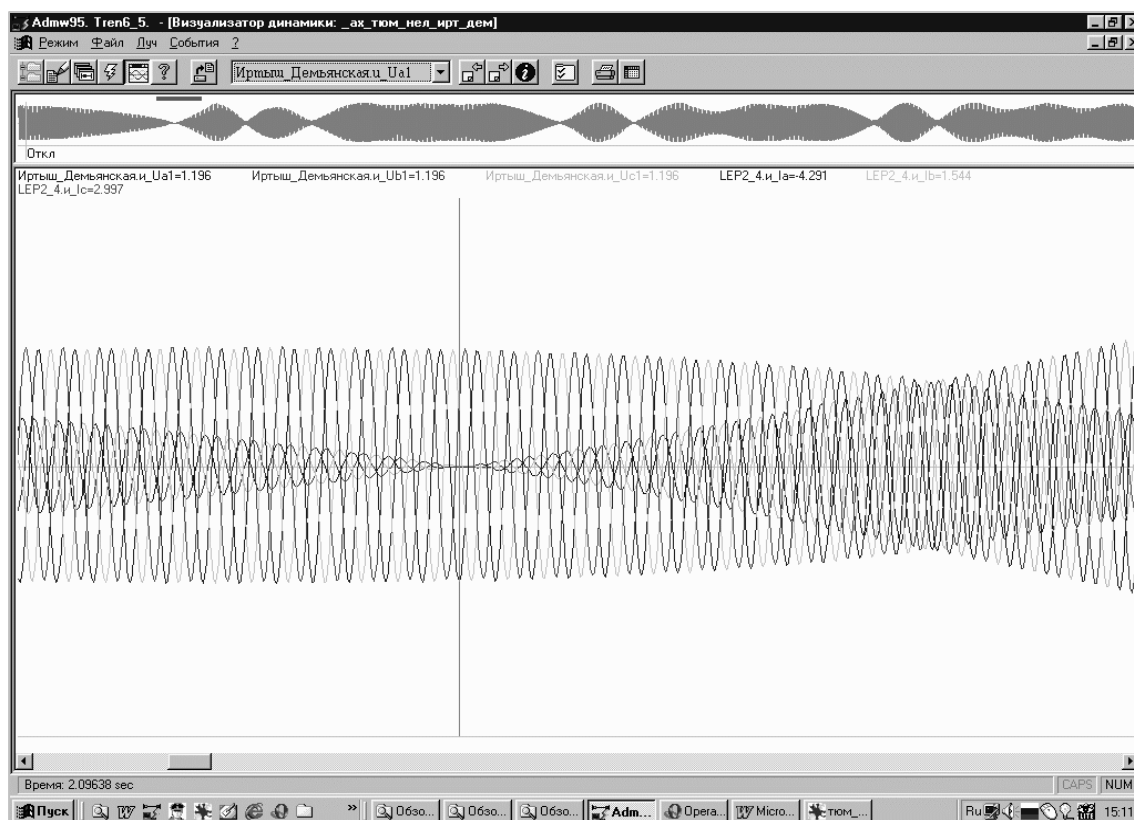


Fig. 4. Phase currents and voltages at modeling asynchronous mode in EES

#### REFERENCES

1. Skopintsev V.A., Moroshkin Yu.V. The analysis and prediction of emergency in electric power systems // *Elektrichestvo*. – 1997. – № 11. – P. 2–8.
2. Control of power energy combinations / Ed. by S.A. Sovalov. – Moscow: Energoatomizdat, 1984. – 256 p.
3. Kheming R.V. Numerical methods: Translated from English / Ed. by R.S. Guter. – Moscow: Nauka, 1968. – 400 p.
4. Babushka I., Vitasek E., Prager M. Numerical processes of solving the differential equations: Translated from English / Ed. by G.I. Marchuk. – Moscow: Mir, 1969. – 368 p.
5. Hall J., Watt J. The modern numerical methods of solving ordinary differential equations: Translated from English / Ed. by A.D. Gorbunov. – Moscow: Mir, 1979. – 312 p.
6. Pogosyan T.A. An error in calculation of electromechanical transients in electric systems // *Elektrichestvo*. – 1984. – № 3. – P. 54–56.
7. Verzhbitskiy V.M. Numerical methods (mathematical analysis and ordinary differential equations). – Moscow: Vysshaya Shkola, 2001. – 382 p.
8. Gusev A.S., Svechkarev S.V., Plodisty I.L. The problem of power system modeling, the concept hybrid solution // *The 10<sup>th</sup> IFAC/IFORS/IMACS/IFIP Symp. in Large Scale Systems: Theory and Applications (LSS 2004)*. – Japan, Osaka, Osaka International Convention Center, July 26–28, 2004. – V. 1. – P. 440–445.
9. Leites L.V., Pintsov A.M. The equivalent circuits of multiwinding transformers. – Moscow: Energiya, 1974. – 192 c.
10. Bernas S., Tsek Z. Mathematical models of electric power system elements. – Moscow: Energoizdat, 1982. – 312 c.
11. The reference book on nonlinear circuits: Translated from English / Ed. by D. Sheingold. – Moscow: Mir, 1977. – 523 p.

Received on 26.12.2005