



# Dynamic Mode II fracture behavior of rocks under hydrostatic pressure using the short core in compression (SCC) method

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## ABSTRACT

The shear failure of rocks under both a static triaxial stress and a dynamic disturbance is common in deep underground engineering and it is therefore essential for the design of underground engineering to quantitatively estimate the dynamic Mode II fracture toughness  $K_{IIc}$  of rocks under a triaxial stress state. However, the method for determining the dynamic  $K_{IIc}$  of rocks under a triaxial stress has not been developed yet. With an optimal sample preparation, the short core in compression (SCC) method was designed and verified in this study to measure the dynamic  $K_{IIc}$  of Fangshan marble (FM) subjected to different hydrostatic pressures through a triaxial dynamic testing system. The formula for calculating the dynamic  $K_{IIc}$  of the rock SCC specimen under hydrostatic pressures was obtained by using the finite element method in combination with secondary cracks. The experimental results indicate that the failure mode of the rock SCC specimen under a hydrostatic pressure is the shear fracture and the  $K_{IIc}$  of FM increases as the loading rate. In addition, at a given loading rate the dynamic rock  $K_{IIc}$  is barely affected by hydrostatic pressures. Another important observation is that the dynamic fracture energy of FM enhances with loading rates and hydrostatic pressures.

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## 1. Introduction

In deep underground space excavations and deep rock engineering practices, rocks are generally subjected to a high in-situ stress (i.e. a static triaxial stress state or a hydrostatic pressure) [1,2], and these natural rocks are also likely to fracture failure induced by dynamic forces, e.g. optional blasting and earthquake. Therefore, it is crucial to quantify the dynamic fracture properties of deep rocks under both the dynamic load and the in-situ stress [3–8].

The fracture toughness of rocks is one of the crucial fracture properties of rocks and many investigations have been performed to assess the rock fracture toughness under different loading conditions [9–12]. There are three primary fracture modes (i.e. Modes I, II, and III) for the determination of rock fracture toughness [13]. A amount of experimental specimens have been proposed to obtain different types of the fracture toughness of rock-like materials under static loading conditions: (1) Mode I (opening): cracked

chevron notched Brazilian disc [14–17], short rod/beam [18–20], chevron bending [18], and notched semi-circular bend (NSCB) method [10,21]; (2) Mode II (shearing): antisymmetric four-point bending specimen [22–29], punch-through shear (PTS) specimen [30–32], and short core in compression (SCC) specimen [33]; and (3) mixed mode I/II: Arcan specimen with a notch for uniform plane stress [34–36], the NSCB with inclined notch [37–39], and cracked straight-through Brazilian disc specimen [14]. Among the methods mentioned above, the International Society for Rock Mechanics and Rock Engineering (ISRM) has suggested the NSCB method to quantify the dynamic Mode I fracture toughness  $K_{Ic}$  of rocks [40].

Generally, the main fracture mode for engineering materials (e.g. alloys and concretes) is Mode I fracture. However, in natural rock structures, Mode II or mixed mode I/II failure frequently happen due to the complex mutual effect between tensile and shear fracture [9,41,42]. For example, discontinuities in rock masses and rocks with pre-existing cracks are commonly failures as a shear mode when they are subjected to compressive/shear mixed mode forces [5,43]. Hence, shearing failure is the normal mode in

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rock engineering and it is essential to study the Mode II fracture toughness  $K_{IIc}$  of rocks [33,44,45].

Several researchers have studied the  $K_{IIc}$  of materials under static loading conditions [39,43,46–48]. Watkins and Liu [46] introduced a short beam in compression (SBC) specimen to quantify the  $K_{IIc}$  of plain concrete due to the simple specimen and loading configuration of the SBC method. Lawn [49] claimed that the shearing fracture toughness probably depends on the normal pressure on the plane of failure. Melin [50] pointed out that under high confining pressures the Mode II fracture is dominated in the rock failure. Whittaker et al. [51] gave a review of various approaches for measuring the  $K_{IIc}$ . Rao et al. [43] measured the pure  $K_{IIc}$  in the mixed mode loading and found the  $K_{IIc}$  for the same rock is higher than the  $K_{Ic}$ . Backers et al. [31] and Backers et al. [30] examined the effect of the confining pressure on the  $K_{IIc}$  by means of the PTS method with confining pressure. Lee [52] employed the rectangular PTS specimens to measure the  $K_{IIc}$  of rocks. Due to the advantages of a core-based specimen with confinement, the PTS method has been accepted in 2012 by the ISRM to quantify the dynamic static  $K_{IIc}$  of rocks under different confining pressures [47]. Furthermore, Jung et al. [53] used the SCC method with a cylindrical rock core to replace the short cuboid beam in the compression method and further measured the shear strength and  $K_{IIc}$  of rock under static loading based on the SCC method. Xu et al. [33] evaluated the validity of the SCC method for measuring the  $K_{IIc}$  and calculated the stress intensity factor (SIF) and the  $K_{IIc}$  of the SCC specimen.

The above investigations emphasized on the  $K_{IIc}$  of rocks under static loading conditions. However, the methods for quantifying the dynamic  $K_{IIc}$  were recently developed. The PTS specimen was recently extended to measure the dynamic rock  $K_{IIc}$  [5] and the SCC specimen in combination with a split Hopkinson pressure bar (SHPB) system was suggested to obtain the dynamic rock  $K_{IIc}$  [54]. Furthermore, the PTS specimen was modified to determine the dynamic rock  $K_{IIc}$  with confining pressures [55].

Although the static and dynamic  $K_{IIc}$  of rocks under confining pressures was extensively determined, there is a lack of a method to quantify the dynamic rock  $K_{IIc}$  over various in-situ stresses or hydrostatic pressures. Therefore, a dynamic SCC method for determining the  $K_{IIc}$  of rocks under a hydrostatic pressure is proposed in this work. Compared with the numerous testing methods for obtaining the dynamic rock  $K_{IIc}$ , the SCC method is easily applicable to the dynamic apparatus with the hydrostatic pressure loading system and has an easy preparation with core-based specimen [33]. Also, the Mode II fractures initiate along the notch-tips in the SCC method [33]. A triaxial SHPB system is utilized to exert both a hydrostatic pressure and a dynamic force to the SCC specimen. The dimensions of the dynamic SCC specimen with the hydrostatic pressure are redesigned to reach the dynamic stress equilibrium [40,56]. The dynamic fracture mode and fracture energy of the rock SCC specimen over various hydrostatic pressures are discussed.

In this study, the dynamic experimental apparatus for the hydrostatic pressure loading and the SCC specimen preparation are presented, following by the quantification of the  $K_{IIc}$  of the SCC specimen under the hydrostatic pressure. After that, the fracture pattern and the dynamic rock  $K_{IIc}$  under different hydrostatic pressures are discussed.

## 2. Experimental methodology

### 2.1. Specimen preparation

The SCC specimen is generally a cylinder with two parallel half-through notches from opposite sides, as shown in Fig. 1a.

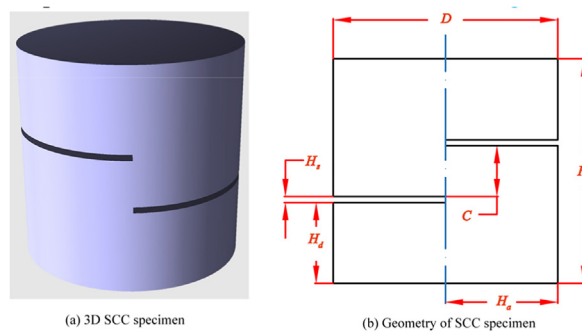


Fig. 1. SCC specimen ( $H_1$  is the length,  $D$  is the diameter,  $H_n = D/2$  is the notch depth,  $H_s$  is the notch thickness,  $C$  is the distance between two notches, which are parallel to the specimen ends.).

The distance from the notch to its nearest core end is the same for both upper and low notches and thus defined as  $H_d$  in Fig. 1b. In addition, the fronts of these two notches are parallel, creating a rectangular rock bridge in the central plane along the core axis (Fig. 1). Under a uniaxial compression, the shear stress is generated in this rectangular bridge of the SCC specimen. Hence, the Mode II fracture is induced in this bridge that can be considered as a fracture plane.

In the previous studies, the SCC specimen with a 38-mm diameter was employed in a static test [33] and the SCC specimen with a 50-mm diameter was used in a dynamic test [53]. Thus, the SCC specimen with a 38-mm diameter is applied in this study because this diameter is compatible with the dynamic loading system.

The existing studies have indicated that the Mode II SIF of the SCC specimen is mainly affected by the geometry factor  $C/D$  and  $C/H_1$  [33,54]. Meanwhile, the dynamic stress equilibrium in the rock sample is a precondition for a valid dynamic rock SHPB test [40]. In such a case, the short rock specimen can easily accomplish the dynamic stress equilibrium. In addition,  $H_1/D = 1$  was successfully employed in the previous dynamic SCC test [54]. Therefore, the height of the SCC specimen is chosen as 38 mm in this study to easily reach the dynamic stress equilibrium in the rock specimen.

Because the small variation of SIF is close to the ideal case for the Mode II shear failure, the small value of  $C/H_1$  is recommended by researchers [33,53,54]. Meanwhile, the studies have indicated that the shear stress depends on the value of  $C/H_1$  [33,46,54]; that is, if  $C/H_1 \geq 0.3$ , the SCC specimen is a tensile failure and invalid for measuring the  $K_{IIc}$  of rocks [33,46]. Thus,  $C/H_1 = 0.2$  is selected in this study to generate shear failure in the fracture plane. In addition, to ensure the symmetry of shear stress around two notch-tips in the SCC specimen, two parallel notches have the identical distance  $H_d$  to the corresponding end surfaces. Also, the fronts of these two notches are parallel to each other (as shown in Fig. 1). As discussed above, the configuration of the dynamic SCC specimen is summarized in Table 1.

Table 1 Configuration of SCC specimen.

Property	Value
Distance between two notches, $C$ (mm)	7.6
Diameter, $D$ (mm)	38.0
Length, $H_1$ (mm)	38.0
Notch depth, $H_n$ (mm)	19.0
Notch thickness, $H_s$ (mm)	1.0
$C/D$	0.2
$C/H_1$	0.2

To manufacture the SCC specimen, rock cylinders with desired diameter and length were machined. Based on the requirements for the dynamic rock specimen in the SHPB test [40], all surfaces of the SCC specimen should be smooth without abrupt irregularities. Henceforth, two half-through notches were made with slow cutting speed to guarantee smooth notch surfaces. The thickness of the notches should be not greater than 1 mm.

In this study, the dynamic SCC specimen is made from fine-grained Fangshan marble (FM). The primary properties of FM are detailed in Table 2 [5,57–59]. The mineral analysis and microscopic observation in the authors' previous studies [5,57] indicated that FM can be considered as a homogeneous and isotropic material, and thus it is suitable for demonstrating the feasibility of the proposed dynamic SCC method with triaxial stresses. The photo of the original SCC specimen made from the FM is shown in Fig. 2.

## 2.2. Dynamic SCC method with hydrostatic pressure

The dynamic Mode II fracture failure experiments with the SCC specimen under the hydrostatic pressure were conducted by using the triaxial dynamic testing system, which was proposed in the authors' earlier study [60]. As shown in Fig. 3, this triaxial dynamic testing system comprises a dynamic loading device and a static triaxial loading apparatus. The dynamic loading system is also a traditional SHPB system (Fig. 3b). This dynamic loading system is undertaken to exert dynamic compressive forces on the SCC specimen. Meanwhile, the static triaxial loading apparatus is utilized to act the hydrostatic pressure on the SCC specimen before dynamic loading. As shown in Fig. 3b, Cylinder 1 produces lateral confinement on the SCC specimen, and Cylinder 2 provides the axial pressure to the SCC specimen. Because the axial pressure and the confinement pressure on the SCC specimen are separately exerted by two cylinders, the dynamic load can be easily applied to the SCC specimen. The hydrostatic pressure on the SCC specimen can be reached when the pressures of these two cylinders are identical. Thus, in this study, both Cylinder 1 and Cylinder 2 are linked to the same oil pressure unit. The SCC specimen is first placed in the dynamic loading system and is then immersed into oil in Cylinder 1. Subsequently, axial forces are acted on the specimen/bar interfaces through the pressure from Cylinder 2 since the rigid frame controls the leftward movement of the incident bar (Fig. 3b) and two tie-rods constrain the relative motion of two cylinders (Fig. 3b). In addition, the lateral pressure on the residual portion of the SCC specimen is acted by the oil pressure  $\sigma_1$  in Cylinder 1. The combination of the pressures in Cylinder 1 ( $\sigma_1$ ) and Cylinder 2 ( $\sigma_2$ ) provides the triaxial stress on the whole SCC specimen [60]. Although the hydraulic pressure in Cylinder 1 provides the axial pressure on the notch surfaces, the axial pressures on the notch surfaces are offset due to the symmetry of the notch surfaces. With the force equilibrium on the specimen/bar interfaces, the SCC specimen can be subjected to a hydrostatic pressure if  $\sigma_1 = \sigma_2 = \sigma_h$  (where  $\sigma_h$  is the hydrostatic pressure, as shown in Fig. 3b).

**Table 2**  
Basic mechanical and physical properties of manufactured FM specimens.

Property	Value
Density (g/cm <sup>3</sup> )	2.85
Young's modulus (GPa)	85
Poisson's ratio	0.3
P-wave velocity (m/s)	5900
Compressive strength (MPa)	155
Tensile strength (MPa)	9.5
$K_{IC}$ (MPa·m <sup>1/2</sup> )	1.5

When the expected level of the hydrostatic pressure is reached on the SCC specimen, the incident stress wave  $\varepsilon_i$  (which is produced by the striker impact) can efficiently propagate rightward because the rightward dynamic stress wave is barely influenced by the small flange [60,61]. Similar to the traditional SHPB test, the reflected stress wave  $\varepsilon_r$  and the transmitted stress wave  $\varepsilon_t$  are generated at the interface between the SCC specimen and the bars. Fig. 4a illustrates the original signals in a typical dynamic SCC test. These three waves were obtained from the strain gauges on bars and recorded by a digital oscilloscope after amplification. In this study, because alternating current (AC) coupling is implemented in an oscilloscope, the dynamic stress strains were merely detected in the Wheatstone bridge circuit. Consequently, one can see from Fig. 4a that the baselines of voltage in the original signals align with zero in the dynamic SCC tests with hydrostatic pressure [4].

According to the rock dynamic testing methods suggested by the ISRM [40], a valid dynamic rock test by using the SHPB system should satisfy the dynamic stress equilibrium on the rock sample before the failure point [56]. Therefore, the pulse shaper (Fig. 3b) was utilized in this study to reach the dynamic stress equilibrium [40]. Here, the dynamic stress equilibrium is expressed as

$$P_1(t) \approx P_2(t) \quad (1)$$

where  $P_1$  is the force on the left loading end of the specimen,  $P_1(t) = AE(\varepsilon_i(t) + \varepsilon_r(t))$ ;  $P_2$  the force on the right loading end of the specimen,  $P_2(t) = AE\varepsilon_t(t)$ ;  $t$  the time; and  $A$  and  $E$  the cross-sectional area and Young's modulus of the bars, respectively. Fig. 4b illustrates these two dynamic forces in a typical SCC test. Before the peak values of these forces are applied to the SCC specimen, the force  $P_1$  is nearly equal to the force  $P_2$ . In addition, it has been verified that the peak value of the dynamic force on the SCC specimen is matched with the specimen shear failure if the dynamic stress equilibrium is reached [54]. Therefore, one can see that the dynamic stress equilibrium is reached before the shear failure of the SCC specimen. The dynamic force equilibrium for each dynamic SCC test has been critically evaluated to ensure that the valid dynamic  $K_{IIc}$  of rock specimens can be obtained under various hydrostatic pressures.

## 2.3. Dynamic fracture energy measurement for the SCC method with hydrostatic pressure

The stress wave energy  $W$  in the dynamic SCC test is expressed as follows [62].

$$W = \int_0^t E(\varepsilon(\tau))^2 A v_p d\tau \quad (2)$$

where  $\tau$  is the time integral variable;  $v_p$  the one-dimensional P wave velocity of the bars; and  $\varepsilon$  the time-resolved strain (i.e.  $\varepsilon_i$ ,  $\varepsilon_r$ , and  $\varepsilon_t$ ). Since the wave impedance of steel bars is massively different from that of the hydraulic oil, the stress waves in bars are mostly transmitted into the SCC specimen and the authors assume that most of the energy is consumed by the specimen during the dynamic SCC test with the hydrostatic pressure [63]. As a result, the energy consumed during the dynamic SCC test can be quantified; that is, the total energy dissipation in the SCC specimen  $\Delta W$  equals the energy difference between the incident energy ( $W_i$ ) and the sum of the reflected energy ( $W_r$ ) and the transmitted energy ( $W_t$ ) [64].

$$\Delta W = W_i - (W_r + W_t) \quad (3)$$

The energy dissipation in the SCC specimen is comprised of two components: the creation of new crack surfaces ( $W_c$ ) and the kinetic energy  $R$  in the two parts of the failed SCC specimen. Namely,  $R = mv^2/2$ , where  $m$  is the fragment mass and  $v$  is the

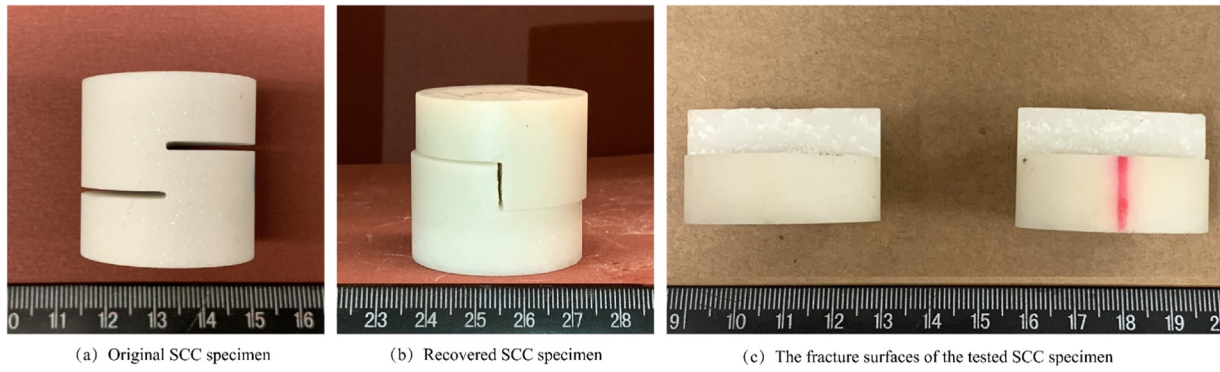


Fig. 2. A typical SCC specimen before and after the test.

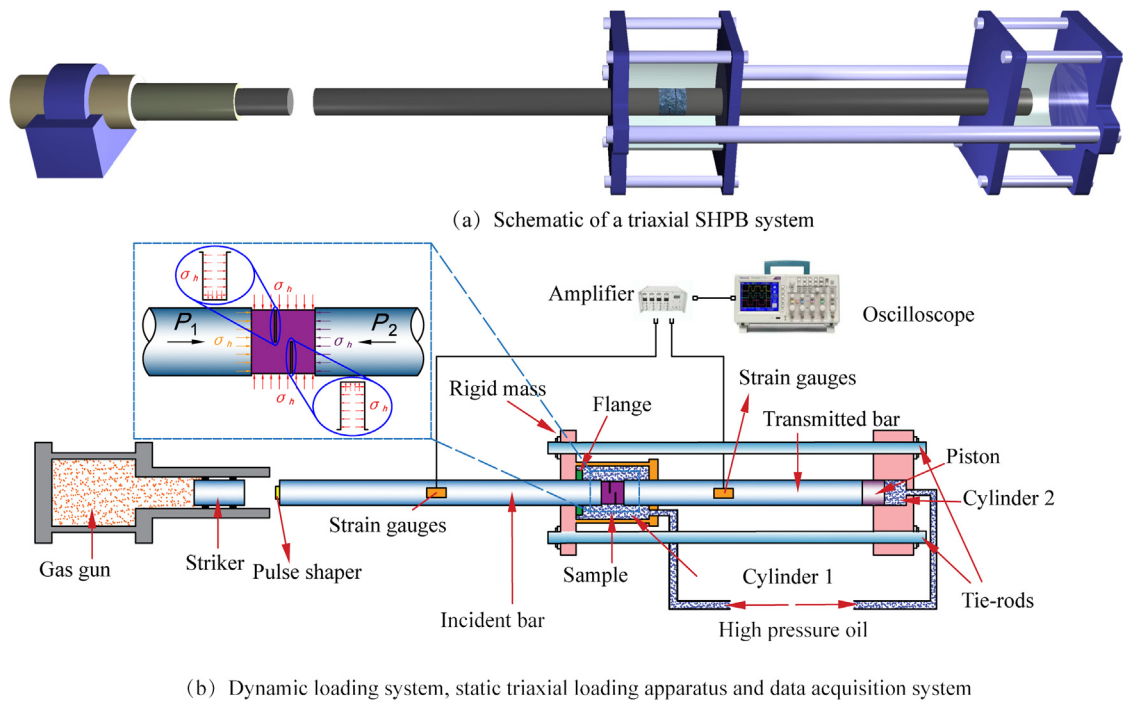


Fig. 3. Triaxial SHPB testing system.

fragment velocity, which was obtained by using the speed of the bar end because the specimen ends are not detached to the bar ends during the dynamic loading period. The velocities at the incident bar end ( $v_1$ ) and the transmitted bar end ( $v_2$ ) are

$$v_1(t) = v_p(\varepsilon_i(t) - \varepsilon_r(t)), \quad v_2(t) = v_p \varepsilon_t(t) \quad (4)$$

In such a case, the energy for the new crack surface created by the dynamic shear loading can be calculated as

$$W_G = \Delta W - R \\ = \Delta W - \left( \frac{1}{2} \int_0^t m_1 (v_1(\tau))^2 d\tau + \frac{1}{2} \int_0^t m_2 (v_2(\tau))^2 d\tau \right) \quad (5)$$

where  $m_1$  and  $m_2$  are the masses of two fragments, respectively. Thus, the dynamic shear fracture energy of the SCC specimen under hydrostatic pressure can be estimated by the above equation.

### 3. Determination of the $K_{IIC}$ in SCC specimens under hydrostatic pressure

#### 3.1. Deduction of the Mode II fracture toughness

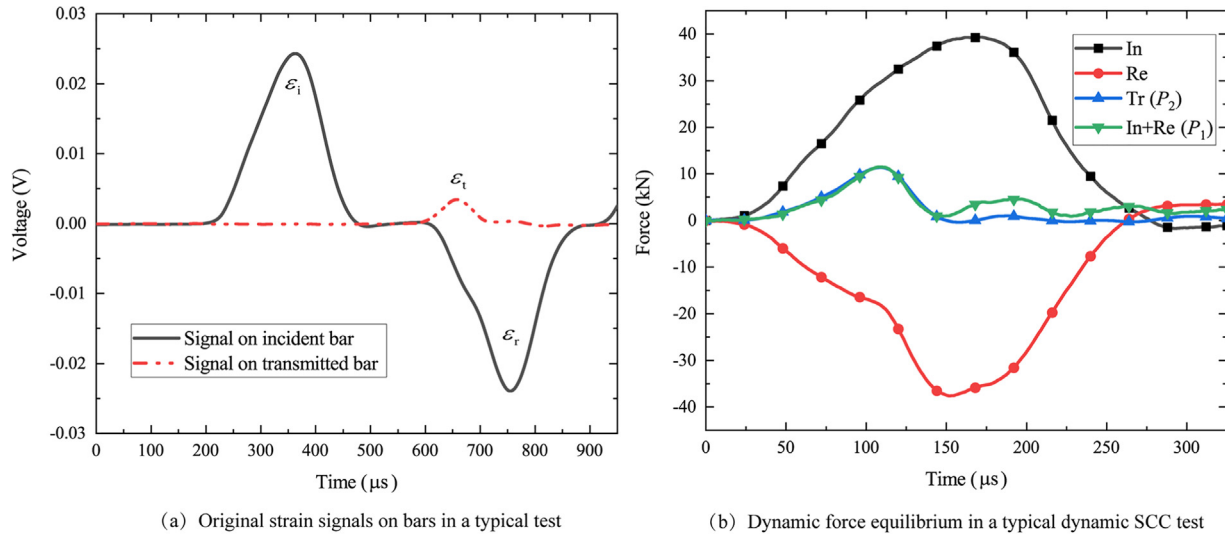
It has been proven that the  $K_{IIC}$  of the SCC specimen under both static and dynamic conditions can be calculated by the peak load on the loading end of the SCC specimen [33,53,54]. Consequently, the formula for determining  $K_{IIC}$  ( $\text{MPa}\cdot\text{m}^{1/2}$ ) of SCC specimens can be generally written as

$$K_{IIC} = \alpha \sigma_{\max} \quad (6)$$

where  $\sigma_{\max}$  is the peak compressive stress on the SCC specimen (MPa); and  $\alpha$  depends on the geometry of the SCC specimen. In addition, according to the ISRM suggested method to obtain the rock  $K_{IIC}$  via the PTS specimen under confinement pressure [47], the  $K_{IIC}$  of rocks through the SCC specimen under hydrostatic pressure can be similarly estimated as follows.

$$K_{IIC} = \alpha \sigma_{\max} + \beta \sigma_h \quad (7)$$





**Fig. 4.** Original strain signals on bars and dynamic force equilibrium in a typical dynamic SCC test with the hydrostatic pressure of 10 MPa (“In”, “Re”, and “Tr” denote “incident”, “reflected”, and “transmitted”, respectively).

where  $\beta$  is the geometry parameter determined by the numerical simulation [33,53,54] in a combination of the  $J$ -integral method [13,33,65,66] and the displacement extrapolation technique [47,67]. The  $J$ -integral method is a simple and effective approach with good accuracy and has been prevalently used to estimate the SIF around the crack-tip. Therefore, the finite element analysis (FEA) in a combination of the  $J$ -integral method is applied to determine the values of two geometry parameters ( $\alpha$  and  $\beta$ ) and to further calculate the  $K_{IIC}$  of the rock SCC specimen under the hydrostatic pressure. In such a case, it is essential to generate a valid finite element model of the SCC specimen and then determine these two parameters with the verified finite element model. The numerical SCC model was constructed to analyze the stress field in the SCC specimen and verify the finite element model for the SIF calculation at the crack tip.

### 3.2. Stress distribution in SCC specimens

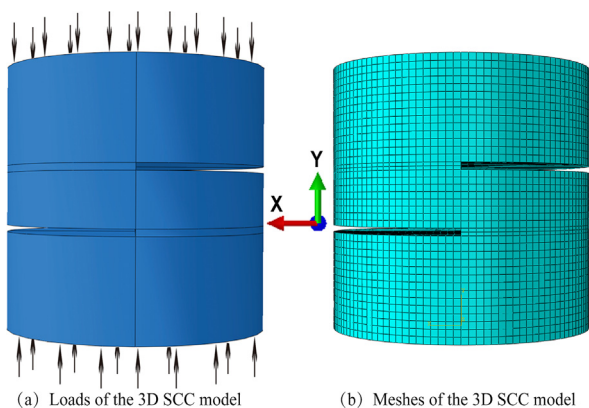
A three-dimensional (3D) finite element model, as shown in Fig. 5, was established via a commercial program ABAQUS to investigate the stress distribution inside the SCC specimen. The SCC specimen geometry for dynamic experiments is used in the 3D model, i.e.  $C/H_t = 0.2$  and  $H_t/D = 1$ . This numerical model is comprised of 423152 nodes and 402960 eight-node quadratic plane-

strain hexahedral elements with linear geometric order. In this model, the SIF was estimated by using the  $J$ -integral method, in which the energy release associated with crack growth was characterized around the crack tip. The energy release rate is given by  $\bar{J} = \int_A \lambda(s) \mathbf{n} \cdot \mathbf{H} \cdot \mathbf{q} dA$ , where  $\lambda(s)$  is a virtual crack advance,  $dA$  is a surface element along a vanishing small tubular surface enclosing the crack tip or crack line,  $\mathbf{H}$  is an equation in terms of the elastic strain energy density and the stress vector,  $\mathbf{n}$  is the outward normal to  $dA$ , and  $\mathbf{q}$  is the local direction of virtual crack extension [67,68]. The energy release can be related to the SIF when the material response is linear. Thus, in this numerical model, only elastic modulus and Poisson's ratio were set as the corresponding values in Table 2. As shown in Fig. 5a, the compressive loading was acted on both the upper and bottom ends of the SCC specimen.

Due to the symmetrical configuration of SCC samples, the XY central plane, which is normal to the failure surface, can represent the shear stress distribution along the failure surface. Thus, based on the shear stress field of the central plane in the 3D specimen in Fig. 6, the peak shear stress is located at the notch-tip, indicating that the shear failure occurs at the notch tips. Also, the hydrostatic pressure applied on the SCC specimen has barely influence on the shear stress. Furthermore, the shear stress on the upper notch-tip of a typical SCC model is given in Fig. 7. One can see that the maximum shear stress is reached at the XY central plane, which can be considered as the critical plane for shear fracture. Consequently, the stress field on the critical plane can characterize the stress state when the shear failure commences in the SCC specimen. The SIF at notch-tips (in the center of which shear fractures occur) is determined based on the stress distribution in the critical plane. In such a case, a 2D model on the critical plane was built to efficiently examine the shear stress distribution and further to determine the SIF at notch-tips where shear fractures initiate.

### 3.3. Numerical model for the SCC method

According to the geometry of the critical plane of the SCC specimen, the 2D SCC model was created by using 5776 nodes and 5548 eight-node quadratic elements. The elastic modulus and Poisson's ratio were set as the corresponding values in Table 2. The axial compressive stress was acted on both the upper and bottom ends of the SCC model. Based on the shear stress field on the central plane of the SCC specimen under different hydrostatic



**Fig. 5.** Configuration of the 3D SCC model.

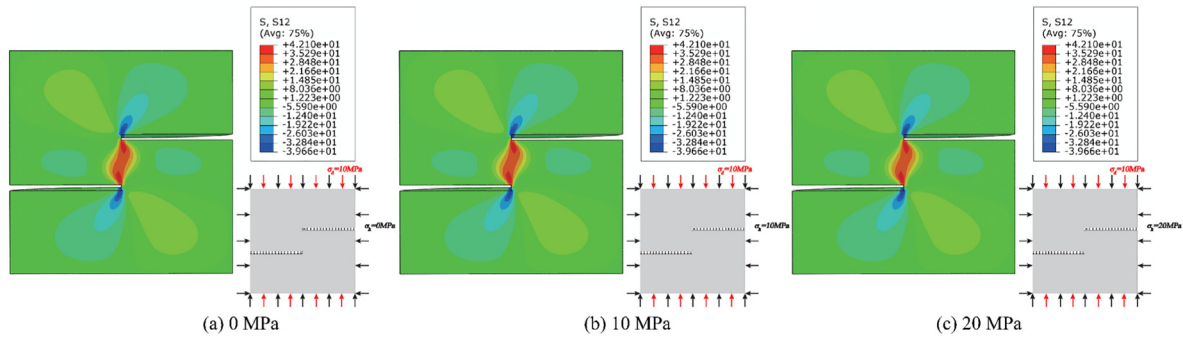


Fig. 6. Shear stress distribution (MPa) on the central plane of the 3D SCC model over the axial compressive stress of 10 MPa with different hydrostatic pressures.

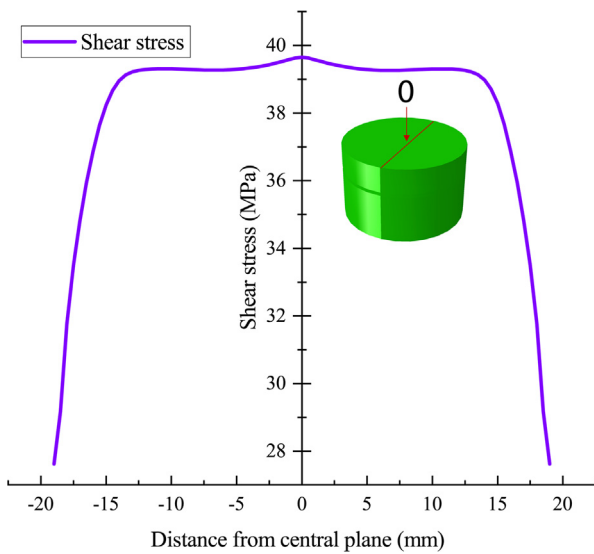


Fig. 7. Shear stress on the upper notch of the 3D SCC sample under the compressive stress of 10 MPa without hydrostatic pressure (the line in the inset illustrates the nodes of the upper notch and 0 is the central plane).

pressures in Fig. 8, the hydrostatic pressure has barely influence on the shear stress in the SCC specimen. A shear zone (red zone in Fig. 8) is formed between these two notch-tips and the maximum shear stresses appear at both two notch-tips. The shear stresses between these two notch-tips are almost constant and a region with a high shear stress is created along the potential shear failure path between these two notch-tips. This shear stress distribution may result in the shear failure that occurs between two notch-tips. Therefore, the SCC geometry in this study is valid for the shear failure under various hydrostatic pressures. In addition, the

distributions of the principal stress for different hydrostatic pressures are shown in Fig. 9. For all hydrostatic pressure conditions, the major principal stress (tensile) is distributed along the bridge of these two notch-tips. However, the principal stress in the residual area is uniformly close to zero. The result of the 2D simulation is consistent with that reported by other researchers [33,46,53]. As a result, the 2D numerical analysis here is valid to determine the stress state and the SIF around the notch-tip in the SCC specimen.

The fracture toughness is determined by the critical value of the SIF at the notch-tip. Based on the static and dynamic SCC experiments and the numerical simulation for the SCC specimen [33,53,54], the shear fracture is normal to the notch plane. This differs from the PTS specimen and the shear box test, in which the shear fractures nearly grow along the notch plan. Hence, a secondary crack in the direction of the notch is unnecessary to obtain the SIFs at the notch-tips in simulation analysis if the crack plane is along the notch plane, such as the PTS test and the shear box test. However, based on fracture mechanics theory, the secondary crack is a precondition for accurately determining the SIF in the fracture process [69–71], and the  $K_{IIC}$  can be further determined precisely when a crack tip exists along the shear fracture plane [72]. The methodology for using a secondary crack was initially used in the wing crack model [70], in which secondary cracks were originated from the wing crack-tips and the SIF is derived from the limit if the length of the secondary cracks approach zero. This model has been widely used in fracture mechanics analysis because it estimated primarily the ultimate strength measured in the experiments and the direction of the general failure plane [70]. Recently, based on this method, Xu et al. [33] introduced secondary cracks at the notch-tips to obtain the SIF of the SCC specimen under static uniaxial compression. Thus, secondary cracks at notch-tips along the shear fracture plane were used in this study to estimate the SIF of the dynamic SCC sample. As shown in Fig. 10a, secondary cracks ( $l_c$ ) are introduced in the 2D SCC finite element model validated above. These two secondary shear cracks are perpendicular to

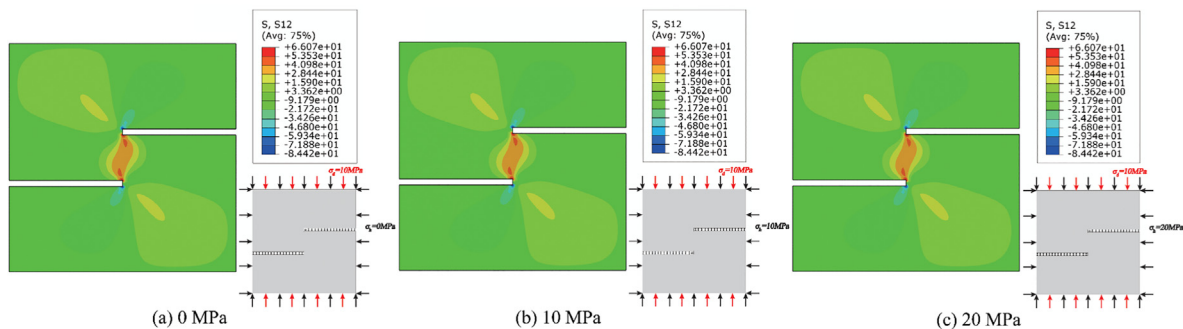


Fig. 8. Shear stress (MPa) distribution of the 2D SCC model under the axial compressive stress of 10 MPa with different hydrostatic pressures.

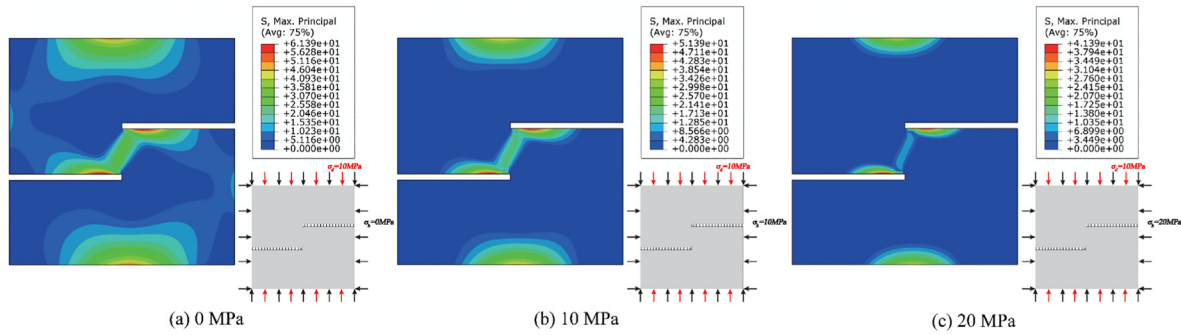


Fig. 9. Principal stress (MPa) distribution of the 2D SCC model under the axial compressive stress of 10 MPa with different hydrostatic pressures.

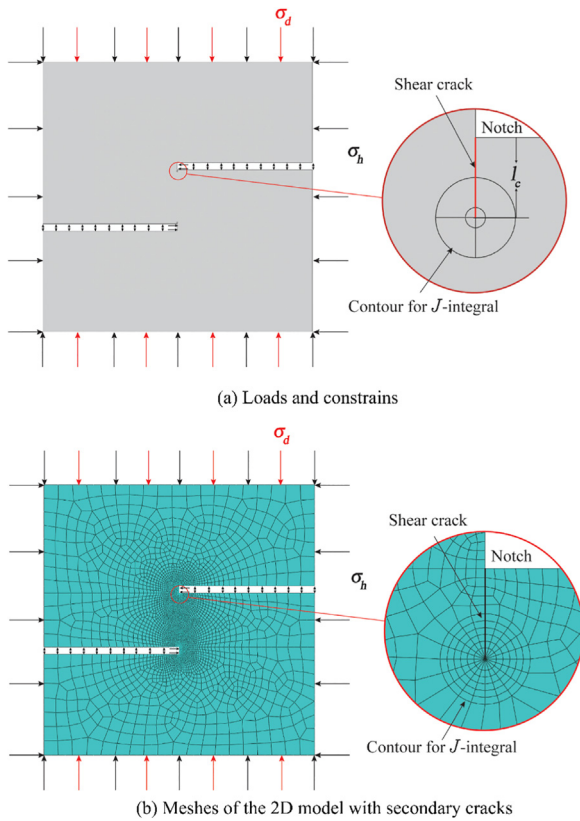


Fig. 10. Loads and meshes of the 2D model under the axial compressive stress ( $\sigma_a$ ) and the hydrostatic pressure ( $\sigma_h$ ).

the notch-tips. This model was constructed by ABAQUS and the singular quadrilateral eight-node elements were employed to mimic the singularity at secondary crack-tips (Fig. 10b). This model with the secondary cracks includes 5438 elements as illustrated in Fig. 10b. The SCC model was under six hydrostatic pressures (i.e. 0, 5, 10, 15, 20, and 25 MPa) and seven axial compressive loads (i.e. 5, 10, 15, 20, 25, 30, and 35 MPa). The SIFs at the shear crack-tips in the SCC specimen were determined by using the  $J$ -integral method [13,65,66], which is embedded in the finite element program ABAQUS and has been extensively utilized by many researchers to determine the SIFs due to its reliability [10,33,73,74].

### 3.4. Determination of Mode II SIF of the SCC specimen under hydrostatic pressure

Based on the energy analysis, the Mode II SIF at the shear crack-tip ( $K_{II}^*$ ) of the SCC specimen and short beam specimen under static

loading condition without hydrostatic pressure can be generally expressed as [33,46]

$$K_{II}^* = Y(C/H_I) \sqrt{\pi H_a} \cdot (P/DC) \tag{8}$$

where  $Y(C/H_I)$  is a geometrical function;  $P$  the compressive force; and  $P/(DC)$  can be considered as a nominal shear stress acting on the shear plane. Hence, the geometrical function  $Y$  can be obtained by

$$Y(C/H_I) = (K_{II}^* / \sqrt{\pi H_a}) / (P/DC) \tag{9}$$

Based on Eq. (9), the  $K_{II}^*$  of SCC specimens under the specific geometry is obtained via the displacement extrapolation technique in the foregoing finite element model with secondary cracks [75]. In the displacement extrapolation technique, the Mode II SIF around the shear crack-tip ( $K_{II}$ ) is calculated by using the FEA with the  $J$ -integral method, and then the  $K_{II}^*$  can be obtained with the extrapolation of the  $K_{II}$  around the crack-tip.

The  $K_{II}$  is illustrated in Fig. 11 in terms of the length of the shear crack  $l_c$  under the axial compressive load of 5 MPa. It demonstrates that  $K_{II}$  enhances almost linearly with  $l_c$  for all hydrostatic pressure conditions. The hydrostatic pressures have no influence on the values of  $K_{II}$ . In addition, the  $K_{II}$  is given as a function of  $l_c$  under the hydrostatic pressure of 10 MPa in Fig. 12. It can be seen that the  $K_{II}$  increases almost linearly with  $l_c$  for all axial compressive loading conditions. Thus, the  $K_{II}^*$  of the SCC specimen is determined by extrapolating  $K_{II}$  to  $l_c = 0$ , and the  $K_{II}^*$  of SCC specimens under differ-

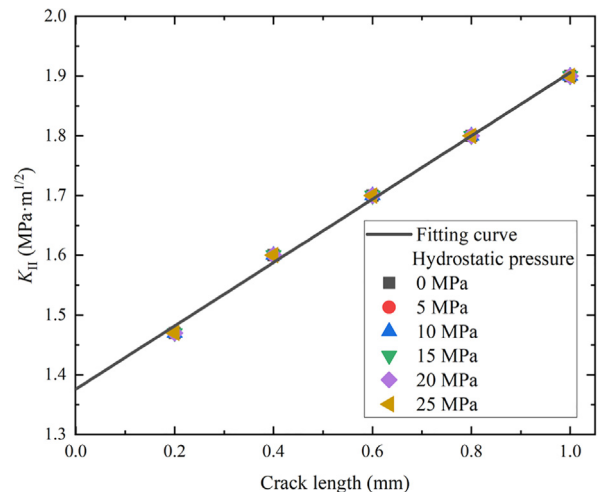


Fig. 11. SIF around crack tip in the SCC specimen under different hydrostatic pressures using the extrapolation method (The axial compressive load is 5 MPa).

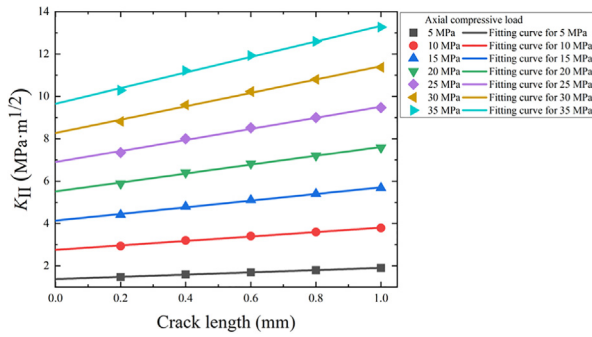


Fig. 12. SIF around crack tip in the SCC specimen under different axial compressive loads using the extrapolation method (The hydrostatic pressure is 10 MPa).

ent compressive loads and hydrostatic pressures can be determined as the intercept for the fitting curves in both Figs. 11 and 12.

According to the method above, the values of  $K_{II}^*$  for the SCC specimen under various hydrostatic pressures and different compressive loads can be obtained (Fig. 13). For each hydrostatic pressure, the  $K_{II}^*$  linearly increases as the axial compressive load and the slopes of the fitting curves for each hydrostatic pressure are identical. Meanwhile, the  $K_{II}^*$  for the SCC specimen without hydrostatic pressure can be written as

$$K_{II}^* = \left\{ \left[ Y(C/H_I) \sqrt{\pi H_a} \right] / DC \right\} \cdot \pi(D/2)^2 \sigma = \alpha \sigma \quad (10)$$

where  $\sigma$  is the axial loading (MPa) on the ends of the SCC specimen. Thus, the value of  $\alpha$  is the slope of the curve without the hydrostatic pressure. Moreover, because the slopes of the curves for each hydrostatic pressure in Fig. 13 are the same, the  $K_{II}^*$  of the SCC specimen with the hydrostatic pressure can be expressed as

$$K_{II}^* = \alpha \sigma + \beta \sigma_h \quad (11)$$

Further, to determine the value of  $\beta$ , the values of  $K_{II}^*$  are replotted in terms of the hydrostatic pressure in Fig. 14. It can be seen that the  $K_{II}^*$  keeps constant as the increase of the hydrostatic pressure and the slopes of the curves for a certain axial compressive load are zero. Hence, the value of  $\beta$  for each hydrostatic pressure is the slope of the arbitrary fitting curve in Fig. 14. Consequently, the values of  $\alpha$  and  $\beta$  are determined as  $\alpha = 0.27 \text{ m}^{1/2}$  and  $\beta = 0 \text{ m}^{1/2}$  for the SCC specimen under hydrostatic pressures in this

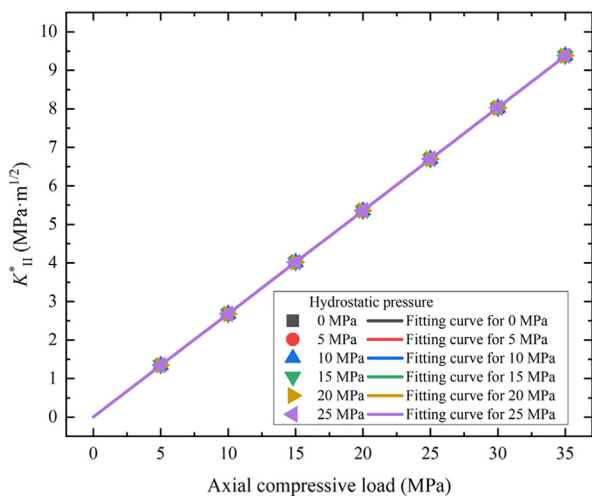


Fig. 13.  $K_{II}^*$  in terms of various axial compressive loads.

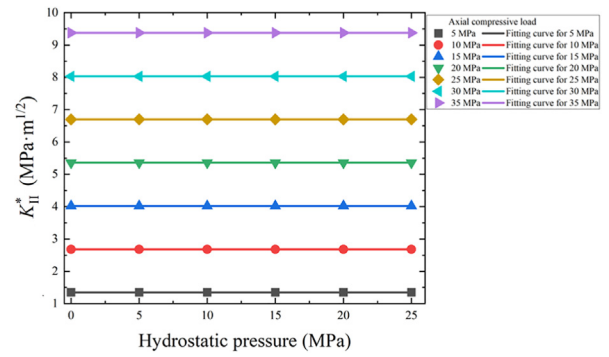


Fig. 14.  $K_{II}^*$  in terms of various hydrostatic pressures.

study. In addition, the formula to determine the  $K_{IIc}$  of rocks through the SCC specimen under hydrostatic pressures can be rewritten as

$$K_{IIc} = \alpha \sigma_{\max} + \beta \sigma_h = 0.27 \sigma_{\max} + 0 \times \sigma_h = 0.27 \sigma_{\max} \quad (12)$$

It implies that the rock  $K_{IIc}$  is barely affected by the hydrostatic pressure. Based on Eq. (12), the  $K_{IIc}$  of rocks under various hydrostatic pressures can be calculated when the geometry of the SCC specimen proposed in this study is used in the dynamic tests.

### 3.5. Determination of loading rate for the SCC specimen under hydrostatic pressure

With the dynamic stress equilibrium for SCC samples, the time evolution of the dynamic  $K_{II}^*$  is deduced from Eq. (11).

$$K_{II}^*(t) = \alpha \sigma(t) + \beta \sigma_h \quad (13)$$

where  $\sigma(t)$  is the dynamic compressive stress (MPa). Since the hydrostatic pressure is consistent during the dynamic shear process, the values of  $\alpha$  and  $\beta$  in Eq. (12) are also applicable to Eq. (13). Based on the definition of the dynamic loading rate suggested by the ISRM [40], the slope (i.e. the dashed-dot line in Fig. 15) of the almost linear rising section in the SIF-time curve is the loading rate for the dynamic SCC test.

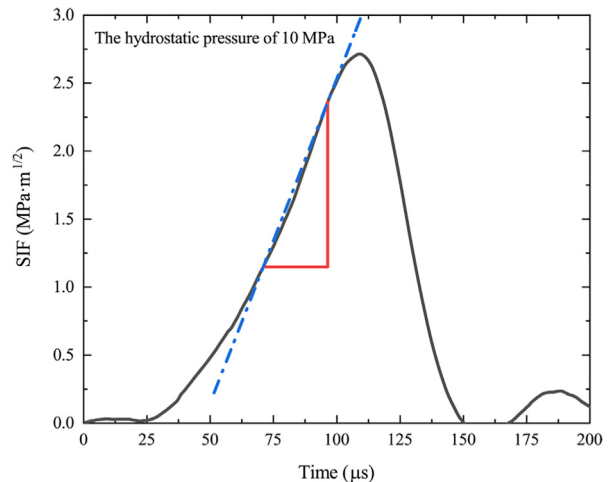


Fig. 15. Dynamic loading rate determination in a typical dynamic SCC test. The loading rate is 48 GPa·m<sup>1/2</sup>/s and  $K_{IIc} = 2.71 \text{ MPa}\cdot\text{m}^{1/2}$  in this typical dynamic SCC test.



### 4. Results and discussions

The SCC specimen failed after a typical dynamic test is shown in Fig. 2b and c. The specimen was sheared separately along the potential fracture plane between these two notch-tips. The failure pattern is consistent with the stress distribution observed in the above numerical analysis and the failure mode reported by other researchers [33,53]. Based on both experimental observation and numerical analysis, the failure mode is Mode II and the dynamic SCC specimen is valid to measure the  $K_{IIc}$  of rocks.

The dynamic  $K_{IIc}$  of FM over different hydrostatic pressures (i.e. 0, 5, 10, 15, and 20 MPa) is shown in Fig. 16. The largest value of dynamic  $K_{IIc}$  (4.76 MPa·m<sup>1/2</sup>) was obtained when the hydrostatic pressure is 20 MPa. It indicates that the dynamic  $K_{IIc}$  over a given hydrostatic pressure rises as the loading rate. This reveals that the dynamic  $K_{IIc}$  of FM has a strong rate dependence, which has been widely found in other mechanical behaviors of rocks (e.g. compressive/tensile strength, Mode I fracture toughness) in literature [76–80]. The  $K_{IIc}$  of FM measured through the PTS specimen in the authors' early study [5] is given in Fig. 16. The dynamic  $K_{IIc}$  measured by both the SCC specimen and the PTS specimen has a consistent trend in terms of the loading rate. Namely, the dynamic  $K_{IIc}$  almost linearly increases with the loading rate for both the SCC specimen and the PTS specimen, and the slope of the linear fitting line based on the dynamic  $K_{IIc}$  data points from the SCC specimen is nearly the same as that of the fitting line based on the dynamic  $K_{IIc}$  data points from the PTS specimen. The dynamic  $K_{IIc}$  of FM under a specific loading rate without hydrostatic pressure in this study has a slight discrepancy with that of FM under the corresponding loading rate by using the PTS specimen. For example, at the loading rate of around 30 GPa·m<sup>1/2</sup>/s, the dynamic  $K_{IIc}$  from the SCC specimen is 0.19 MPa·m<sup>1/2</sup> higher than that from the PTS specimen. This little difference between the values of  $K_{IIc}$  derived from these two testing methods is acceptable and may be caused by the diversity of the FM. In addition, at a given loading rate the dynamic rock  $K_{IIc}$  is barely affected by the hydrostatic pressure. This is probably attributed to the constant shear stress field around the crack-tips under various hydrostatic pressures. Moreover, Fig. 16 shows the dynamic  $K_{Ic}$  of FM in the references [58,59]. One can see that the dynamic  $K_{IIc}$  under various hydrostatic pressures are bigger than the dynamic  $K_{Ic}$  over a similar loading rate. This phenomenon was discovered in other types of rocks as well [47].

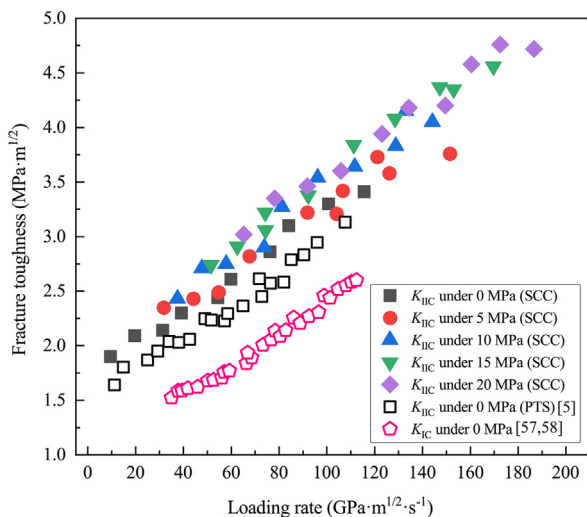


Fig. 16. Fracture toughnesses of FM with different hydrostatic pressures.

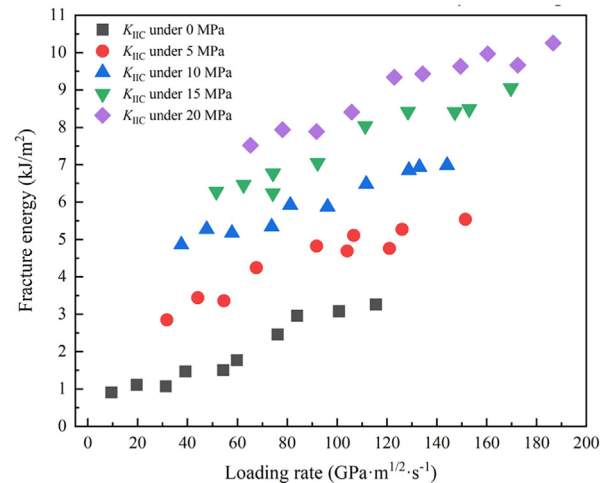


Fig. 17. Dynamic fracture energy of the FM SCC specimen with various hydrostatic pressures.

Fig. 17 gives the dynamic fracture energy of FM over various hydrostatic pressures. At a given hydrostatic pressure, the fracture energy of FM demonstrates a loading rate dependence. This phenomenon is consistent with the observation in the authors' early study by using the dynamic PTS method [5]. Another finding is that the fracture energy of FM under a certain loading rate increases with the hydrostatic pressure. This reveals that the hydrostatic pressure has an apparent effect on the fracture energy in the dynamic SCC tests; that is, during the dynamic shear failure process in the SCC test under a certain loading rate, the more the hydrostatic pressure, the more energy consumed by the creation of the new shear fracture surface.

### 5. Conclusions

- (1) The dynamic rock  $K_{IIc}$  over different hydrostatic pressures was studied via a dynamic SCC method. The dynamic SCC specimen was designed following the requirement of the valid dynamic rock test. The hydrostatic pressure was applied to the SCC specimen by two hydraulic cylinders in the dynamic loading system. Pulse shaper was utilized to facilitate the dynamic stress equilibrium in SCC specimens.
- (2) The FM was employed in dynamic SCC experiments with hydrostatic pressures. The rock sample was sheared separately along the potential fracture plane between these two notch-tips. The SIF of the dynamic SCC sample was determined by using the FEA with the secondary cracks. The equation for calculating the  $K_{IIc}$  of the SCC sample was obtained from the FEA, and the dynamic  $K_{IIc}$  of FM can be obtained from the peak dynamic stress and the hydrostatic pressure.
- (3) The results of SCC tests indicate that the dynamic  $K_{IIc}$  of FM under a certain hydrostatic pressure increases as the loading rate. In addition, at a given loading rate the dynamic rock  $K_{IIc}$  is barely affected by the hydrostatic pressure. This is probably attributed to the constant shear stress field around the crack-tips under various hydrostatic pressures.
- (4) The  $K_{IIc}$  of FM under different hydrostatic pressures is consistently higher than the  $K_{Ic}$  of FM under the corresponding loading rate. Furthermore, at a given hydrostatic pressure, the fracture energy of FM demonstrates a loading rate dependence.

- (5) Another important finding is that the fracture energy of FM under a certain loading rate increases as hydrostatic pressures. This reveals that the hydrostatic pressure has an apparent effect on the fracture energy of rocks in dynamic SCC tests.

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