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On the modelling of internal soil erosion

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ABSTRACT: Suffusion is the process whereby fine particles are moved locally and transported through a coarse layer. This is a bulk erosion, which corresponds to a source term in the mass balance equations. This bulk erosion is counter intuitive, as erosion is basically an interfacial process. We suggest a framework within which suffusion in a clayey sand can be viewed as a clay/water interface erosion at the microscopic scale. The coefficient of surface erosion of the clay matrix can therefore be used to quantify the suffusion rate. The proposed erosion law depends on the clay volume fraction, and accounts for the influence of the hydraulic gradient. Comparisons with published experimental data show that this approach gives good results over the full range of hydraulic gradient explored.

1 Introduction

The internal erosion of soil resulting from seepage flow is the main cause of serious failure at hydraulic works (dykes, levees, dams). Internal erosion is also liable to occur in natural soils. Overviews of current research on internal erosion in the field of soil mechanics and earthdam industry are given in Fell *et al.* (2003), Fell and Fry (2007), or Wan and Fell (2004). In the oil petroleum industry, the sand production processes occurring in wellbores are similar to internal erosion. An overview of the research on this topic is available in Papamichos *et al.* (2006).

Suffusion is an internal erosion process by which finer soil particles are detached by seepage flow from the solid matrix, and transported through the pore domain. The term suffusion (or suffosion, or suffosion) was introduced in 1898 by Pavlov. The internal instability that results from suffusion gives rise to a wide range of hazards including piping and sinkholes (Fell and Fry, 2007; Gutiérrez *et al.*, 2008).

Several models for internal erosion have been proposed in the framework of continuum mixture theory (Papamichos *et al.*, 2001; Vardoulakis *et al.*, 1996; Wan and Wang, 2002). The process of erosion is assumed in these studies to involve a smooth transition from solid-like to fluid-like behaviour. This transition is described with a three-phase model (comprising solid, fluid, and fluidized phases). These three phases interact while being constrained by balance equations. A sink term in the mass balance equations describes the detachment of sand particles via an erosion law. It can also be assumed that erosion follows a path of least resistance, given by the porosity gradient (Papamichos and Vardoulakis, 2005).

These models have been mainly applied to granular materials, and not to sand/clay mixture. The erosion constitutive law has been introduced as usual by phenomenological analysis, at the macroscale. However, it is not clear whether the driving force associated to the volume flux of eroded material is the pressure gradient, or the seepage velocity. In addition, the coefficient of volume erosion has little mechanical significance. Finally, as erosion is the removal of material caused by the eroding power of the flow, it is basically an interfacial process, and not a volume process.

The philosophy underlying the present approach differs from previous theories on suffusion in that our description deals with sharp fluid/soil interfaces and interfacial erosion, by means of a multi-scale approach. Two scales are considered: a micro-scale defined at the scale of the pore constrictions, and a macro-scale corresponding to the scale of the Representative Elementary Volume (REV), corresponding to a sample in the laboratory. Apart from Bouddour *et al.* (1996), few attempts have been made to relate macroscopic and microscopic quantities in a multi-scale description of the suffusion process. The present study was intended to provide a step towards the modelling of suffusion in a clayey sand.

2 The suffusion model

2.1 Seepage flow with volume erosion

Internal erosion is similar to sand production. The modelling of the sand production process is well established (Papamichos *et al.*, 2001; Vardoulakis *et al.*, 1996; Wan and Wang, 2002). Let us consider a saturated soil Ω in one-dimensionnal evolution. By the dilute suspension flow assumption, the concentration does not affect the behaviour and becomes a secondary unknown. In one-dimensional evolution, the set of equations can be written as follows:

Balance equations

$$\frac{\partial \phi}{\partial t} = \frac{r}{\rho_{Clay}} \text{ (solid mass balance equation) ,} \quad (1)$$

$$\frac{\partial q}{\partial x} = 0 \text{ (pore-fluid mass balance equation) ,} \quad (2)$$

$$-\frac{\partial p}{\partial x} = F \text{ (pore-fluid momentum balance equation) ,} \quad (3)$$

Constitutive equations

$$F = \frac{\eta_w}{\lambda} q \text{ (Darcy constitutive law) .} \quad (4)$$

In these equations, ϕ is the porosity, r the volume flux of eroded material, ρ_{Clay} the density of eroded material, q the seepage velocity, p the pressure, F the mechanical interaction between the pore-fluid and the matrix, η_w the water viscosity and λ the geometric permeability. This system needs to be supplemented by a constitutive equation for r . This equation is now introduced by means of an homogenization argument.

2.2 Pore-scale flow with surface erosion of the clay matrix

Let us consider a representative elementary volume $\hat{\Omega}$ (REV) of a soil partitioned into three disjoint regions $\hat{\Omega} = \hat{\Omega}_{Pore} \cup \hat{\Omega}_{Clay} \cup \hat{\Omega}_{Sand}^s$ (Figure 1): a connected meso-pore domain $\hat{\Omega}_{Pore}$ (volume fraction $\phi = \hat{\Omega}_{Pore} / \hat{\Omega}$), a clay matrix domain $\hat{\Omega}_{Clay}$ (volume fraction $\phi_V = \hat{\Omega}_{Clay} / \hat{\Omega}$) and a sand grain domain $\hat{\Omega}_{Sand}^s$ (volume fraction $1 - \phi_{Sand} = \hat{\Omega}_{Sand}^s / \hat{\Omega}$). The meso-pore domain contains an incompressible fluid, namely a mixture of water and clay particles, assumed to be a dilute suspension of constant density ρ_w and viscosity η_w . The clay matrix is a continuum consisting in two phases: clays particles $\hat{\Omega}_{Clay}^s$ of constant density ρ_s (relative volume fraction $1 - \phi_{Clay} = \hat{\Omega}_{Clay}^s / \hat{\Omega}_{Clay}$), and water of constant density ρ_w . The sand grains are homogeneous and impervious of constant density ρ_s . From the Figure 1, we infer the important relation:

$$\phi_V = \phi_{Sand} - \phi . \quad (5)$$

The porosity may be divided into macro-porosity ϕ corresponding to the meso-pore domain, and micro-porosity ϕ_{Clay} in the clay matrix. This type of heterogeneous soil is similar to the classical double-porosity model for modeling flow in fractured porous media, developed in the early 1960s. However, the fluid exchange between the clay matrix and the meso-pore domain is not considered here. We assume that the soil is a clayey sand (Revil *et al.*, 1999; Revil *et al.*, 2002): the clay volume fraction ϕ_V is smaller than the porosity of the sand matrix alone ϕ_{Sand} , and the sand matrix is connected while the clay matrix may not.

This description accounts for two observation scales : (i) local (microscopic) scale, which is associated to the characteristic size of heterogeneities (clay aggregates, sand grains, meso-pores) and (ii) the macroscopic scale, which is the scale of entire soil layers and is most important for practical purposes. The scale corresponding to clay particles and micro-pores in the clay matrix is not considered.

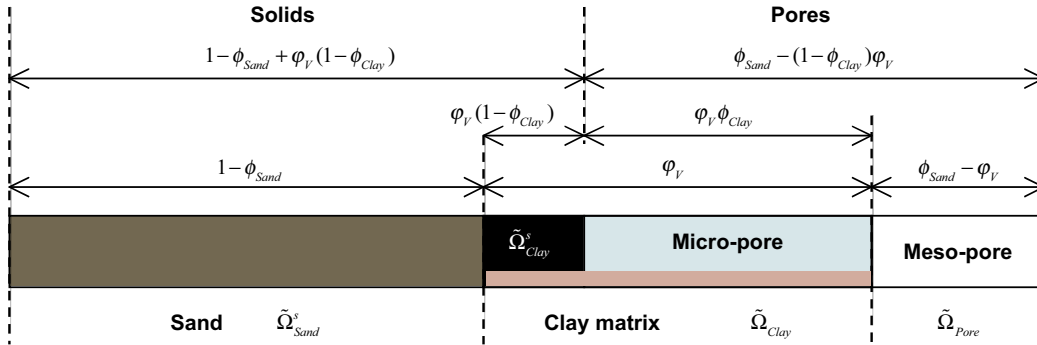


Figure 1. Clayey sand with relative proportions of each constituent.

Only the clay matrix is erodible; the sand grains are not erodible. The size of the meso-pores is assumed much bigger than the size of the micro-pores, which implies that the clay volume fraction ϕ_V is not too close to its maximum value ϕ_{Sand} . By this assumption, while the meso-pore domain makes up only a small percentage of the total pore volume $\phi + \phi_V \phi_{Clay}$, it transmits the major portion of the flow through the REV. Consistent with this situation, we assume that erosion occurs only at the interface of the clay matrix and the meso-pore domain $\hat{\Gamma}_{Clay} \cap \hat{\Gamma}$ where $\hat{\Gamma}_{Clay} = \partial\hat{\Omega}_{Clay}$ and $\hat{\Gamma} = \partial\hat{\Omega}_{Pore}$ (Figure 2). The erosion process is therefore external to the clay matrix, and does not affect its porosity ϕ_{Clay} , nor its density $\rho_{Clay} = (1 - \phi_{Clay})\rho_s + \phi_{Clay}\rho_w$. As illustrated by Figure 3, suffusion may occur when $0 < \phi_V < \phi_{Sand}$, while no suffusion occurs when $\phi_V = 0$ (no clay) or $\phi_V = \phi_{Sand}$ (the pore constrictions are too small).

The mechanical state is such that matrix deformations are negligible. Physico-chemical effect (like dissolution) are not considered. The porosities ϕ_{Sand} and ϕ_{Clay} can be considered as constant. The Reynolds number is small and inertia is neglected. We take $\hat{a}(\bar{x})$ to denote any quantity a varying at the micro-scale. The mass and momentum balance equations for the pore flow mixture within $\hat{\Omega}_{Pore}$, as well as the constitutive law accounting for the viscosity can be written as follows:

$$\hat{\nabla} \cdot \hat{\mathbf{u}} = 0, \quad \hat{\nabla} \cdot \hat{\mathbf{T}} = 0, \quad \hat{\mathbf{T}} = -\hat{p}\mathbf{I} + \eta_w(\hat{\nabla}\hat{\mathbf{u}} + {}^T\hat{\nabla}\hat{\mathbf{u}}) \quad \text{in } \hat{\Omega}_{Pore}. \quad (6)$$

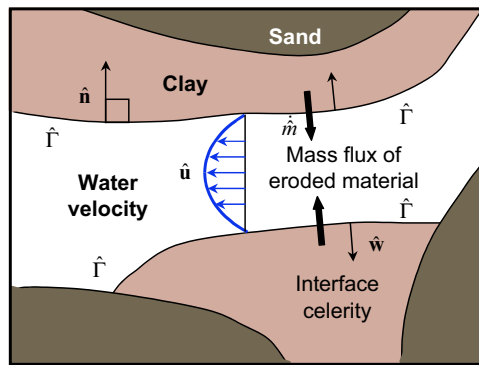


Figure 2. Schematic of the suffusion process viewed as the surface erosion of the clay matrix.

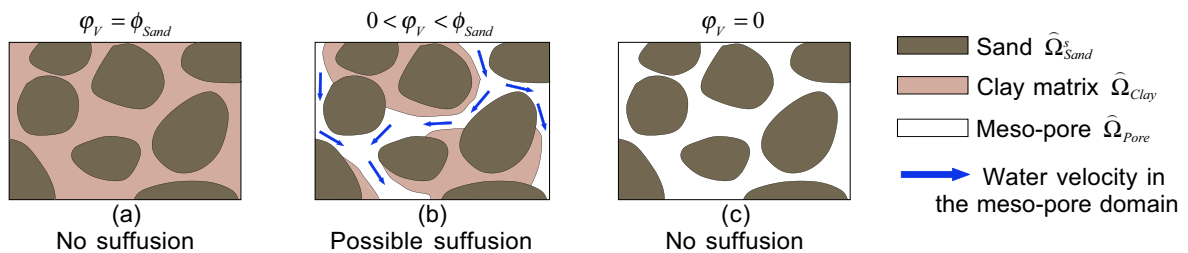


Figure 3. Schematic of the Sand-Clay mixture in the clayey sand domain.

In these equations, $\hat{\mathbf{u}}$ is the velocity, $\hat{\mathbf{T}}$ the Cauchy stress tensor, and \hat{p} the pressure.

A mass flux crosses each interface separating fluid and solid, where erosion occurs. The jump equations over $\hat{\Gamma}$ are therefore

$$\llbracket \hat{\rho}(\hat{\mathbf{w}} - \hat{\mathbf{u}}) \cdot \hat{\mathbf{n}} \rrbracket = 0, \quad \llbracket \hat{\rho} \hat{\mathbf{u}}(\hat{\mathbf{w}} - \hat{\mathbf{u}}) \cdot \hat{\mathbf{n}} \rrbracket = -\llbracket \hat{\mathbf{T}} \cdot \hat{\mathbf{n}} \rrbracket \quad \text{on } \hat{\Gamma}_{Clay} \cap \hat{\Gamma}, \quad (7)$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{n}} = 0, \quad \llbracket \hat{\mathbf{T}} \cdot \hat{\mathbf{n}} \rrbracket = 0 \quad \text{on } \hat{\Gamma}_{Sand} \cap \hat{\Gamma}, \quad (8)$$

In these equation, $\hat{\rho}$ is the density, $\hat{\mathbf{w}}$ the interface velocity, $\hat{\mathbf{n}}$ the normal unit vector of $\hat{\Gamma}$, and $\llbracket [a] \rrbracket$ the jump of any quantity a across $\hat{\Gamma}$. The coordinate system depends on the solid matrix ($\hat{\mathbf{u}} = 0$ in the solids). The surface flux of eroded material (clay particles+water) crossing the interface is therefore $\hat{m} = \hat{\rho}_{Clay} \hat{\mathbf{w}} \cdot \hat{\mathbf{n}}$ (Brivois *et al.*, 2007; Bonelli and Brivois, 2008). As this is a surface erosion due to tangential flow, the constitutive law may be chosen as

$$\hat{m} = \hat{k}_{er} H\hat{\tau} - \hat{\tau}_c. \quad (9)$$

In Eq. (9), $\hat{\tau}_c$ (Pa) is the threshold stress, \hat{k}_{er} (s/m) the coefficient of surface erosion, $H(\cdot)$ the Heaviside step function ($H(x) = 0$ if $x < 0$; $H(x) = 1$ if $x > 0$), and $\hat{\tau}$ the norm of the tangential shear stress vector at the interface:

$$\hat{\tau} = \|\hat{\boldsymbol{\tau}}\|, \quad \hat{\boldsymbol{\tau}} = [\mathbf{I} - \hat{\mathbf{n}} \otimes \hat{\mathbf{n}}] \cdot \hat{\mathbf{T}} \cdot \hat{\mathbf{n}} \quad \text{on } \hat{\Gamma}. \quad (10)$$

An additional (and usual) constitutive assumption on the interface is that all tangential velocities are continuous across $\hat{\Gamma}$, (implying the continuity of $\hat{\boldsymbol{\tau}}$ across $\hat{\Gamma}$):

$$\llbracket \hat{\mathbf{u}}_T \rrbracket = 0, \quad \hat{\mathbf{u}}_T = [\mathbf{I} - \hat{\mathbf{n}} \otimes \hat{\mathbf{n}}] \cdot \hat{\mathbf{u}} \quad \text{on } \hat{\Gamma}. \quad (11)$$

The use of these equations is extended here to study suffusion in a clayey sand soil.

2.3 Formulation of a suffusion erosion law

The complete set of equations (6-11) was already used to study different systems by means of periodic homogenization by Bouddour *et al.* (1996). Under the dilute flow assumption, the obtained system is formally similar to Eqs. (1-4). Bouddour *et al.* (1996) established that the global volume flux of eroded material $r(x)$ Eq. (1) is related to the local surface flux $\hat{m}(\hat{\mathbf{x}})$ Eq. (9) as follows:

$$r = \frac{1}{|\hat{\Omega}|} \int_{\hat{\Gamma}} \hat{m} d\hat{\Gamma}. \quad (12)$$

In addition, the local shear stress $\hat{\boldsymbol{\tau}}(\hat{\mathbf{x}})$ Eq. (10) is linear with respect to the global pressure gradient $\partial p / \partial x$ Eq. (3), and the macro shear stress is linear with respect to the global pressure gradient. From these results, we infer that

$$\frac{1}{|\hat{\Omega}|} \int_{\hat{\Gamma}} \hat{\boldsymbol{\tau}} d\hat{\Gamma} = \phi \zeta \frac{\partial p}{\partial x}. \quad (13)$$

The parameter $\zeta \geq 1$ accounts for the tortuosity of the meso-pore domain:

$$\zeta = \left(\frac{\int_{\hat{\Gamma}} \hat{\boldsymbol{\tau}}(\hat{\mathbf{x}}) d\hat{\Gamma}}{\int_{\hat{\Gamma}} \|\hat{\boldsymbol{\tau}}(\hat{\mathbf{x}})\| d\hat{\Gamma}} \right)^{-1}. \quad (14)$$

A first order approximation of the surface inhomogeneities yields

$$\int_{\hat{\Gamma}} H\hat{\tau} - \hat{\tau}_c d\hat{\Gamma} = \frac{|\hat{\Gamma}_{er}|}{|\hat{\Gamma}|} \int_{\hat{\Gamma}} \hat{\tau} d\hat{\Gamma}, \quad (15)$$

were we assume that the mean shear stress on $\hat{\Gamma}_{Clay} \cap \hat{\Gamma}$ and on $\hat{\Gamma}$ are of the same order. In Eq. (15), $\hat{\Gamma}_{er}$ denotes the surface of eroded material (the clay matrix/meso-pore water interface where $\hat{\tau} > \hat{\tau}_c$):

$$\hat{\Gamma}_{er} = \{\hat{\mathbf{x}} \in \hat{\Gamma} / \hat{m}(\hat{\mathbf{x}}) > 0\}. \quad (16)$$

The mean threshold stress does not appear in Eq. (15), as the average of a local threshold law involving local stresses is not a threshold law involving the average of the local stresses. For simplification, we assume a constant local coefficient of erosion \hat{k}_{er} . Eq (12) may be now written as follows:

$$r = \gamma_w \hat{k}_{er} \phi \zeta \frac{|\hat{\Gamma}_{er}|}{|\hat{\Gamma}|} i_w, \quad (17)$$

where $\gamma_w = \rho_w g$ is the water specific weight, and i_w is the hydraulic gradient

$$i_w = \frac{1}{\gamma_w} \left| \frac{\partial p}{\partial x} \right|. \quad (18)$$

The dimensionless quantity $|\hat{\Gamma}_{clay} \cap \hat{\Gamma}| / |\hat{\Gamma}|$ is equal to zero if $\varphi_V = 0$ ($\hat{\Gamma}_{clay} = \emptyset$), and is close to one if φ_V is close to its maximum value ϕ_{sand} . A first order approximation yields:

$$\frac{|\hat{\Gamma}_{clay} \cap \hat{\Gamma}|}{|\hat{\Gamma}|} = \left(\frac{\varphi_V}{\phi_{sand}} \right)^{2/3}. \quad (19)$$

The fact that a surface quantity ($|\hat{\Gamma}_{clay} \cap \hat{\Gamma}| / |\hat{\Gamma}|$) is related to a volume quantity (φ_V / ϕ_{sand}) explains the value $2/3$ for the exponent. For example, the surface of a sphere scales with its volume with this exponent.

The amount of eroded material (the clay) is limited, and depends on the hydraulic gradient, as established by experimental results (Sterpi, 2003; Bendahmane *et al.*, 2006; Bendahmane *et al.*, 2008). The parameter $|\hat{\Gamma}_{er}|$ appears to be the relevant quantity accounting for this material information. We assume that the surface fraction of eroded material $|\hat{\Gamma}_{er}|$ scales with the clay volume fraction as follows

$$\frac{|\hat{\Gamma}_{er}|}{|\hat{\Gamma}_{clay} \cap \hat{\Gamma}|} = H \left[\varphi_V - \varphi_V^{stop} \right] \left[1 - \left(\frac{\varphi_V^{stop}}{\varphi_V} \right)^{2/3} \right]. \quad (20)$$

In addition, we assume that the final clay volume fraction φ_V^{stop} depends on the hydraulic gradient:

$$\varphi_V^{stop} = \varphi_V^0 \left(\frac{i_c^0}{i_c^0 + i_w} \right)^{3/2}. \quad (21)$$

The parameter i_c^0 is a phenomenological parameter which is not explicitly connected to the local threshold $\tilde{\tau}_c$. It plays the role of a threshold hydraulic gradient to stop erosion, as erosion occurs if $\varphi_V > \varphi_V^{stop}$, which means that

$$i_w > i_c^0 \left[\left(\frac{\varphi_V^0}{\varphi_V} \right)^{2/3} - 1 \right]. \quad (22)$$

3 Comparison with experimental results

The suffusion law is now compared with previously published data, giving the mass fraction of eroded material as a function of time for several hydraulic gradients (Sterpi, 2003). The mass fraction of eroded material is a function of the clay volume fraction as follows:

$$\square(t) = \frac{(1 - \phi_{clay}) [\varphi_V^0 - \varphi_V(t)]}{(1 - \phi_{clay}) \varphi_V^0 + (1 - \phi_{sand})}. \quad (23)$$

The sample is assumed homogeneous. The hydraulic gradient is therefore homogeneous, as dilute suspension flow is assumed, and no deposition occurs. Consequently, the behaviour law Eq (4), which involves the permeability, is not necessary in a pressure driven problem. The dimensionless expression of the differential equation Eqs (1), (17), and (19-21) shows that this is a two-parameters problem:

$$\frac{d}{dt} \tilde{\varphi}_V = - \frac{1}{t_{er}} (\tilde{a} \tilde{\varphi}_V^{2/3} - 1) (1 - \tilde{\varphi}_V), \quad (24)$$

$$\tilde{\varphi}_V = \frac{\varphi_V}{\phi_{sand}}, \quad (25)$$

$$t_{er} = \frac{\rho_{clay}}{\zeta \rho_w g k_{er}} \left(\frac{1}{i_c^0} + \frac{1}{i_w} \right) \left(\frac{\varphi_V^0}{\phi_{sand}} \right)^{-2/3}, \quad (26)$$

$$\tilde{a} = \left(1 + \frac{i_w}{i_c^0} \right) \left(\frac{\phi_{sand}}{\varphi_V^0} \right)^{2/3}, \quad (27)$$

as the two basic unknown are t_{er} and \tilde{a} . This expression suggests that it is possible to estimate the parameter i_c^0 from the total amount of eroded material, which corresponds to $\tilde{\varphi}_V = \tilde{a}^{-2/3}$ if at least equilibrium is attained, which was not the case of the available data.

Table 1 summarizes the parameters of the model. Parameters ϕ_{sand} and ϕ_V^0 were inferred from Sterpi (2003), while a characteristic value of ϕ_{clay} was chosen at this value was not available. The identification gives mean values and standard deviations for i_c^0 and \hat{k}_{er} . The coefficient of erosion is about 10^{-5} s/m. This is a low value, which corresponds to a very slow erodable soil for surface erosion (Bonelli and Brivois, 2008; Wand and Fell, 2004). This finding is consistent with previous experimental results: internal erosion rates were found to be smaller than surface erosion rates (Reddi *et al.*, 2000). Figure 4 gives the increase in the mass fraction of eroded material as a function of time. The data are well-described by this approach over the full range of hydraulic gradient explored.

Table 1. Parameters.

ϕ_{sand}	Sand porosity	0.49
ϕ_{clay}	Clay porosity	0.60
ϕ_V^0	Initial clay volume fraction	38%
ρ_w	Water density	1 000 kg.m ⁻³
ρ_{Clay}	Clay saturated density	1 688 kg.m ⁻³
ζ	Tortuosity	2
g	Gravitational constant	9.81 m.s ⁻²
\hat{k}_{er}	Coefficient of erosion	2×10^{-5} s.m ⁻¹ ± 0.37
i_c^0	Threshold hydraulic gradient	3.16 ± 0.28

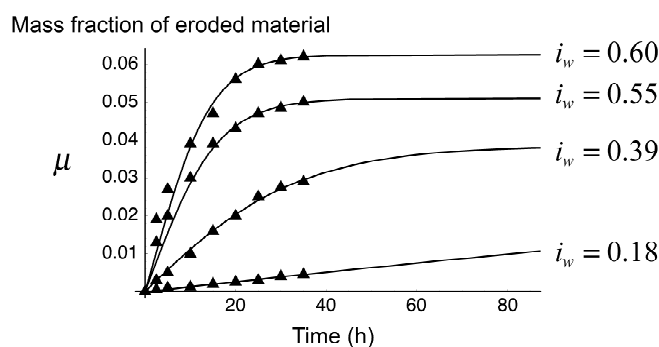


Figure 4. Suffusion tests with a constant pressure drop, test (symbols) versus model (continuous lines). Mass fraction of eroded material is shown as a function of time (Experimental results from Sterpi (2003)).

4 Conclusion

The present study was intended to provide a step towards the modelling of suffusion in a clayey sand. Erosion is basically an interfacial process, which can be considered as shear stress driven. A multi-scale approach shows that :

- i) The macroscopic bulk erosion is driven by the macroscopic pressure gradient (which is related to the microscopic shear stress), and not by the seepage velocity (as assumed by Einstein in 1937).
- ii) The coefficient of surface erosion is a relevant parameter to quantify the rate of bulk erosion.

The two main assumptions of our model are:



- i) The eroded clay/water interface are related to the clay volume fraction.
- ii) The amount of erodable clay is related to the hydraulic gradient.

Comparisons with published experimental data show that good results are obtained over the full range of hydraulic gradient explored. Other important phenomena should be considered in a more comprehensive modelling of internal erosion: particle transport and filtration, deposition and clogging, two-phase seepage flow and concentrated flow, dissolution and physico-chemical effects.

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