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ABELIAN GROUPS AS ARTINIAN OR NOETHERIAN MODULES ABOVE ENDOMORPHISM RINGS. P. 3

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The A and B Abellian groups, such that the Hom(A, B) homomorphism group is the Artin module over the ring of the B group endomorphism, are described. Description of the A and B group for which the Hom(A,B) group is the Artin module over the ring of the A group endomorphism is reduced to the case when the A group has no torsion and the B group is either a quasi-cyclic group or a divisible group without torsion. The A and B Abellian groups for which the Hom(A,B) group is the Neter module over the E(A) or E(B) ring are characterized. The research of arbitrary Abellian group with the link Neter ring of endomorphisms is reduced to the research of the group without torsion with the link Neter ring of endomorphisms. The research of the right Neter ring of endomorphisms remained uncompleted. The separable Abele groups without torsion with the link and right Neter rings of endomorphisms are described.

The problem of description of the A and B groups such that $E(B)$-module of $\mathrm{Hom}(A,B)$ is the Neter one, is reduced in the certain sense in the cases of the A cyclical group of the simple order and the torsion-free A and B groups (the 5th theorem [1. P. 69]).

Suggestion 9. If $E(B)$ module of $\mathrm{Hom}(A,B)$ is Neter one,

$$\overline{A} = H \oplus \sum_{n}{}^{\oplus} Q \oplus \sum_{j=1}^{m}{}^{\oplus} Z(p_j^{\infty}) \oplus G,$$

where H is the finite group, G is reduced torsion-free group, $n,m \in N$,

$$S = C \oplus \sum_{\eta}{}^{\oplus} Q \oplus \sum_{i=1}^{k}{}^{\oplus} \sum_{\eta_i}{}^{\oplus} Z(p_j^{\infty}) \oplus G',$$

where C is limited group, G is reduced torsion-free group, $k \in N$, η, η_i are some cardinals.

Let`s note the large significance of the consequence 27.3 from [2] for proof of this suggestion. It follows from the 9th Suggestion that the Neter character of the module to be studied is equivalent to one of the following four $E(B)$-modules: $\mathrm{Hom}(H,B)$, $\mathrm{Hom}(Z(p^{\infty}),B)$, $\mathrm{Hom}(G,B)$, $\mathrm{Hom}(Q,B)$, where H is the finite group, G is the reduced torsion-free group.

The 10th and 11th Suggestions answer the question: when $E(B)$-module of $\mathrm{Hom}(Q,B)$ is the Neter one?

The Suggestion 10. Assume that $\sum_{\eta}^{\oplus} Z(p^{\infty})$ is divisible p-component of a B – group. Then the $E(B)$-module of $\mathrm{Hom}(Q,\sum_{\eta}^{\oplus} Z(p^{\infty}))$ is not the Neter one.

The Suggestion 11. Assume that the B group has no quasicyclic group. Than the $E(B)$-module of $\mathrm{Hom}(Q,\sum_{\eta}^{\oplus} Q)$ is the Neter one.

The Neter character of the $E(B)$ – module of $\mathrm{Hom}(Z(p^{\infty}),B)$ is stated in the Suggestion 12.

The Suggestion 12. Let $\sum_{\eta}^{\oplus} Z(p^{\infty})$ is divisible part of the p-component of a B-group. Than the $E(B)$-module of $\mathrm{Hom}(Z(p^{\infty}),\sum_{\eta}^{\oplus} Z(p^{\infty}))$ is the Neter one.

The proof of this suggestion is interesting: the descending chain of its submodules is built and it is proof that this module has no other own submodules.

As to the $E(B)$-module of $\mathrm{Hom}(H,B)$, its Neter character, clearly, is equal to the Neter character of the $E(B)$-modules of $\mathrm{Hom}(Z(p^k),B)$ form for p numbers belonging to a H group (let`s remember that H is a finite group). The Neter character of the $E(B)$-modules of the such form is equal to the one of the $E(B)$-module of $\mathrm{Hom}(Z(p),B)$ according to the Suggestion 13.

The Suggestion 13. The $E(B)$-module of $\mathrm{Hom}(Z(p^k),B)$ is the Neter one if and only if when the module of $\mathrm{Hom}(Z(p),B)$ is the Neter one.

It is known, that canonic isomorphism of the $E(B)$-modules:

$$\mathrm{Hom}(Z(p),B) \cong B[p].$$

is true. Therefore the problem of the Neter character of the low level of $B[p]$ as the $E(B)$-module arises in connection with studying of the $E(B)$-module of $\mathrm{Hom}(Z(p),B)$. In other words, when is stabilized each chain of quite characteristic subgroups of B-group laying in $B[p]$?

The Suggestion 14. Let D_p is divisible p-component of the B group, G is reduced group without torsion. The $E(B)$-module of $\mathrm{Hom}(G,D_p)$ is not Neter one.

This fact is used at research of mixed group with the endomorphis ring being Neter one on the left.

The theorem 5 is the result of investigation of the A and B group such, that the $E(B)$-module of $\mathrm{Hom}(A,B)$ is the Neter one.

The theorem 5. Let A and B are groups. $E(B)$ module of $\mathrm{Hom}(A,B)$ is the Neter one if and only if, when

$$\overline{A} = H \oplus \sum_{n}{}^{\oplus} Q \oplus \sum_{j=1}^{m}{}^{\oplus} Z(p_j^{\infty}) \oplus G,$$

Where H is finite, G is a reduced group without torsion, $n,m \in N$,

$$S = C \oplus \sum_{\eta}{}^{\oplus} Q \oplus \sum_{i=1}^{k}{}^{\oplus} D_{p_i} \oplus G',$$

Where C is the limited group, D_{p_i} is a divisible p_i-component of a S group, G' – the reduced group without torsion, $k \in N$, η is some cardinal; for anyone p, belonging to a H group, $E(B)$ module of $\mathrm{Hom}(Z(p),B)$ is the Neter one, and:

a) if $k \neq 0$,

$$\overline{A} = H \oplus \sum_{j=1}^{m}{}^{\oplus} Z(p_j^{\infty}), \quad S = C \oplus \sum_{i=1}^{k}{}^{\oplus} D_{p_i};$$

b) if $k=0$, but $\eta \neq 0$,

$$\overline{A} = H \oplus \sum_{n}{}^{\oplus} Q \oplus G, \quad S = C \oplus \sum_{\eta}{}^{\oplus} Q \oplus G',$$

where $r(G)<\infty$ and $E(B)$ module of $\mathrm{Hom}(G,S/M)$ is the Neter one, where $M=C\oplus\sum_{\eta}^{\oplus}Q$;

c) if $k=\eta=0$,

$$\overline{A}=H\oplus G,\ S=C\oplus G',$$

For anyone p, belonging to a S group, $r_p(G)<\infty$ and the $E(B)$ module of $\mathrm{Hom}(G,S/C)$ is the Neter one.

The basic idea of its proof is the same, as one of the theorem 3: construction of induced exact sequences of the $E(B)$ modules. The proof of necessity of the theorem 5 also is based on the Suggestion 9 and Suggestion 13. If both groups A and B are torsion-free, it is possible to hope for some partial results only in a question about the Neter character of the $E(B)$ module of $\mathrm{Hom}(A,B)$.

Let's give some consequences of the theorem 5.

The Consequence 14. Let A and B are periodic groups. The $E(B)$ module of $\mathrm{Hom}(A,B)$ is the Neter one if and only if, when

$$\overline{A} = H \oplus \sum_{j=1}^{m}{}^{\oplus} Z(p_j^{\infty}), \quad S = C \oplus \sum_{i=1}^{k}{}^{\oplus} D_{p_i},$$

Where $m,k \in N$, H is a finite group, C is a limited group, D_{p_i} is a divisible p_i-component of a S group, for anyone p, belonging to H-group, the $E(B)$ module of $\mathrm{Hom}\ (Z(p),B)$ is the Neter one.

The Consequence 15. Let A and B are divisible groups. The $E(B)$ module of $\mathrm{Hom}(A,B)$ is the Neter one if and only if when groups $\overline{A}$ and S either are both periodic, or both are torsion-free, and:

in the first case

$$\overline{A} = \sum_{j=1}^{m}{}^{\oplus} Z(p_j^{\infty}), \quad S=\sum_{i=1}^{k}{}^{\oplus} D_{p_i};$$

in the second case

$$\overline{A} = \sum_{n}{}^{\oplus} Q, \quad S = \sum_{\eta}{}^{\oplus} Q,$$

where $m,n,k \in N$, η is a some cardinal.

The following three consequences follow from the Suggestion 14 and the theorem 5.

The Consequence 16. Let A is a reduced torsion-free group, B is a periodic group. The $E(B)$ module of $\mathrm{Hom}(A,B)$ is the Neter one if and only if when a S trace is the limited group and for anyone p, belonging to a A group, $r_p(A)<\infty$.

The Consequence 17. Let A is a reduced torsion-free group, B is a divisible group. The $E(B)$ module of $\mathrm{Hom}(A,B)$ is the Neter one if and only if when a S trace is torsion-free and $r(A)<\infty$.

The Consequence 18. Let A is the reduced torsion-free group, and B group is those, that its torsion-free part is a divisible group. The $E(B)$ module of $\mathrm{Hom}(A,B)$ is the Neter one if and only if when a S trace is equal to the direct sum of the limited group and divisible group without torsion, and: a) if a S trace contains even one Q group $r(A)<\infty$; b) if a S trace is the limited group, for anyone p, belonging to a S group, $r_p(A)<\infty$.

From the theorem 3 and theorem 5 it is possible to deduce conditions, at which the $E(B)$ module of $\mathrm{Hom}\ (A,\ B)$ is the Artin and Neter one simultaneously.

The Consequence 19. Let A and B are periodic groups. The $E(B)$ module of $\mathrm{Hom}(A,B)$ is the Artin and Neter one if and only if when a S trace is the limited group, and for anyone p, belonging to a S group, the reduced p-component of a B group is limited. The group $\overline{A}$ is final and for anyone p, belonging to a group $\overline{A}$, the $E(B)$ module of $\mathrm{Hom}\ (Z(p),B)$ is the Neter one.

The Consequence 20. Let A and B are divisible groups. The $E(B)$ module of $\mathrm{Hom}(A,B)$ is the Artin and Neter one if and only if when $\overline{A}$ and S groups are torsion-free, and the rank of group $\overline{A}$ is finite.

The Consequence 21. Let A is the torsion-free reduced group, B is a periodic group. The $E(B)$ module of $\mathrm{Hom}(A,B)$ is the Artin and Neter one if and only if when a S trace is the limited group; for anyone p, belonging to a S group, the reduced p-component of a B group is limited and for anyone p, belonging to a S group, $r_p(A)<\infty$.

The Consequence 22. Let A is the reduced torsion-free group, B is divisible group. $E(B)$ module $\mathrm{Hom}(A,B)$ is the Artin and Neter one if and only if when a S trace is torsion-free and the rank of group A is finite.

The Consequence 23. Let A is the reduced torsion-free group, and B group is such, that its torsion-free part is divisible group. The $E(B)$ module of $\mathrm{Hom}(A,B)$ is the Artin and Neter one if and only if when a S trace is equal to the direct sum of the limited group and divisible torsion-free group; for anyone p, belonging to a S group, the reduced p-component of a B group is limited, and: a) if a S trace contains even one Q group $r(A)<\infty$, b) if a S trace is the limited group for anyone p, belonging to a S group, $r_p(A)<\infty$.

The description *of A* and *B* groups, such, that the $E(A)$ module of Hom(A,B) is the Neter one, is reduced to the cases of an A group with unlimited p-component even for one p, belonging to a trace of an A group in a B group and A and B torsion-free groups (the 49[th] theorem [3. S. 69]).

The theorem 6. Let A and B are some groups and let a reduced p-component of an A group is limited for anyone p, belonging to a S trace of an A group in a B group. The $E(A)$ module of Hom(A,B) is the Neter one if and only if when

$$\overline{A}=\sum_{i=1}^{n}{}^{\oplus}D_{p_i}\oplus\sum_{\eta}{}^{\oplus}Q\oplus C\oplus G,$$

$$S=\sum_{j=1}^{n}{}^{\oplus}Z(p_j^{\infty})\oplus\sum_{m}{}^{\oplus}Q\oplus H\oplus G',$$

where n, $m\in N$, η is a some cardinal, is a divisible p_i-component of A group, C is a limited group, H is a finite group, G and G' are the reduced torsion-free groups, and:

a) if $n\neq 0$,

$$\overline{A}=\sum_{i=1}^{n}{}^{\oplus}D_{p_i}\oplus C\oplus G,$$

$$S=\sum_{j=1}^{n}{}^{\oplus}Z(p_j^{\infty})\oplus\sum_{m}{}^{\oplus}Q\oplus H\oplus G',$$

and the $E(A)$ module of Hom$(\overline{A}/M,S)$, where $M=\sum_{i=1}^{n}{}^{\oplus}D_{p_i}\oplus C$, is the Neter one;

b) if $n=0$, but $m\neq 0$,

$$\overline{A}=\sum_{\eta}{}^{\oplus}Q\oplus C\oplus G,$$

$$S=\sum_{m}{}^{\oplus}Q\oplus H\oplus G',$$

and the $E(A)$ module of Hom$(\overline{A}/L,S)$, where $L=\sum_{\eta}{}^{\oplus}Q\oplus C$ is the Neter one;

c) if $n=m=0$, or

$$S=H\oplus G',$$
$$\overline{A}=C\oplus G$$

and the $E(A)$ module of Hom$(\overline{A}/C,S)$ is the Neter one.

The proof of the theorem 6 which belongs to the basic results of the work, is based on construction of induced exact sequences of $E(A)$ modules, the theorem 1 and the following suggestions.

The suggestion 15. Let the $E(A)$ module of Hom(A,B) is the Neter one. Then a S group is the direct sum of finite number of items:

$$S=\sum_{j=1}^{n}{}^{\oplus}Z(p_j^{\infty})\oplus\sum_{m}{}^{\oplus}Q\oplus H\oplus G',$$

where $n,m\in N$, H is finite group, G' is the torsion-free reduced group.

The Suggestion 15 gives the information on a structure of an A group trace in a B group for the Neter $E(A)$ module of Hom(A,B). We shall emphasize a significant role of the consequence 27.3 from the book [2] in its proof. From the Suggestion 15 (if to take into account a kind of some submodules of the $E(A)$ module of Hom(A,B)) it follows, that the Neter character of the $E(A)$ module of Hom(A,B) is equivalent to its following submodules: Hom$(A,Z(p^k))$, Hom(A,Q), Hom$(A,Z(p^{\infty}))$, Hom(A,G'), where G' is the reduced group without torsion. Besides knowing a structure of the trace of the A group in the B group, it is easy to make conclusion on a structure of the trace of the B group in the A group.

The Suggestion 16. Let D_p is a divisible p-component of an A group. The $E(A)$ module of Hom$(D_p,Z(p^{\infty}))$ is the Neter one.

The idea of the proof of the Suggestion 16 is interesting: the decreasing sequence of submodules of $E(A)$ module of Hom$(DD_p,Z(p^{\infty}))$ which existence based earlier the non-Artin character of this module, is written out, and it is shown, that this module has no other own submodules.

The Suggestion 17. Let $V=\sum_{\eta}^{\oplus}Q$, where η is some cardinal. The $E(V)$ module Hom$(V,Z(p^{\infty})$ is not Neter and not Artin one.

The Suggestion 18. Let a S trace of an A group in a B group contains if only one quasi-cyclic group, $\sum_{\eta}^{\oplus}Q$ is divisible part of an torsion-free A group, η is some cardinal, D_p is a divisible p-component of an A group and $D=D_p\oplus\sum_{\eta}^{\oplus}Q$. The $E(A)$ module of Hom$(D,Z(p^{\infty}))$ is not Neter and not Artin one.

The Suggestion 19. Let D is a divisible part of an A group, D_p is the periodic part of a D group, that is $D=D_p\oplus\sum_{\eta}^{\oplus}Q$ for some cardinal η. The $E(A)$ module of Hom(D,Q) is the Neter and Artin one.

Let's give consequence of the theorem 6.

The Consequence 24. Let A and B is divisible groups. The $E(A)$ module of Hom(A,B) is the Neter one if and only if when a S trace of the A group in the B group is divisible group of a finite rank, and:

a) if the S trace contains a quasi-cyclic group, $\overline{A}$ is a periodic group with finite number of the p-components;

b) if the S trace does not contain a quasi-cyclic group, $\overline{A}$ is the group without torsion.

The Consequence 25. Let A and B are periodic groups and let a reduced p-component of the A group is limited for any p, belonging to the S trace. The $E(A)$ module of Hom(A,B) is the Neter one if and only if when the S trace is equal to the direct sum of a finite group and a divisible periodic group of a finite rank, and $\overline{A}$ is the direct sum of a limited group and a divisible periodic group with final number of p-components.

It follows from the theorem 6 and the consequences 15.

The Consequence 26. Let A and B are divisible groups. The Hom(A,B) group is the Neter $E(A)$ module and the Neter $E(B)$ if and only if when the $\overline{A}$ and S groups have a finite rank, and they are either periodic, or torsion-free.

The following consequence follows from the theorem 6 and consequence 14.

The Consequence 27. Let A and B are periodic groups and a reduced p-component of the A group is limited for any p, belonging to a S trace. The Hom(A,B) group is the Neter $E(A)$ module and simultaneously the Neter $E(B)$ module if and only if when $\overline{A}$ and S groups are equal to the direct sum of a finite group and a divisible periodic group of a finite rank (ranks of $\overline{A}$ and S groups are finite, but they are not obliged to coincide) and for each p, belonging to the reduced part of an $\overline{A}$ group, the $E(B)$ module of Hom($Z(p)$,B) is the Neter one.

Let's give consequences of the theorems 6 and 4.

The Consequence 28. Let A and B are divisible groups. The $E(A)$ module of Hom(A,B) is the Artin and Neter one if and only if when $\overline{A}$ and S groups are torsion-free and the S group rank is finite.

The Consequence 29. Let A and B are periodic groups. The $E(A)$ module of Hom(A,B) is the Artin and Neter one if and only if when the S trace is equal to the direct sum of finite group and divisible periodic group of a finite rank, the $\overline{A}$ group is limited and for any p, belonging to B group, a reduced p-component of the A group is limited.

The Consequence 30. Let A and B groups are such, that their parts without torsion are divisible groups. The $E(A)$ module of Hom(A,B) is the Artin and Neter one if and only if when for any p, belonging to B group, the reduced p-component of the A group it is limited; the trace S is equal to the direct sum of finite group and divisible group of the finite rank, and:

a) if the S group contains a quasi-cyclic group it is periodic, and the $\overline{A}$ group is limited;
b) if the S group does not contain a quasi-cyclic group, but contains finite number of Q group copies it is equal to the direct sum of a finite group and a divisible torsion-free group of a finite rank, and the $\overline{A}$ group is the direct sum of the limited group and divisible torsion-free group;
c) if the S trace is finite group the $\overline{A}$ group is limited.

The structure any Abele groups with Artin rings of endomorphisms is known. The ring of $E(A)$ endomorphisms of A groups is the Artin one on the left (or on the right) if and only if when A=B⊕D, where B is a finite group, D is divisible torsion-free group of a finite rank (the theorem 111.3 from [4]). Also the periodic Abele groups with the Neter rings of endomorphisms on the right (or on the left) are described. The ring $E(A)$ of the periodic A group is the Neter one on the right (or at the left) if and only if when A is the direct sum of finite number of co-cyclic groups (the Suggestion 111.4 from [4]). Let's remind, that a co-cyclic group is either cyclic p-group, or quasi-cyclic group. As opposed to condition of minimality condition of maximality imposed on a ring of endomorphisms, limits group structure not too much.

The Lemma 6. Let A is a mixed Abele group with the Neter ring of endomorphisms. Then A=T⊕G, where T is the direct sum of finite number of co-cyclic groups, G is a group without torsion.

From this lemma it follows, that if a ring of endomorphisms of the A mixed group is the Neter one it can be presented by the ring of matrixes:

$$E(A) \cong \begin{pmatrix} E(T) & \mathrm{Hom}(G,T) \\ 0 & E(G) \end{pmatrix},$$

where G and T are such groups, as in the lemma 6. According to exc. 6 [5. P. 165] such ring of matrixes is the Neter one on the left (accordingly on the right) if and only if when $E(T)$ and $E(G)$ rings are the Neter on the left (accordingly on the right) and the $E(T)$ module of Hom(G,T) is the Neter one (accordingly the $E(G)$ of module Hom(G,T) is the Neter one).

Thus, studying *of* an *A group* with a Neter ring of endomorphisms $E(A)$ is closely connected to studying of the Neter module Hom(G,T) above rings of endomorphisms of G and T groups, where G and T groups are such, as in the lemma 6.

Research of arbitrary Abele groups with the $E(A)$ Neter ring of endomorphisms on the left has been successful to reduce completely to research of torsion-free groups with the Neter ring of endomorphisms on the left [6].

The Suggestion 20. Let G is a torsion-free group, T is the direct sum of finite number of co-cyclic groups. The $E(T)$ module of Hom(G,T) is the Neter one if and only if when T is the reduced group and for any p, belonging to T, $r_p(G)$ is finite.

The theorem 7. Let A is the mixed group. The ring is the Neter one on the left if and only if when A=T⊕G where T is a finite group, G is a torsion-free group such, that the $E(G)$ ring is the Neter one on the left and for any p, belonging to T, $r_p(G)$ is finite.

Research of the mixed groups with the Neter rings of endomorphisms on the right remained uncompleted; it was impossible to reduce their studying to studying of torsion-free groups with the Neter rings of endomorphisms on the right. It was connected with the fact that it was impossible in general case to answer a question about the Neter character of the right $E(G)$ module of Hom(G,T) where G is the reduced torsion-free group, T is the direct sum of finite number of co-cyclic groups. The answer has been obtained with some restrictions on the G group ([6]).

The Suggestion 21. Let G is a torsion-free group, p is a simple number, is a divisible p-group. If G is either not p-divisible group, or G is a p-divisible group and the $E(G)$ ring is countable, the $E(G)$ module of Hom(G,D) is not the Neter one.

The Suggestion 22. Let G is a torsion-free group, T is a finite group. If $r_p(G)$ is finite for each p, belonging to the T group, the $E(G)$ module of Hom(G,T) is the Neter one.

The suggestion 21 and 22 allow to make some conclusions on a structure of the mixed groups with the Neter rings of endomorphisms on the right.

The Consequence 31. Let A=T⊕G group where G is torsion-free group with the Neter ring $E(G)$ on the right, *T* is finite group, and $r_p(G)$ is finite for every *p*, belonging to the *T* group. Then the *E*(*A*) ring is the Neter one on the right.

The Consequence 32. Let $A=T\oplus G$ group, where *G* is a torsion-free group with accounting ring *E*(*G*) (for example, the *G* group has a finite rank), *T* is the direct sum of finite number of co-cyclic groups. If the ring *E*(*A*) is the Neter one on the right, *T* is the reduced group (or, what here is equivalent, *T* is final group).

The Consequence 33. Let *A* is the mixed torsion-free group of a finite rank. The *E*(*A*) ring is the Neter one on the right if and only if when $A=T\oplus G$, where *G* is a torsion-free group of a finite rank with the Neter *E*(*G*) ring on the right, *T* is a final group.

Let's remind, that the torsion-free *A* group is named as separable if each finite subset of elements from *A* is contained in some quite decomposable direct summand of the *A* groups.

The exhaustive description of separable torsion-free Abele groups with rings of endomorphisms being the Neter one on the left or on the right [7] is obtained.

The theorem 8. A *G* ring of endomorphisms of separable torsion-free groups is the Neter one on the right if and only if when *G* is quite decomposable group of a finite rank and types its homogeneous component are incomparable in pairs.

The theorem 9. Let *G* is the separable torsion-free group. The ring *E*(*G*) is the Neter one on the left if and only if when the *G* group is quite decomposable group of a finite rank and types of its various homogeneous components are either incomparable or comparable due to infinities.

This description essentially leans on research of the Hom(*A*,*B*) group, where *A* and *B* are groups without torsion of a rank 1, as the Neter *E*(*B*) or *E*(*A*) module.

Let's give the consequences of the theorems 8 and 9.

The Consequence 34. A ring of endomorphisms of a separable torsion-free groups of a finite rank, types of all straight line items of the rank 1 which are idempotent is the Neter one on the left.

The Consequence. **35** If a ring of endomorphisms of a separable group without torsion is the Neter one to the right, it is the Neter one to the left.

The well-known fact, that a ring of endomorphisms of $Z\oplus Q$ groups, being isomorphic to a ring of matrixes

$$\begin{pmatrix} Z & 0 \\ Q & Q \end{pmatrix},$$

is the Neter one to the left, but is not the Neter one to the right, follows from the theorem 8 and the consequences 34. The same is true for the $Q_p\oplus Q$, $Z\oplus Q_p$ groups.

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