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ON PAIR OF *m*-SURFACES WITH THE GIVEN NETWORK IN MULTIVARIANT PROJECTIVE SPACE

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The two m-dimensional surfaces in n-dimensional projective space between which points a point conformity is establishedis studied. The network of lines is given on surfaces. Some geometrical images connected with the network are considered. Consideration has everywhere local character. All functions considered in the given work are assumed analytical.

The multidimensional differential geometry of various varieties for a long time draws attention of mathematicians in connection with its various applications. In particularly, multidimensional surfaces and networks of lines on them [1, 2] are studied. In the middle of the twentieth century one began to study pairs of surfaces and various conformity between them [3]. The given work belongs to this direction and is devoted to pair of m-dimensional surfaces in n-dimensional projective space.

1. Let S_m^1 and S_m^2 – are two surfaces in projective space P_n and Π: $S_m^1 \to S_m^2$ – is smooth one-to-one conformity between them.

Let's attach to the considered pair of m-surfaces some projective reference point $R=\{A_0,A_1,...,A_n\}$ with derivational formulas $dA_I=\omega_I^J A_J$ $(I,J,...=0,1,...,n)$ and the structural equations $D\omega_I^J=\omega_I^K \wedge \omega_K^J$, and $\omega_I^I=0$.

Let's carry out the following partial canonization of a reference point: let's place points A_0 and $A_n=\Pi(A_0)$ in corresponding points of the surfaces and of the pairs; points $A_1,...,A_m$ – in the m-plane $L_m^1=(A_0,...,A_m)$ being tangent to the m-surface S_m^1 in point A_0, and points $A_{n-m},...,A_{n-1}$ – in the m-plane $L_m^2=(A_{n-m},...,A_{n-1})$ being tangent to the m-surface S_m^2 in point A_n.

Point conformity Π induces projective conformity between binders of tangents the directions, associated to two corresponding points A_0 and A_n.

Let's choose a reference point of pair so that directions A_0A_i corresponded in this projectivitat to directions A_nA_{n-i}. Then the basic equations of our problem become

$$\omega_0^\alpha = 0, \quad \omega_0^n = 0, \quad \omega_0^{n-i} = 0, \tag{1}$$

$$\omega_n^\alpha = 0, \quad \omega_n^0 = 0, \quad \omega_n^i = 0,$$
$$\omega_0^i = \omega_n^{n-i}. \tag{2}$$

$(i,j, ...=1,2,...,m;\ a,b,c,...=2,3,...,m;$
$\alpha,\beta,...=m+1,\ m+2,\ ...,n-m-1).$

Let's designate for brevity further $\omega_0^i=\omega_n^{n-i}$.

Continuing the equations (1, 2), we obtain

$$\omega_i^\alpha = \Lambda_{ij}^\alpha \omega^j, \omega_i^n = \Lambda_{ij}^n \omega^j, \omega_k^{n-i} = \Lambda_{kj}^{n-i} \omega^j,$$
$$\omega_{n-i}^0 = \Lambda_{n-i,j}^0 \omega^j, \omega_{n-k}^i = \Lambda_{n-k,j}^i \omega^j, \omega_{n-i}^\alpha = \Lambda_{n-i,j}^\alpha \omega^j,$$
$$\omega_{n-j}^{n-i} - \omega_j^i + \delta_j^i(\omega_0^0 - \omega_n^n) = A_{jk}^i \omega^k,$$
$$\nabla\Lambda_{ij}^\alpha = \Lambda_{ijk}^\alpha \omega^k, \quad \nabla\Lambda_{ij}^n + \Lambda_{ij}^\alpha \omega_\alpha^n + \Lambda_{ij}^{n-k} \omega_{n-k}^n = \Lambda_{ijk}^n \omega^k,$$
$$\nabla\Lambda_{kj}^{n-i} + \Lambda_{kj}^\alpha \omega_\alpha^{n-i} + \Lambda_{kj}^{n-l} \omega_{n-l}^{n-i} = \Lambda_{kjl}^{n-i} \omega^l,$$
$$\nabla\Lambda_{n-i,j}^\alpha = \Lambda_{n-i,jk}^\alpha \omega^k, \tag{3}$$

$$\nabla\Lambda_{n-i,j}^0 + \Lambda_{n-i,j}^k \omega_k^0 + \Lambda_{n-i,j}^\alpha \omega_\alpha^0 = \Lambda_{n-i,jk}^0 \omega^k,$$
$$\nabla\Lambda_{n-k,j}^i + \Lambda_{n-k,j}^\alpha \omega_\alpha^i - \Lambda_{n-l,j}^i \omega_{n-k}^{n-l} = \Lambda_{n-k,jl}^i \omega^l,$$
$$\nabla A_{jk}^i + \delta_j^i(\omega_k^0 - \omega_{n-k}^n) - \delta_k^i(\omega_{n-j}^n - \omega_j^0) -$$
$$-\Lambda_{jk}^\alpha \omega_\alpha^i + \Lambda_{n-j,k}^\alpha \omega_\alpha^{n-i} = A_{jkl}^i \omega^l. \tag{4}$$

Here the symbol ∇ designates the covariant differentiation operator.

From the equations (3) it follows, that systems of functions Λ_{ij}^α and $\Lambda_{n-i,j}^\alpha$ are tensors in the G. F. Laptev sense [4, 5].

Continuing the equations (3, 4), we obtain the system of the differential equations of a sequence of fundamental objects: Λ_{ij}^α, Λ_{ij}^n, Λ_{ij}^{n-k}, $\Lambda_{n-i,j}^\alpha$, $\Lambda_{n-i,j}^0$, $\Lambda_{n-i,j}^i$, A_{jk}^i; Λ_{ijk}^α, Λ_{ijk}^n, Λ_{kjl}^{n-i}, $\Lambda_{n-i,jk}^\alpha$, $\Lambda_{n-i,jk}^0$, $\Lambda_{n-i,jl}^i$, A_{jkl}^i; Λ_{ijkl}^α, Λ_{ijkl}^n,...

2. Let`s the first and second normals of surfaces and [1, 3] are given, S_m^1 and S_m^2 surfaces are defined by points

$$L_{n-m}^1 = (A_0, A_n, A_{n-i}, A_\alpha), \quad L_{m-1}^1 = (A_1, A_2, ..., A_m)$$

and

$$L_{n-m}^2 = (A_n, A_0, A_i, A_\alpha), \quad L_m^2 = (A_{n-m}, A_{n-m+1}, ..., A_{n-1}),$$

accordingly.

Let one-dimensional distribution Δ_1 and additional to it distribution Δ_{m-1} are prescribed on a surface S_m^1 then if the main parameters are fixed, apex A_1 can move over the straight line $\Delta_1(A_0)$, and apexes A_a – in the plane $\Delta_{m-1}(A_0)$.

Hence, forms ω_1^a and ω_a^1 are main

$$\omega_1^a = \Lambda_{1i}^a \omega^i, \omega_a^1 = \Lambda_{ai}^1 \omega^1. \tag{5}$$

Continuing the equations (5), we obtain

$$\nabla\Lambda_{ai}^1 - \delta_i^1 \omega_a^0 = \Lambda_{aij}^1 \omega^j,$$
$$\nabla\Lambda_{1i}^a - \delta_i^a \omega_a^0 = \Lambda_{1ij}^a \omega^j.$$

Hence, each of systems of functions Λ_{1ij}^a and Λ_{aij}^1 forms quasi-tensor [4].

Let`s find on the straight line $\Delta_1(A_0)$ point $F=\lambda A_0+A_1$ such that at displacement of point A_0 in the direction $\Delta_{m-1}(A)$ displacement of point F did not leave from $(n-m+1)$ planes $L_{n-m+1}=(A_0,A_1,A_{m+1},...,A_n)$. Relation

$$dF \in L_{n-m+1}(A) \tag{6}$$

is fulfilled if and only if

$$\lambda\omega^a + \omega_1^a = 0.$$

Since relation (6) has to be carried out at $\omega^1=0$, that, using the equations (5), we

$$(\Lambda_{1a}^{b}+\lambda\delta_{a}^{b})\omega^{a}=0. \quad (7)$$

So far as not all forms ω^a are simultaneously equal to zero λ has to satisfy to the equation

$$\det\left\|\Lambda_{1a}^{b}+\lambda\delta_{a}^{b}\right\|=0. \quad (8)$$

Let's assume, that all roots of the equation (8) are simple, real. Then the system of the equations (7) defines $m-1$ linearly independent one-dimensional distributions Δ_1^a belonging to distribution Δ_m. Integral curves of distributions Δ_1,Δ_1^a form a network of lines on the surface S_m^1 which we shall designate Σ_m. Locating each of apex A_a of a reference point on the corresponding straight line $\Delta_1^a(A_0)$, we obtain $\Lambda_a^b=0$, $a\neq b$. On the straight line $\Delta_1(A_0)$ we obtain the $m-1$ point

$$F_1^a=\Lambda_{1a}^{a}A_0+A_1.$$

(to not summarize on a)

3. The point F_i^j $(i\neq j)$ is named pseudo-focus [7] of the straight line A_0A_i, if at displacement of the point $A0$ in direction A_0A_j the tangent to a line described by the point F_i^j, belongs to a hyperplane

$$L_{n-1}^{j}=(A_0A_1\ldots A_{j-1}A_{j+1}\ldots A_m\ldots A_n).$$

Let the point

$$F_i^j=x_i^jA_0+A_i \quad (i\neq j)$$

is a pseudo-focus of the straight line A_0A_α. Then, from

$$(dF_i^j,L_{n-1}^j)\big|_{\omega^1=\omega^2=\ldots=\omega^{j-1}=\omega^{j+1}=\ldots=\omega^m=0}=0$$

We obtain

$$[x_i^j\omega^j+\omega_i^j,\omega^1,\omega^2,\ldots,\omega^{j-1},\omega^{j+1},\ldots,\omega^m]=0.$$

Hence

$x_i^j=-\Lambda_{ij}^j$ ($i\neq j$, to not summarize on j)

and

$F_i^j=-\Lambda_{ij}^jA_0+A_i$ (to not summarize on j). (9)

From formula (9) it follows, that the point is the pseudo-focus of straight line A_0A_1 corresponding to direction $\Delta_1^a(A_0)$. Points

$$F_i=\frac{1}{m-1}\Lambda_{ij}^jA_0+A_i \text{ (to not summarize on } j)$$

are named harmonic poles of the point A_0 in relation to pseudo-focuses of the straight line A_0A_i.

If $\Lambda_{ij}^j=0$ (to summarize on j) apexes Ai of the reference point are placed in harmonic poles of the straight lines A_0A_i.

By virtue of given projectivitet Π between pairs of surfaces S_m^1 and S_m^2, on the surface S_m^2 similar constructions take place which we shall not give here.

4. Let's designate through L_{2m+1} the $(2m+1)$ -dimensional plane stretched on tangents of the m-plane of both surfaces of pair. Let's note, that L_{2m+1} is a tangent $(2m+1)$-dimensional subspace of m-parametrical variety which element is straight line A_0A_n, i.e. it contains straight line A_0A_n and all its first differential vicinity. Crossing of equipping planes of each from surface pair we designate L_{n-2m-2}. This plane is equipping plane of m-surface pair. Equipping planes of surfaces and can be given by the equations

$$x^0-\lambda_{i_1}^0x^{i_1}=0;x^i-\lambda_{i_1}^ix^{i_1}=0; \quad (10)$$

$$x^n-\lambda_{i_2}^nx^{i_2}=0,x^{i_3}-\lambda_{i_2}^{i_3}x^{i_2}=0, \quad (11)$$

accordingly, and normals of the first kind of surfaces and can be given by the equations

$$x^i-\lambda_{i_1}^ix^{i_1}=0; \quad (12)$$

$$x^{i_3}-\lambda_{i_2}^{i_3}x^{i_2}=0. \quad (13)$$

$(i_1,j_1,\ldots=m+1,\ldots,n;\ i_2,j_2,\ldots=0,1,2,\ldots,n-m-1;$
$i_3,j_3,\ldots=n-m,\ldots,n-1).$

accordingly.

Here objects of equipment are covered by fundamental geometrical object of pair m-surfaces and satisfy to the following differential equations:

$$\nabla\lambda_{i_1}^i=-\omega_{i_1}^i+\lambda_{i_1j}^i\omega^j,$$

$$\nabla\lambda_{i_2}^{i_3}=-\omega_{i_2}^{i_3}+\lambda_{i_2n+j}^{i_3}\omega^j,$$

$$\nabla\lambda_{i_2}^n=-\lambda_{i_2}^{i_3}\omega_{i_3}^n-\omega_{i_2}^n-\lambda_{i_2}^n\omega_n^n+\lambda_{i_2n+j}^n\omega^j,$$

$$\nabla\lambda_{i_1}^0=-\lambda_{i_1}^0\omega_0^0-\lambda_{i_1}^i\omega_i^0-\omega_{i_1}^0+\lambda_{i_1j}^0\omega^j.$$

Components $\lambda_{i_1}^i(\lambda_{i_2}^{i_3})$ of object of equipment form independent subobject which defines a field of invariant $(n-m)$-dimensional planes being the field of normals of the first kind surface

From (10) – (13) it follows, that the $(n-2m-2)$ plane L_{n-2m-2} is given by the equations (12), (13), and $(n-2m)$-plane L_{n-2m},, attached invariantly to the pair and having with the $(2m+1)$ plane L_{2m+1} the common points A_0 and A_n, is given by the equations (10), (11).

5. The fields of hyperquadric having the second order contact with surfaces S_m^1 and S_m^2 can be attached to surfaces of the pair

$$a_{ij}x^ix^j-2b_{i_1}x^0x^{i_1}+2b_{i_1}c_i^{i_1}x^ix^{i_1}+b_{i_1}c_{j_1k_1}^{i_1}x^{j_1}x^{k_1}=0; \quad (14)$$

$$a_{i_3j_3}x^{i_3}x^{j_3}-2b_{i_2}x^nx^{i_2}+2b_{i_2}c_{i_3j_2}^{i_2}x^{i_3}x^{j_2}+$$
$$+b_{i_2}c_{j_2k_2}^{i_2}x^{j_2}x^{k_2}=0, \quad (15)$$

where

$$b_{i_1}=\lambda_{i_1i}^i+m\lambda_{i_1}^0-\Lambda_{ij}^{j_1}\lambda_{j_1}^i\lambda_{i_1}^j,$$

$$b_{i_1}=\lambda_{i_1i_3}^{i_3}+m\lambda_{i_1}^n-\Lambda_{i_3j_3}^{j_1}\lambda_{j_1}^{i_3}\lambda_{i_1}^{j_3},$$

$$a_{ij}=b_{i_1}\Lambda_{ij}^{i_1},\quad a_{i_3j_3}=b_{i_1}\Lambda_{i_3j_3}^{i_1}.$$

If to consider that

$$c_{ii_1}^{j_1}=\Lambda_{ij}^{j_1}\lambda_{i_1}^j-\delta_{i_1}^{j_1}\lambda_i^0,\quad c_{i_1j}^{k_1}=\Lambda_{ij}^{i_1}\lambda_{i_1}^i\lambda_{j_1}^j-\lambda_{(i_1}^0\delta_{j_1)}^{k_1}+c_{i(i_1}^{k_1}\lambda_{j_1)}^i,$$

$$c_{i_3j_1}^{i_1}=\Lambda_{i_3j_3}^{i_1}\lambda_{j_1}^{j_3}-\delta_{j_1}^{i_1}\lambda_{i_3}^n,$$

$$c_{i_1j_1}^{k_1}=\Lambda_{i_3j_3}^{k_1}\lambda_{i_1}^{i_3}\lambda_{j_1}^{j_3}-\lambda_{(i_1}^n\delta_{j_1)}^{k_1}+c_{i_3(i_1}^{k_1}\lambda_{j_1)}^{i_3},$$

than from (14), (15) we obtain unique adjoining the hyperquadrics of surfaces S_m^1 and S_m^2, accordingly.

These hyperquadrics have the following property: polara of the first (second) normal of the surface $S_m^1(S_m^2)$ in relation of the hyperquadric (14), (15) passes through the second (first) normal of the surface $S_m^1(S_m^2)$.

Consequrntly, the hyperquadric (14), (15) establishes quasi-polar conformity [8, 9] between the normals of the surface $S_m^1(S_m^2)$.

In m-planes L_m^1 and L_m^2 the tensors and the quasitensors define the quadric

$$(a_{ij}+\lambda_i^0\lambda_j^0)x^i x^j - 2\lambda_i^0 x^i x^0 + (x^0)^2 = 0, \quad x^{i_2}=0; \quad (16)$$

Accordingly, and

$$(a_{i_3 j_3}+\lambda_{i_3}^n\lambda_{j_3}^n)x^{i_3}x^{j_3} - 2\lambda_{i_3}^n x^{i_3}x^n + (x^n)^2 = 0, \quad x^{i_2}=0. \quad (17)$$

Polara of the point A_0 (A_n) in relation to the quadric (27), (28) is the second normal *of a* m-surface $S_m^1(S_m^2)$.

6. The point $X=x^i(A_i+\lambda_i^0 A_0)$ belonging to the second normal L_{m-1}^1 of the m-surface S_m^1, along the 1[st]-family

$$\omega^i = t^i\theta, \quad D\theta=\theta\Lambda\theta,$$

$$dt^i - t^i\omega_0^0 + t^j\omega_j^i = t_j^i\omega^j \quad (18)$$

describes a line with a tangent $TX(t)$. The linear space stretched L_{n-m}^1 on and $TX(t)$, is crossed with L_{n-m}^1 in the point Y. The point Y describes alongside (18) a line with tangent $TY(t)$. The linear space stretched on L_{n-m}^1 and $TY(t)$, is crossed with in the point $Z=z^i(A_i+\lambda_i^0 A_0)$, where

$$z^i = \{\delta_i^j(\lambda_{kp}^0 - \lambda_k^0\lambda_p^0 + \Lambda_{kp}^l\lambda_l^q\lambda_q^0) +$$

$$+\Lambda_{kp}^{i_2}(\lambda_{i_2 j}^i - \Lambda_{qj}^{j_2}\lambda_{i_2}^q\lambda_{j_2}^i)\}t^j t^p x^k. \quad (19)$$

Relationship (19) defines projective transformation of (m–1)-plane L_{m-1}^1 in itself which is defined by a matrix Π_i^j, where

$$\Pi_i^j = \{\delta_p^j(\lambda_{iq}^0 - \lambda_i^0\lambda_q^0 + \Lambda_{iq}^{i_2}\lambda_{i_2}^k\lambda_k^0) +$$

$$+\Lambda_{iq}^{i_2}(\lambda_{i_3 p}^j - \Lambda_{kp}^{j_2}\lambda_{i_2}^k\lambda_{j_2}^j)\}t^p t^q.$$

This transformation will be transformation W, if $\Pi_i^j=0$.

Thus, in the (m–1)-plane L_{m-1}^1 we obtain the quadric, which each point is corresponded by transformation W of the (m–1)-plane L_{m-1}^1 in itself [10]. This quadric can be given by the equations

$$\{\lambda_{ij}^0 - \lambda_i^0\lambda_j^0 + \Lambda_{ij}^{i_2}\lambda_{i_2}^k\lambda_k^0 +$$

$$+\Lambda_{pj}^{i_2}(\lambda_{i_2 i}^p - \Lambda_{kl}^{j_2}\lambda_{i_2}^k\lambda_{j_2}^p)\}x^i x^j = 0,$$

$$x^{i_2}=0, \quad x^0 - \lambda_i^0 x^i = 0. \quad (20)$$

The quadric (20) in the m-plane L_m^1 is corresponded by a cone

$$\{\lambda_{ij}^0 - \lambda_i^0\lambda_j^0 + \Lambda_{ij}^{i_2}\lambda_{i_2}^k\lambda_k^0 + \Lambda_{pj}^{i_2}(\lambda_{i_2 i}^p - \Lambda_{kl}^{j_2}\lambda_{i_2}^k\lambda_{j_2}^p)\}x^i x^j = 0,$$

$$x^{i_2}=0.$$

The similar cone we obtain in the m-plane L_m^2.

7. Let the point $X=x^{i_1}(A_{i_1}+\lambda_{i_1}^0 A_0+\lambda_{i_1}^i A_i)$ belonging to equipping plane L_{n-m-1}^1 of the m-surface is given. The space stretched on L_{n-m-1}^1 and $TX(t)$, is crossed with L_m^1 in the point

$$Y=(L_{n-m-1}^1, TX(t))\cap L_m^1 = y^0 A_0 + y^i(A_i+\lambda_i^0 A_0).$$

Then

$$X^* = (L_m^1, TY(t))\cap L_{n-m-1}^1 = x^{*i_1}(A_{i_1}+\lambda_{i_1}^0 A_0 + \lambda_{i_1}^i A_i),$$

where

$$x^{*i_1} = \Lambda_{ip}^{i_1}(\lambda_{j_1}^0\delta_j^i + \lambda_{j_1 j}^i - \Lambda_{kj}^{k_1}\lambda_{j_1}^k\lambda_{k_1}^i)t^j t^p x^{j_1}. \quad (21)$$

Hence, we obtain transformation (21) of the (n–m–1)-plane L_{n-m-1}^1 in itself which is transformation W if

$$\Lambda_{ij}^{i_1}(\lambda_{i_1}^1\delta_k^i + \lambda_{i_1 k}^i - \Lambda_{pk}^{j_1}\lambda_{i_1}^p\lambda_{j_1}^i)t^k t^j = 0.$$

Thus, we obtain a cone

$$\Lambda_{ij}^{i_1}(\lambda_{i_1}^0\delta_k^i + \lambda_{i_1 k}^i - \Lambda_{pk}^{j_1}\lambda_{i_1}^p\lambda_{j_1}^i)x^j x^k = 0, \quad x^{i_1}=0,$$

in the m-plane: which generatrixes are corresponded by the 1[st]-families (18) giving transformations W of the (n–m–1)-plane L_{n-m-1}^2 in itself.

Similarly in the m-plane L_m^2 we obtain a cone

$$\Lambda_{i_3 j_3}^{i_2}(\lambda_{i_2}^n\delta_{k_3}^{i_3} + \lambda_{i_2 k_3}^{i_3} - \Lambda_{k_3 l_3}^{j_2}\lambda_{i_2}^{l_3}\lambda_{j_2}^{i_3})x^{j_3}x^{k_3} = 0, \quad x^{i_1}=0,$$

which generatrixes are corresponded by transformations W of the (n–m–1) – L_{n-m-1}^2 in itself.

8. Let\`s consider the point $X=x^0 A_0+x^{i_1}(A_{i_1}+\lambda_{i_1}^0 A_0+\lambda_{i_1}^i A_i)$ belonging to the (n–m–1)-plane L_{n-m}^1. We have alongside (18)

$$Y=(L_{n-m}^1, TX(t))\cap L_{m-1}^1 = y^i(A_i+\lambda_i^0 A_0),$$

where

$$y^i = x^0 t^i + x^{i_1}(\lambda_{i_1}^0\delta_j^i + \lambda_{i_1 j}^i - \Lambda_{jk}^{j_1}\lambda_{i_1}^k\lambda_{j_1}^i)t^j.$$

Let\`s find

$$X^* = (L_{m-1}^1, TX(t))\cap L_{n-m}^1 =$$

$$= x^{*0}A_0 + x^{*i_1}(A_{i_1}+\lambda_{i_1}^0 A_0 + \lambda_{i_1}^i A_i),$$

where

$$x^{*0} = (\lambda_{ij}^0 - \lambda_i^0\lambda_j^0 - \Lambda_{ij}^{i_1}\lambda_{i_1}^0 + \Lambda_{ij}^{i_1}\lambda_{i_1}^k\lambda_k^0)t^i t^j x^0 +$$

$$+(\lambda_{i_1}^0\delta_k^i + \lambda_{i_1 k}^i - \Lambda_{jk}^{j_1}\lambda_{i_1}^j\lambda_{j_1}^i)(\lambda_{ip}^0 - \lambda_i^0\lambda_p^0 - \Lambda_{ip}^{j_1}\lambda_{j_1}^k\lambda_k^0)t^k t^p x^{i_1}, \quad (22)$$

$$x^{*i_1} = \Lambda_{ij}^{i_1}t^i t^j x^0 + \Lambda_{ij}^{i_1}(\lambda_{j_1}^0\delta_k^i + \lambda_{j_1 k}^i - \Lambda_{pk}^{k_1}\lambda_{j_1}^p\lambda_{k_1}^i)t^j t^k x^{j_1}.$$

Hence, (22) defines projective transformation of the (n–m)-plane L_{n-m}^1 in itself which will be transformation W if

$$\{\lambda_{ij}^0 - \lambda_i^0\lambda_j^0 + \Lambda_{ij}^{i_1}\lambda_{i_1}^k\lambda_k^0 + \Lambda_{ik}^{i_1}(\lambda_{i_1 j}^k - \Lambda_{pj}^{j_1}\lambda_{i_1}^p\lambda_{j_1}^k)\}t^i t^j = 0.$$

Thus, we obtain the cone

$$\{\lambda_{ij}^0 - \lambda_i^0\lambda_j^0 + \Lambda_{ij}^{i_1}\lambda_{i_1}^k\lambda_k^0 + \Lambda_{ik}^{i_1}(\lambda_{i_1 j}^k - \Lambda_{pj}^{j_1}\lambda_{i_1}^p\lambda_{j_1}^k)\}x^i x^j = 0,$$

$$x^{i_1}=0,$$

in the m-plane L_m^1 which generatrixes are corresponded by the 1[st] – families (18) giving transformations W of the (n–m)-plane L_{n-m}^1. Transformation of the (n–m)-plane L_{n-m}^1 in itself and the corresponding cone in the m-plane L_m^2 which generatrixes are corresponded by transformations W (n–m) of the (n–m)-plane L_{n-m}^2 in itself are obtained by similar way.

9. Let\`s take the point $X=x^0 A_0+x^i(A_i+\lambda_i^0 A_0)$ belonging to the tangent of a m-plane to the m-surface S_m^1. We have alongside (18)

$$Y=(L_m^1, TX(t))\cap L_{n-m-1}^1 = y^{i_1}(A_{i_1}+\lambda_{i_1}^0 A_0 + \lambda_{i_1}^i A_i),$$

where

$$y^{i_1}=\Lambda^{i_1}_{ij}x^i x^j,$$

тогда

$$X^*=(L^1_{n-m},TY(t))\cap L^1_m=x^{*0}A_0+x^{*i}(A_i+\lambda^0_i A_0),$$

where

$$x^{*0}=\{\lambda^0_{i_1 j}-\lambda^0_{i_1}\lambda^0_j-\lambda^0_i\lambda^i_{i_1 j}+$$

$$+\Lambda^{j_1}_{pj}(\lambda^0_i\lambda^p_{i_1}\lambda^i_{j_1}-\lambda^p_{i_1}\lambda^0_{j_1})\}\Lambda^{i_1}_{kq}t^j t^q x^k, \qquad (23)$$

$$x^{*i}=\Lambda^{i_1}_{kj}(\lambda^0_{i_1}\delta^i_p+\lambda^i_{i_1 p}-\Lambda^{j_1}_{pq}\lambda^q_{i_1}\lambda^i_{j_1})t^p t^j x^k.$$

Hence, (23) defines projective transformation of the m-plane L^1_m in itself, which is transformation W if

$$\Lambda^{i_1}_{ij}(\lambda^0_{i_1}\delta^i_p+\lambda^i_{i_1 p}-\Lambda^{j_1}_{kp}\lambda^k_{i_1}\lambda^i_{j_1})t^p t^j=0$$

and we have the cone

$$\Lambda^{i_1}_{ij}(\lambda^0_{i_1}\delta^j_p+\lambda^i_{i_1 p}-\Lambda^{j_1}_{kp}\lambda^q_{i_1}\lambda^i_{j_1})x^i x^j=0,$$

$$x^{i_1}=0,$$

in the m-plane L^1_m, which generatrixes are corresponded by the 1st families, giving transformations W of the m-plane L^1_m in itself. Similar transformation is obtained in L^2_m.

The theorem. If transformation (19), (22) is transformation W of the plane $L^1_{m-1}(L^1_{n-m})$ in itself, then and transformation (22), (19) is transformation W of the plane $L^1_{n-m}(L^1_{m-1})$ in itself.

If transformation (21) is transformation W of the plane L^1_{n-m-1} in itself, transformation (23) is transformation W of the plane L^1_m in itself and on the contrary if transformation (23) is transformation W of the plane L^1_m in itself, (21) is transformation W of the plane L^1_{n-m-1}.

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