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ON THE CHOICE OF TEMPERATURE PROFILE AT SOLVING THE HEAT CONDUCTION EQUATION IN SPHERICAL COORDINATES BY THE METHOD OF THERMAL BALANCE INTEGRAL

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Solutions of the heat conduction equation for a sphere and an area limited from within by a spherical cavity have been obtained by means of the integrated method. The influence of the choice of the temperature profile on efficiency of the approached analytical solution is shown. The variant of solution specification in transitive area is offered.

Introduction

Exact solutions of heat conduction problems are rather cumbersome and laborious. Moreover they are almost absent in problems on radial heat flux in spherical coordinates at changing aggregate state [1, 2]. Therefore the diagrams obtained by numerical or approximate methods are usually used for solving practical problems [3]. One of the approximate analytical methods is the method of heat balance integral (HBI), in which its physical clearness, simplicity and rather high accuracy of the results are, first of all attractive, that T. Goodmen shows obviously [4] by numerous examples. The main difficulty which one can face using the method of HBI consists in correct specification of the temperature profile which influences greatly the results accuracy, in T. Goodmen's opinion.

There are several approaches in selecting temperature profiles. A.I. Veinik in the work [5] proposes using temperature profiles in the form of common polynomials for the problems of any geometry that should simplify the solution of the set problem.

Referring to the work of F. Poll and T. Lardner [6] and without solution, T. Goodmen suggests in his article [4] using the temperature profile of the form:

$$T(r,t)=\text{polynomial}/r, \tag{1}$$

is time in the case of spherical symmetry; where $T(r,t)$ is the temperature of the body; r is the reference radius, t.

It is substantiated by the fact that the exact solution of the problems is proportional to the magnitude $1/r$, and use of the profile in the form of a common polynomial at large times gives a considerable error.

For the external problem (area limited from within by a spherical cavity) with the restricted conditions of the second kind of G. Karslow and D. Eger [3] the following solution:

$$T(r,t)=$$
$$=\frac{R^2q}{\lambda r}\left\{\begin{matrix}\Phi^*\left(\dfrac{r-R}{2(at)^{1/2}}\right)-\\ -\exp\left(\dfrac{r-R}{R}+\dfrac{at}{R^2}\right)\Phi^*\left(\dfrac{r-R}{2(at)^{1/2}}+\dfrac{(at)^{1/2}}{R}\right)\end{matrix}\right\}, \tag{2}$$

is given, where $\Phi^*(x)=\dfrac{2}{\sqrt{\pi}}\cdot\int\limits_x^\infty \exp(-\xi^2)d\xi$ is the error function; q is the heat flux; R is the sphere radius; a, λ are the coefficients of temperature- and heat conduction respectively.

It is seen that solution is proportional to the value $1/r$, and the profile recommended by T. Goodmen is possible to function efficiently here (1). But is it suitable for the inner problem (sphere area) with the same boundary conditions of the second kind in general solution of which only one summand is proportional to $1/r$?

In the book of A.V. Lykov [4] this solution is introduced in the form of series and has the form:

$$T(r,t)=$$
$$=\frac{qR}{\lambda}\left[\begin{matrix}3\text{Fo}-\dfrac{1}{10}\left(3-5\dfrac{r^2}{R^2}\right)-\\ -\sum\limits_{n=1}^{\infty}\dfrac{2}{\mu_n^2\cos\mu_n}\cdot\dfrac{R\sin\mu_n\dfrac{r}{R}}{r\mu_n}\exp(-\mu_n^2\text{Fo})\end{matrix}\right]+T_0, \tag{3}$$

where Fo$=at/R^2$ is the Fourier number; μ_n are the roots of characteristic equation $\text{tg}(\mu)=\mu$.

Let us carry out the solution by the method HBI of inner and outer heat conduction problems with different temperature profiles and estimate their efficiencies for answering the raised question.

Inner problem

According to the physical concept of the method the process of temperature change in the body are usually divided into two stages. At the first, *initial*, stage the depth of penetration of heat pulse $\delta(t)$ achieves the center (i.e. $\delta(t)\le R$), at the second stage temperature starts changing in the body center.

Thus, the mathematical set of the problem for the initial stage of the process has the form [1, 2]:

$$\frac{\partial T}{\partial t}=a\frac{1}{r^2}\frac{\partial}{\partial}\left(r^2\frac{\partial T}{\partial r}\right),\quad 0\le t<\infty,\quad 0\le r\le R,\qquad (4)$$

$$r=R,\quad \lambda\frac{\partial T(R,t)}{\partial r}=q,\qquad (5)$$

$$r=R-\delta,\quad T(R-\delta,t)=T_0,\qquad (6)$$

$$r=R-\delta,\quad \frac{\partial T(R-\delta,t)}{\partial r}=0.\qquad (7)$$

The detailed course of solution is introduced in the works [7, 8], therefore, let us show here only the main stages of solution. According to this method the equation (4) is multiplied by differential volume r^2dr and integrated in the range from $r=R$ to $r=R-\delta$. As a result we obtain the heat balance integral:

$$\frac{d}{dt}\left[\Theta-\frac{T_0}{3}(R-\delta)^3\right]=-aR^2\frac{q}{\lambda},\qquad (8)$$

where $\Theta=\int\limits_R^{R-\delta}Tr^2dr$. Let us assign the temperature profile (1) by the equation:

$$T=(\beta_0+\beta_1 r+\beta_2 r^2)/r,\qquad (9)$$

where the coefficients β_0, β_1 and β_2 in general case depend on time and determined by the boundary conditions (5–7). Then the temperature profile looks as follows in an explicit form:

$$T(r,\delta(t))=\frac{qR^2}{\lambda[R^2-(R-\delta)^2]}\cdot\frac{(R-\delta-r)^2}{r}+T_0.\qquad (10)$$

Substituting (10) into (8) we obtain an ordinary differential equation relative to δ and find its solution:

$$\frac{(R-\delta)^4-6R^2(R-\delta)^2+8R^3(R-\delta)-3R^4}{12(R^2-(R-\delta)^2)}=-at.\qquad (11)$$

The expressions (10) and (11) are the approximate solutions of the assigned problem.

It should be mentioned that the boundary conditions (5–7) are enough only for determining three coefficients β_i in the temperature profiles. Therefore, it is necessary to assign the additional boundary conditions which may be obtained in the following way

$$r=R-\delta,\quad \frac{\partial T^n(R-\delta)}{\partial r^n}=0\qquad (12)$$

for using polynomials of high degree.

Taking into account (12), the solution of the problem with polynomials of the degree $n\ge1$ take the form:

1) for $T(r,t)$=polynomial/r:

$$\frac{\vartheta_R}{\mathrm{Ki}}=(-1)^n\cdot\frac{(1-\delta-\bar{r})^n}{\bar{\delta}^{n-1}\cdot(n-\bar{\delta})\cdot\bar{r}},\qquad (13)$$

$$\mathrm{Fo}=\frac{(n+2)\cdot\bar{\delta}^2-\bar{\delta}^3}{(n+1)\cdot(n+2)\cdot(n-\bar{\delta})},\qquad (14)$$

2) for $T(r,t)$=polynomial:

$$\frac{\vartheta_R}{\mathrm{Ki}}=(-1)^n\cdot\frac{(1-\bar{\delta}-\bar{r})^n}{n\cdot\bar{\delta}^{n-1}},\qquad (15)$$

$$\mathrm{Fo}=\frac{\bar{\delta}^2}{n\cdot(n+1)}\cdot\left[1-\frac{2\bar{\delta}}{n+2}+\frac{2\bar{\delta}^2}{(n+2)\cdot(n+3)}\right],\qquad (16)$$

where $\vartheta_R=(T(r,t)-T_0)/(T_c-T_0)$ is the dimensionless temperature; $\mathrm{Ki}=qR/(l(T_c-T_0))$ is the Kirpichev criterion; $\bar{\delta}=\delta(t)/R$ is the relative heat pulse depth of penetration, $\bar{r}=r/R$ is the relative sphere radius; T_c is the source temperature.

Besides temperature distribution the body average temperature and heat quantity transferred to the body have to be determined most often in engineering design [3]:

$$\bar{T}=\frac{1}{V}\int\limits_V TdV,\quad Q=c\rho V\int\limits_V TdV,$$

where V, c, ρ are the volume, heat capacity and body density, respectively.

It should be noted that in [4, 5] the comparison of the approximate solutions with the accurate ones for similar problems was carried pout only by the moment of the initial stage termination of body heating process. In this work the obtained approximate solutions are compared with the accurate ones by dimensionless characteristics: average body temperature, temperature of the surface and center (denoted by subscripts «*ср*», «*пов*» and «0» respectively) for different time points:

$$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{ср}=\frac{\int\limits_0^1\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)\cdot r^2dr}{\int\limits_0^1 r^2dr},$$

$$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{пов}=\frac{\vartheta_R(1,\mathrm{Fo})}{\mathrm{Ki}},\quad \left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_0=\frac{\vartheta_R(0,\mathrm{Fo})}{\mathrm{Ki}}.\qquad (17)$$

The values of these characteristics and errors of their determination (Δ, %) for different time points are introduced in the Table 1.

At the second stage of the process of sphere heating by a constant heat flux the value $\delta(t)$ – the depth of heat pulse penetration looses its physical sense and new boundary conditions are required instead of those (6)

Table 1. *Comparison of exact and approximate solutions at the initial stage of body heating process**

Exact solution (3)		Approximate solution (13)				Approximate solution (15)			
$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{ср}$	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{пов}$	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{ср}$	Δ, %	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{пов}$	Δ, %	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{ср}$	Δ, %	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{пов}$	Δ, %
Fo=0,0265									
79,539	213,88	79,539	0,0003	213,441	0,2068	79,539	0,0003	209,659	1,9740
Fo=0,0150									
45,001	154,70	44,999	0,0024	154,126	0,371	45,000	0,0024	151,647	1,9728
Fo=0,0050									
15,002	85,062	14,999	0,0148	84,558	0,592	14,999	0,1471	83,620	1,6953
Fo=0,0005									
1,499	25,889	1,499	−0,0456	25,530	1,388	1,500	−0,0455	25,428	1,7802
Fo=0,00005									
0,148	8,029	0,149	−1,0650	7,958	0,890	0,150	−1,0767	7,947	1,0178

**The rated values of representative temperatures are multiplied by 10^3 and obtained at n=4*

and (7) that were used before for describing the process by the approximate method HBI.

One of them is the condition in the center of sphere:

$$r = 0, \quad \frac{\partial T}{\partial r}(0,t) = 0. \tag{18}$$

Let us note that the profile of the type $T(r,t)$=polynomial/r does not give an opportunity to use this condition. Therefore, for this stage of the process the course of solution is introduced for the profile in the form of quadratic polynomial $T(r,t)=\beta_0+\beta_1 r+\beta_2 r^2$.

One more additional boundary condition is required for recovering constants β_0, β_1 and β_2. As one of the variants let us suppose that the temperature on sphere surface is a certain time function:

$$T(R,t) = z(t). \tag{19}$$

It is obvious that in this case the temperature profile and heat balance integral take the form:

$$T = z - \frac{qR}{2\lambda}\left(1 - \frac{r^2}{R^2}\right). \tag{20}$$

$$\frac{d\Theta}{dt} = aR^2\frac{q}{\lambda}, \tag{21}$$

where $\Theta = \int_0^R Tr^2 dr.$ Using (20), let us determine the efficient temperature:

$$\Theta = \frac{zR^3}{3} - \frac{qR^4}{15\lambda}. \tag{22}$$

Let us substitute the expression (22) in to (21) and solve the obtained differential equation relative to $z=f(t)$. For this purpose let us multiply (24) by dt and integrate the right part in the range from t_1 (time of the first stage termination) to t, and the left one – in the range from $z(t_1)=T(R,t_1)=qR/2\lambda+T_0$ до $z(t)$:

$$\int_{z(t_1)}^{z(t)} d\left(\frac{zR^3}{3} - \frac{qR^4}{15\lambda}\right) = \int_{t_1}^{t} aR^2\frac{q}{\lambda}dt.$$

After integration let us determine the function:

$$z(t) = \frac{3aq}{\lambda R}(t - t_1) + \frac{qR}{2\lambda} + T_0,$$

and after its substitution into (3) we obtain:

$$T(r,t) = \frac{qR}{\lambda}\left[\frac{3at}{R^2} - \frac{3at_1}{R^2} + \frac{1}{2}\frac{r^2}{R^2}\right] + T_0. \tag{23}$$

In this expression $at_1/R^2=\mathrm{Fo}^1$ – the Fourier number by the moment of the process initial stage termination (for the profiles in the form of polynomial it is determined by (16) at $\bar{\delta}=1$). Taking into account this fact the approximate solution for temperature distribution in the form of quadratic polynomial at the second stage of the process will be determined by the expression:

$$T(r,t) = \frac{qR}{\lambda}\left[\frac{3at}{R^2} - \frac{1}{10}\left(3 - 5\frac{r^2}{R^2}\right)\right] + T_0, \quad t > t_1, \tag{24}$$

or in dimensionless form

$$\frac{\vartheta_R}{\mathrm{Ki}} = 3\mathrm{Fo} - 0{,}3 + \frac{\bar{r}^2}{2}, \quad \mathrm{Fo} > \mathrm{Fo}^1. \tag{25}$$

If the polynomial of n-degree is used for assigning the temperature profile the solution is written down in the following way:

$$\frac{\vartheta_R}{\mathrm{Ki}} = 3\mathrm{Fo} - 3\mathrm{Fo}^1 + \frac{\bar{r}^n}{n}. \tag{26}$$

It is seen that the obtained approximate solution (24) coincides with the first members of the exact solution (3). At time point Fo=0,25 the divergence of the approximate and the exact solutions amounts less than 1 %, and at high magnitudes of tends to zero. It is explained by the fact that starting from Fo=0,25, the body heating process becomes *qazisteady*: the temperature of any point increases by the linear law and the temperature distribution follows the parabolic law [2]. The temperature values on the surface and in the center of the sphere as well as average volume temperature and their divergences with the same magnitudes obtained from the exact solution are introduced in Table 2.

If the results of the approximate solutions are estimated (Table 1, 2), then one can notice that they are of asymptotic character at low and high values of Fo. The area where the error of these approximations starts increasing appears. Let us give one of possible variants of accuracy improvement at this part of solution. Let us call the part of the body heating process from the time of initial stage termination by the exact solution Fo≈0,0265 to the value Fo=0,25 justified before the *transition* stage. In this time interval the accuracy of the approximate solution (26) is mainly determined by the value of polynomial degree index which depends in this case on Fourier number. As a result of studying this regularity the approximation dependence $n=n(\mathrm{Fo})$ was obtained:

$$n(\mathrm{Fo})=\sum_{i=0}^{5}\frac{A_i}{\mathrm{Fo}^i},\quad 0{,}0265<\mathrm{Fo}<0{,}25, \qquad (27)$$

where A_0=2,03554159548730; A_1=4,44270901418236·10^{-2}; A_2=1,05859605570089·10^{-2}; A_3=3,61289304594781·10^{-4}; A_4=5,41024194251893·10^{-6}; A_5=2,78893049968760·10^{-8}.

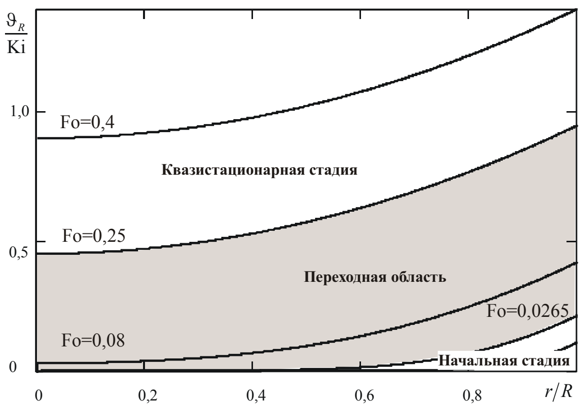

Figure. *Characteristic stages of the sphere heating process Квазистационарная стадия – Quazisteady stage; Переходная область – Transition area; Начальная стадия – Initial stage*

Thus, the expressions (26, 27) will be the approximate solution of the problem at the transition stage of the sphere heating process. The numerical magnitudes of the representative temperatures for different values of are introduced in the Table 3. The solution behavior in the mentioned areas is shown in the Figure.

Exterior problem

It differs from the previous one by the fact that constant thermal flux q acts from within on the surface of spherical cavity of radius R. At reference time the whole system is at temperature T_0. The position of temperature front $R+\delta(t)$ in outer content relative to the cavity, i.e. at $R\le r\le\infty$ should be determined. Thus, the mathematical statement of the problem is like the statement (4–7). Let us carry out the solution with different temperature profiles and estimate their efficiency.

The solution procedure is similar with the exterior problem solution for the initial stage of the sphere heating process. As a result, using the polynomials of the degree $n\ge1$ we obtain the following solution:

1) for $T(r,t)$=polynomial/r:

$$\frac{\vartheta_R}{\mathrm{Ki}}=\frac{(1+\bar\delta-\bar r)^n}{\bar\delta^{\,n-1}\cdot(n+\bar\delta)\cdot\bar r}, \qquad (28)$$

$$\mathrm{Fo}=\frac{(n+2)\cdot\bar\delta^{\,2}+\bar\delta^{\,3}}{(n+1)\cdot(n+2)\cdot(n+\bar\delta)}; \qquad (29)$$

2) for $T(r,t)$=polynomial:

$$\frac{\vartheta_R}{\mathrm{Ki}}=\frac{(1+\bar\delta-\bar r)^n}{n\bar\delta^{\,n-1}}, \qquad (30)$$

$$\mathrm{Fo}=\frac{\bar\delta^{\,2}}{n(n+1)}\left[1+\frac{2\bar\delta}{n+2}+\frac{2\bar\delta^{\,2}}{(n+2)(n+3)}\right]. \qquad (31)$$

Table 2. *Comparison of the exact and approximate solutions at quazisteady stage of the body heating process**

Exact solution (3)			Approximate solution (24)					
$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{ср}$	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{пов}$	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_0$	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{пов}$	Δ, %	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{ср}$	Δ, %	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_0$	Δ, %
Fo=0,25								
750,0	949,3636	452,9292	750,0000	≈0	950,0	0,0670	450,0	0,6467
Fo=0,50								
1500,0	1699,9959	1200,0188	1500,0000	≈0	1700,0	0,0002	1200,0	0,0016
Fo=1,00								
3000,0	3200,0	2700,0	3000,0	≈0	3200,0	5,3·10^{-9}	2700,0	2,9·10^{-8}

**The rated values of representative temperatures are multiplied by 10^3 and obtained at n=2*

Table 3. *Comparison of the exact and approximate solutions at the transition stage**

Exact stage (3)			Approximate stage (26, 27)					
$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{ср}$	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{пов}$	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_0$	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{пов}$	Δ, %	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_{ср}$	Δ, %	$\left(\frac{\vartheta_R}{\mathrm{Ki}}\right)_0$	Δ, %
Fo=0,03								
90,0	229,8518	6,95·10^{-2}	90,0	≈0	223,9166	2,58	6,96·10^{-2}	0,0104
Fo=0,08								
240,0	420,0202	28,4737	240,0	≈0	420,1091	0,0212	28,2889	0,6492
Fo=0,15								
450,0	645,2032	172,0275	450,0	≈0	645,5753	0,0577	172,3412	0,1824
Fo=0,24								
720,0	919,2213	423,5845	720,0	≈0	919,6138	0,0427	422,0205	0,3692

**Rated values of the representative temperatures are multiplied by 10^3*

Table 4. *Comparison of the exact and approximate solutions for the exterior problem**

Exact solution (2)		Approximate solution (28, 29)				Approximate solution (30, 31)			
$\left(\frac{\vartheta_R}{\text{Ki}}\right)_{cp}$	$\left(\frac{\vartheta_R}{\text{Ki}}\right)_{пов}$	$\left(\frac{\vartheta_R}{\text{Ki}}\right)_{0}$	Δ, %	$\left(\frac{\vartheta_R}{\text{Ki}}\right)_{пов}$	Δ, %	$\left(\frac{\vartheta_R}{\text{Ki}}\right)_{cp}$	Δ, %	$\left(\frac{\vartheta_R}{\text{Ki}}\right)_{0}$	Δ, %
Fo=0,03									
5,4295	168,9426	5,4295	$1{,}01\cdot10^{-4}$	171,6655	1,6117	5,4295	$5{,}15\cdot10^{-4}$	179,9424	6,51
Fo=1,00									
7,4216	572,4164	7,4216	$3{,}08\cdot10^{-4}$	567,7413	0,8167	7,4216	$2{,}80\cdot10^{-4}$	766,1399	33,84
Fo=10									
1,5942	829,4223	1,5940	$8{,}77\cdot10^{-3}$	816,5046	1,5574	1,5940	$8{,}62\cdot10^{-3}$	1681,6613	102,75

** Rated values of the representative temperatures are multiplied by 10^3 and obtained at n=3*

The results of computing the representative temperatures for various values of Fo are summarized in the Table 4.

The analysis of the results

The efficiencies of temperature profiles of the type: $T(r,t)$=polynomial and $T(r,t)$=polynomial/r with polynomials of the degree n were for the first time estimated in the work when solving the heat conduction problem in spherical geometry by the method of HBI.

The asymptotic character of the obtained approximate analytic solutions for the process stages accepted in the method of HBI: initial and quazisteady is emphasized. The transition area, for which the approximation specifying the solution is proposed, is suggested to be taken into account.

The following things:

- at the *initial stage* of the process the least divergence of the approximate and exact solutions ($\Delta<2$ %) is achieved at polynomial degree index n=4 for both studied profiles but the least error is given by application of profile for $T(r,t)$= polynomial /r;
- at quazisteady stage of the process the polynomial of the type for $T(r,t)$=polynomial/r losses physical sense and profile for $T(r,t)$=polynomial supports the error of the approximate solution $\Delta<1$ % at magnitudes Fo≥0,25 may be mentioned in the problem of sphere heating (inner problem).

In the problem of array heating from within spherical cavity (exterior problem) application of various type profiles gave the following results:

- profile assignment in the form of a simple polynomial provides the required accuracy at n=3 only at very low Fourier numbers (Fo<<1), and at Fo=1 even the value n=100 gives a divergence more than 4 %;
- the solution obtained by the profile for $T(r,t)$=polynomial/r is more preferable as its highest accuracy is achieved at n=3 for any Fourier number.

The obtained results show that solving the heat conduction problem for different geometry by the approximate method of HBI the sufficient accuracy is not always achieved using the temperature profiles of one and the same type.

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