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Abstract. This article has been withdrawn due to a mistake which is explained in version 2.

We consider a 3-simplex $\sigma$ in an affine space $E$. Let $x_1, x_2, x_3, x_4$ be its vertices; the edges are oriented by the ordering of the vertices. Let $F_i$ be the 2-face opposite to $x_i$. We are looking at germs of codimension-one foliation along $\sigma$ (or along a subcomplex of $\sigma$) which are transversal to $\sigma$ and to all its faces of positive dimension.

If such a foliation $\mathcal{H}$ is given along the three 2-faces $F_2, F_3, F_4$ through $x_1$ and if $\mathcal{H}$ does not trace spiralling leaves on $F_2 \cup F_3 \cup F_4$, then $\mathcal{H}$ extends to $\sigma$ transversally to $F_1$. If $\mathcal{H}$ is only given along $F_2 \cup F_4$ (resp. $F_3 \cup F_4$), then $\mathcal{H}$ extends to $F_3$ (resp. $F_2$) with no spiralling on $F_2 \cup F_3 \cup F_4$, and hence to $\sigma$.

But, on contrary of what is claimed on version 1 of this paper, it is in general not true when $\mathcal{H}$ is given along $F_2 \cup F_3$. It is only true when an extra condition is fulfilled: *The separatrices of $x_2$ in $F_3$ and of $x_3$ in $F_2$ cross $F_2 \cap F_3 = [x_1, x_4]$ respectively at points $y_2$ and $y_3$ which lie in the order $y_2 < y_3$.*

The first place where this extension argument is misused is corollary 4.5. Moreover the statement of this corollary is wrong. Let us explain why.

Let $\sigma^{pl} \subset E$ be a so-called *pleated* 3-simplex associated to $\sigma$ and $\mathcal{H}$ be a germ of codimension-one foliation transversal to its simplices. We recall that $\sigma^{pl}$ and $\sigma$ have the same boundary and we assume that $\mathcal{H}$ traces spiralling leaves on $\partial\sigma$, making the pleating necessary according to the Reeb stability theorem. Let $x * \sigma^{pl}$ be the (abstract) cone on $\sigma^{pl}$. If $\dim E$ is large enough, it embeds into $E$. Certainly $\mathcal{H}$ does not extend to $x * \sigma^{pl}$, contradicting the statement of corollary 4.5. Indeed, if it does, then we get a foliation of $x * \partial\sigma^{pl} = x * \partial\sigma$ transversal to all faces. Proposition 4.4 states that, if all 3-faces through $x$ in the 4-simplex $x * \sigma$ are foliated, then the foliation extends to the face opposite to $x$, which is $\sigma$ itself. But this is impossible due to the spiralling leaves on $\partial\sigma$.

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