in n-plane L_n^1. 1-families (23) giving transformations W of 2-plane L_2^1 into themselves correspond to its generators.

If the reference point is canonized then the equations (29) are reduced to the equations (27).

3. Let point $X=x^\alpha A_\alpha$, belonging to equipping straight line $l_1=(A_{n+1},A_{n+2})$ of n-surface S_n^1 be given. Reasoning similarly, we obtain transformation $\tilde{x}^\alpha=x^\beta\omega_\beta^i\omega_i^\alpha$ of straight line into itself which is defined by matrix (Π_α^β): $\Pi_\alpha^\beta=\Lambda_{\alpha j}^i\Lambda_{ik}^\beta t^k t^k$.

If this is transformation W, then we obtain cone $(\Lambda_{ij}^{n+1}+\Lambda_{ij}^{n+2})x^i x^j=0$; $x^\alpha=0$ in n-plane L_n^1. 1-families (23) giving transformations W of straight line l_1 correspond to its generators. If the reference point is canonical then the equations take on form

$$(\Lambda_{ii}^{n+1}+\Lambda_{ii}^{n+2})(x^i)^2=0;\ x^\alpha=0.$$

If the reference point is canonized (accurate within normalization), then the similar constructions with the same properties may be also obtained for surfaces S_n^2 and S_n^3.

Conclusion

It was shown that the third surface possessing the same properties that the initial surfaces S_n^1 and S_n^2 may be attached to a pair of n-surfaces. The canonical reference point was constructed provided that the coordinate net of lines is conjugate on these surfaces. The private class of a pair of n-surfaces when the coordinate net is conjugate on all surfaces was noted. A series of invariant geometric images for each of surfaces of constructed triplet of n-surfaces was found.

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DETERMINATION OF AMBIGUITY ELLIPSE PARAMETERS AT TWO DIMENSIONS USING GENERALIZED METHOD OF UNCERTAINTY CENTRE

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Definition of empirical dependence parameters at two dimensions using ambiguity ellipse algorithm in a generalized method of uncertainty centre has been considered. The algorithm of optimal parameters definition is offered.

To ascertain the laws of some phenomena the experimental investigations are carried out. During them the values of one or another physicochemical magnitude are measured. At physical experiment processing the empirical models or formula are often used. They include experimentally inaccurately measured magnitudes; inaccuracy is as a rule taken into account in output variables. The case when the output and input variables of the model are timed measured, especially in the case of developing concrete techniques of physical experiment data analysis, is insufficiently represented in scientific literature and is urgent. Statement and investigation of problem solvability of parameters uncertainty set immersion of two-dimensional linear parameter-oriented model at accurate measurement of input variables and inaccurate measurement of output variables were studied in detail in papers [1–4].

Algorithm of parameter uncertainty set immersion of two-dimensional linear dependence into ambiguity ellipse at interval assignment of input and output variables was examined in [5–8]. In these papers inaccuracies which influenced the result of the algorithm operation were made when ambiguity ellipse parameters being determined by generalized method of uncertainty centre.

Let us examine the algorithm [5–8] more detailed. At two dimensions the experimental points should meet the system of interval equations

$$[y]_1=[a]+[b][x]_1,$$
$$[y]_2=[a]+[b][x]_2.$$

The range of possible dimension of linear function parameters is of the form of irregular quadrilateral the angular points of which may be determined using the rules of interval arithmetic. In case of increasing function $[y]=[a]+[b][x]$ i. e. at $x_1^+<x_2^-\Rightarrow y_1^+<y_2^-$, the angular points of uncertainty quadrilateral are determined as

$$A_1=(a_1,\ b_1)=\left(\frac{y_1^-x_2^- - y_1^+x_1^+}{x_2^- - x_1^+};\ \frac{y_2^+ - y_1^-}{x_2^- - x_1^+}\right);$$

$$A_2=(a_2,\ b_2)=\left(\frac{y_1^-x_2^+ - y_2^-x_1^+}{x_2^+ - x_1^+};\ \frac{y_2^- - y_1^-}{x_2^+ - x_1^+}\right);$$

$$A_3=(a_3,\ b_3)=\left(\frac{y_1^+x_2^- - y_2^+x_1^-}{x_2^- - x_1^-};\ \frac{y_2^+ - y_1^+}{x_2^- - x_1^-}\right);$$

$$A_4=(a_4,\ b_4)=\left(\frac{y_1^+x_2^+ - y_2^-x_1^-}{x_2^+ - x_1^-};\ \frac{y_2^- - y_1^+}{x_2^+ - x_1^-}\right). \qquad (1)$$

In case if the function of the form $y=[a]+[b][x]$ is decreasing i.e. at $x_1^+<x_2^-\Rightarrow y_1^+>y_2^-$ then the angular points of uncertainty quadrilateral are determined from the ratios

$$A_1=(a_1,\ b_1)=\left(\frac{y_1^-x_2^- - y_2^-x_1^-}{x_2^- - x_1^-};\ \frac{y_2^- - y_1^-}{x_2^- - x_1^-}\right),$$

$$A_2=(a_2,\ b_2)=\left(\frac{y_1^-x_2^+ - y_2^+x_1^-}{x_2^+ - x_1^-};\ \frac{y_2^+ - y_1^-}{x_2^+ - x_1^-}\right),$$

$$A_3=(a_3,\ b_3)=\left(\frac{y_1^+x_2^+ - y_2^+x_1^+}{x_2^+ - x_1^+};\ \frac{y_2^+ - y_1^+}{x_2^+ - x_1^+}\right),$$

$$A_4=(a_4,\ b_4)=\left(\frac{y_1^+x_2^- - y_2^-x_1^+}{x_2^- - x_1^+};\ \frac{y_2^- - y_1^+}{x_2^- - x_1^+}\right). \qquad (2)$$

To determine the uncertainty quadrilateral centre of gravity the least-squares procedure is used. Then for searching centre of gravity coordinates we obtain the ratio

$$\bar{a}_0=\frac{\bar{x}_2\bar{y}_1-\bar{x}_1\bar{y}_2}{\bar{x}_2-\bar{x}_1};\ \bar{b}_0=\frac{\bar{y}_2-\bar{y}_1}{\bar{x}_2-\bar{x}_1},$$

where $\bar{x}_1$, $\bar{x}_2$, $\bar{y}_1$, $\bar{y}_2$ are the average values of measured input and output variables respectively.

The centre of gravity of the given uncertainty quadrilateral may be also determined in different ways. Any combination of three angular points defined by the ratios (1) and (2) forms the uncertainty triangle. There are four of such triangles: $A_1A_2A_3$, $A_1A_2A_4$, $A_1A_3A_4$, $A_2A_3A_4$. It is known that the centre of gravity of any triangle is on the crossing of its medians. Thus, for centre of gravity coordinates of triangles there is a ratio

$$B_1=\left(\frac{a_1+a_2+a_3}{3};\ \frac{b_1+b_2+b_3}{3}\right);$$

$$B_2=\left(\frac{a_1+a_2+a_4}{3};\ \frac{b_1+b_2+b_4}{3}\right);$$

$$B_3=\left(\frac{a_1+a_3+a_4}{3};\ \frac{b_1+b_3+b_4}{3}\right);$$

$$B_4=\left(\frac{a_2+a_3+a_4}{3};\ \frac{b_2+b_3+b_4}{3}\right).$$

As a result four points B_1, B_2, B_3, B_4, which define new uncertainty quadrilateral are obtained. Let us suppose that $A_1=B_1$, $A_2=B_2$, $A_3=B_3$, $A_4=B_4$. Then the procedure of determining the centers of gravity of new uncertainty quadrilaterals is repeated till the moment while the required accuracy of defining the centre of gravity of the given uncertainty quadrilateral is achieved.

Let us denote quadrilateral centre of gravity by a_0, b_0. At parameter uncertainty set immersion of linear parameter-oriented function into the ambiguity ellipse the centre of gravity of the given quadrilateral is considered to coincide with the centre of ambiguity ellipse circumscribed around it. Then the angular points of uncertainty quadrilateral are arranged in order of parameter b magnitude that is $b_1<b_2<b_3<b_4$.

Segment A_3A_4 is a chord of ambiguity ellipse. Then the parameters of the line A_3A_4

$$k_{34}=\frac{a_4-a_3}{b_4-b_3};\ \alpha_{34}=\frac{a_3b_4-a_4b_3}{b_4-b_3}.$$

Let us draw a straight line parallel to A_3A_4. For this purpose let us determine the coordinates of bisecting point of the segment A_3A_4:

$$a_{34}=\frac{a_3+a_4}{2};\ b_{34}=\frac{b_3+b_4}{2}.$$

The parameters of the line $a=kb+\alpha$ passing through the bisecting point of the segment A_3A_4 and ellipse centre are found as

$$k=\frac{a_0-a_{34}}{b_0-b_{34}};\ \alpha=\frac{\alpha_{34}b_0-a_0b_{34}}{b_0-b_{34}}. \qquad (3)$$

After that the distance from chord A_3A_4 midpoint to the ambiguity ellipse centre is determined

$$l=\sqrt{a_{01}^2+b_{01}^2}.$$

where $a_{01}=a_0-a_{34}$; $b_{01}=b_0-b_{34}$.

The chord of ellipse $A_1^*A_2^*$ is parallel to chord A_3A_4 and symmetric to it with respect to ellipse centre [5–8]. The midpoint of chord $A_1^*A_2^*$ is on the line with parameters (3) its coordinates are determined by the ratios

$$a_{12}=a_0+a_{01};\ b_{12}=b_0+b_{01}.$$

As chords $A_1^*A_2^*$ and A_3A_4 are symmetrical with respect to the ellipse centre then let us calculate the additional magnitudes for defining the coordinates of points A_1^* and A_2^*

$$\Delta a_{34}=\Delta a_{12}=a_4-a_{34}=\frac{a_4-a_3}{2},$$

$$\Delta b_{34}=\Delta b_{12}=b_4-b_{34}=\frac{b_4-b_3}{2}.$$

Then the coordinates of point A_1^* are calculated as

$$a_1^*=a_{12}-\Delta a_{12};\ b_1^*=b_{12}-\Delta b_{12},$$

and coordinates of point A_2^*

$$a_2^*=a_{12}+\Delta a_{12};\ b_2^*=b_{12}+\Delta b_{12}.$$

Thus, the parallelogram $A_1^*A_2^*A_3A_4$ circumscribes the uncertainty quadrilateral of the parameters *a, b.* The centre of gravity of parallelogram $A_1^*A_2^*A_3A_4$ coincides with the centre of ellipse containing it.

The ambiguity ellipse circumscribed next to the parallelogram $A_1^*A_2^*A_3A_4$ is searched in the form of

$$F(a-a_0)^2+2D(a-a_0)(b-b_0)+Q(b-b_0)^2=1,$$

where a_0, b_0 are the coordinates of ambiguity ellipse centre of the parameters.

To define the parameters of ambiguity ellipse let us make a set of equations

$$F\Delta a_1^2+2D\Delta a_1\Delta b_1+Q\Delta b_1^2=1,$$
$$F\Delta a_2^2+2D\Delta a_2\Delta b_2+Q\Delta b_2^2=1, \qquad (4)$$

where the following notations are accepted:

$$\Delta a_1=\frac{a_1^*-a_4}{2};\ \Delta a_2=\frac{a_2^*-a_3}{2};$$
$$\Delta b_1=\frac{b_4-b_1^*}{2};\ \Delta b_2=\frac{b_3-b_2^*}{2}.$$

System (4) is rewritten in the form

$$F\Delta a_1^2+Q\Delta b_1^2=1-2D\Delta a_1\Delta b_1,$$
$$F\Delta a_2^2+Q\Delta b_2^2=1-2D\Delta a_2\Delta b_2. \qquad (5)$$

In papers [5–8] parameters Q and F were inaccurately determined. Let us solve system (5) with respect to Q and F:

$$Q=Q_0+Q_1D;\ F=F_0+F_1D, \qquad (6)$$

where

$$Q_0=\frac{\Delta a_2^2-\Delta a_1^2}{\Delta a_2^2\Delta b_1^2-\Delta a_1^2\Delta b_2^2};\ Q_1=\frac{-2\Delta a_1\Delta a_2}{\Delta a_2\Delta b_1+\Delta a_1\Delta b_2};$$

$$F_0=\frac{\Delta b_2^2-\Delta b_1^2}{\Delta a_1^2\Delta b_2^2-\Delta a_2^2\Delta b_1^2};\ F_1=\frac{-2\Delta b_1\Delta b_2}{\Delta a_1\Delta b_2+\Delta a_2\Delta b_1}.$$

The area of ambiguity ellipse with parameters F, Q, D is calculated as

$$\nu=\frac{\pi}{\sqrt{FQ-D^2}}. \qquad (6)$$

To determine the ellipse of minimal area the maximum of function f_D is found

$$f_D(F,Q,D)=FQ-D^2. \qquad (7)$$

Subject to (6), the expression (7) may be presented in the form of

$$f_D(F,Q,D)=F_0Q_0+(F_0Q_1+F_1Q_0)D+D^2(F_1Q_1-1).$$

Let us calculate the derivative $f_D'(F,Q,D)$:

$$f_D'(F,Q,D)=F_0Q_1+F_1Q_0+2D(F_1Q_1-1).$$

Then

$$f_D'(F,Q,D)=0.$$

Having solved this equation with respect to parameter D we obtain the expression

$$D=\frac{F_0Q_1+F_1Q_0}{2(F_1Q_1-1)}.$$

In this case using the obtained ratios for parameters F, Q, D the ambiguity ellipse area is minimal.

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