The use of infinitesimal dipoles and the spherical wave expansion for planar antennas modeling

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The Use of Infinitesimal Dipoles and the Spherical Wave Expansion for Planar Antennas Modeling

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Abstract— In this paper, a method to derive an equivalent radiation model for planar antennas is presented. This method uses the spherical near-field (NF) data (measured or computed) to ascertain an equivalent set of infinitesimal dipoles placed over the main antenna surface. The spherical wave expansion (SWE) of the NF data is written in terms of infinitesimal dipoles using a transition matrix. This matrix expresses the linear relations between the spherical wave coefficients of the antenna and the spherical wave coefficients of each dipole. Once the excitation of each dipole is known, the field at any angle and distance from the antenna is rapidly calculated even inside the minimum sphere. Computations with EM simulation data of an antenna array illustrate the reliability of the method.

I. INTRODUCTION

The antenna modelling in its operating environment is still a challenge using full-wave solutions. Indeed, the cost of such solutions grows exponentially with the size and the complexity of the problem. To cope with these difficulties, a hybrid approach is proposed. The antenna is separately characterized (measured or simulated) and the resulted near field data are exploited to define an equivalent set of point sources that reproduce the antenna radiated field everywhere. For further applications, the antenna equivalent sources can be incorporated into an existing electromagnetic algorithm to simulate the antenna behaviour in a multiple interaction environment.

In this paper, we present a method to derive simple equivalent radiation sources for planar antennas. This method uses spherical near-field (NF) data to determine an equivalent set of tangential infinitesimal dipoles that reproduce accurately the antenna radiation field, both inside and outside the minimum sphere enclosing the antenna. To this end, the spherical wave expansion (SWE) of the measured NF data are rewritten in terms of infinitesimal dipoles using the translational and rotational additional theorems. The a priori information concerning the antenna geometry are used to set the spatial distribution of the equivalent dipoles.

In the literature different approaches were adopted for the antenna description by an equivalent current/charge distribution. In [1]-[3] an electric field integral equation method (EFIE) is developed to relate the measured electric field to equivalent magnetic /electric current on the antenna aperture. Equivalent currents are written as linear combinations of two-dimensional pulse basis functions, which can be approximated by Hertzian dipoles distributed over any enclosing arbitrary surface. This method has been investigated for near field to far field transformation, inverse electromagnetic radiation applications and antenna diagnosis.

In [4]-[5] evolutionary algorithms were introduced. Using Genetic Algorithm (GA) technique, the antenna is substituted by a set of infinitesimal dipoles. The choice of infinitesimal dipole sources is justified by the simplicity of its implementation in any EM code, and the GA shows its ability to determine the type (electric or magnetic), the position, the orientation and the excitation of each dipole.

In a previous paper [6], the authors have introduced a modeling technique, which consists in the substitution of the antenna with a set of infinitesimal dipoles distributed over the minimum sphere circumscribing the antenna. These equivalent dipoles distribution is not convenient to provide the near-field within the minimum sphere for planar antennas for example.

In the present paper, an improvement of the analysis presented in [6] is proposed. Using the a priori knowledge of the antenna geometry, the equivalent dipoles are placed over the main antenna surface. Once assessed, this set of tangential dipoles allows the knowledge of the antenna electric and magnetic fields rapidly at any distances using the free space Green's function. Also, it has been shown that the use of the antenna spherical wave coefficients (SWC) to elaborate the antenna equivalent model allows a robust matching procedure, where the whole antenna intrinsic radiation properties are contained in a finite number of SWC.

To achieve the above goal, we first describe the problem formulation. Then, in Section III, we present the application process of the method for an antenna array. Finally, concluding remarks summarizing the potential of this method are provided in Section IV. All theoretical results are expressed with $e^{-j\omega t}$ time dependence.

II. FORMULATION

Here, a current source is defined as 4 tangential uncoupled and co-localized infinitesimal dipoles: 2 electric and 2 magnetic. We attach a local coordinate system $(o_i, \vec{x_i}, \vec{y_i}, \vec{z_i})$ for $1 \leq i \leq L_{source}$ to each current source, where the origin $o_i(r_i, \theta_i, \varphi_i)$ coincides with the position of the ith source in the

global coordinate system $(o, \vec{x}, \vec{y}, \vec{z})$ (Fig. 1), and L_{source} is the antenna equivalent sources number.

A. Spherical wave expansion

In source free region outside the minimum sphere of radius $r_{\min}$ circumscribing the antenna, the SWE of the radiated electric field $\vec{E}_{AUT}$ in spherical coordinates system is expressed in terms of truncated series of spherical vector wave functions [7] as

$$\vec{E}_{AUT}(\vec{r}) = \frac{k}{\sqrt{\eta}} \sum_{j=1}^{J_{max}} Q_j \vec{F}_j^{(3)}(\vec{r}) \qquad (1)$$

where, Q_j are the spherical wave coefficients, and $\vec{F}_j^{(3)}(\vec{r})$ are the power-normalized spherical vector wave functions. The truncation number $J_{max} = 2(N_{tr}(N_{tr}+2))$ with, $N_{tr} = [k\, r_{\min}] + 10$ depends on the antenna dimensions and the operating frequency [7]. The Q_j are determined from the knowledge of the tangential components of either $\vec{E}$ or $\vec{H}$ on the measurement sphere. Once Q_j are known, the field outside the minimum sphere is completely characterized by (1).

The SWE of the $\overline{E_i^{e,x}}$ field radiated by an infinitesimal x-directed electric dipole placed at the origin of $(o_i, \vec{x}_i, \vec{y}_i, \vec{z}_i)$ is expressed in the spherical coordinate system $\vec{r}_i(r_i, \theta_i, \varphi_i)$ associated with $(o_i, \vec{x}_i, \vec{y}_i, \vec{z}_i)$ as

$$\overline{E_i^{e,x}}(\vec{r}_i) = \frac{k}{\sqrt{\eta}} \frac{\sqrt{2}}{2} \alpha_i^{e,x} (\vec{F}_2^{(3)}(\vec{r}_i) - \vec{F}_6^{(3)}(\vec{r}_i)) \qquad (2)$$

with $\quad \alpha_i^{e,x} = -\frac{k}{\sqrt{6\eta\pi}} I_i^{e,x} l$.

Expressions for x, y or z-directed dipole either electric or magnetic ones are provided in [6]. Hence, the (ith) current source (4 dipoles) is characterized in $(o_i, \vec{x}_i, \vec{y}_i, \vec{z}_i)$ by 4 SWC $S_i = \left[\alpha_i^{e,x}, \alpha_i^{m,x}, \alpha_i^{e,y}, \alpha_i^{m,y} \right]$.

Explicitly, the proposed modeling technique intends to rewrite the spherical wave expansion (SWE) of the antenna near-filed expressed in $(o, \vec{x}, \vec{y}, \vec{z})$ by means of multiple and local SWE that are expressed in $(o_i, \vec{x}_i, \vec{y}_i, \vec{z}_i)$ for $1 \le i \le L_{source}$ in order to reach the E-field inside the minimum sphere.

B. The problem synthesis

Using translational and rotational addition theorems [7]-[9], we express the SWE of the field radiated by the ith current source in the antenna coordinate system $(o, \vec{x}, \vec{y}, \vec{z})$. Thus, the row vector S_i is written in $(o, \vec{x}, \vec{y}, \vec{z})$ as

$$\left[M_{i,j} \right]_{j=1..J_{max}} = \left[\alpha_i^{e,x} A_{j,i}^{e,x}, \alpha_i^{m,x} A_{j,i}^{m,x}, \alpha_i^{e,y} A_{j,i}^{e,y}, \alpha_i^{m,y} A_{j,i}^{m,y} \right]_{j=1..J_{max}}$$

where the coefficients $A_{j,i}^{e,x}$, $A_{j,i}^{m,x}$, $A_{j,i}^{e,y}$, $A_{j,i}^{m,y}$ are developed in the Appendix of [6].

Let define $\mathcal{A}_{j,i} = \left[A_{j,i}^{e,x}, A_{j,i}^{m,x}, A_{j,i}^{e,y}, A_{j,i}^{m,y} \right]$. The superposition of the fields radiated by all equivalent sources ($\vec{E}_{mod}$) is written in $(o, \vec{x}, \vec{y}, \vec{z})$ as
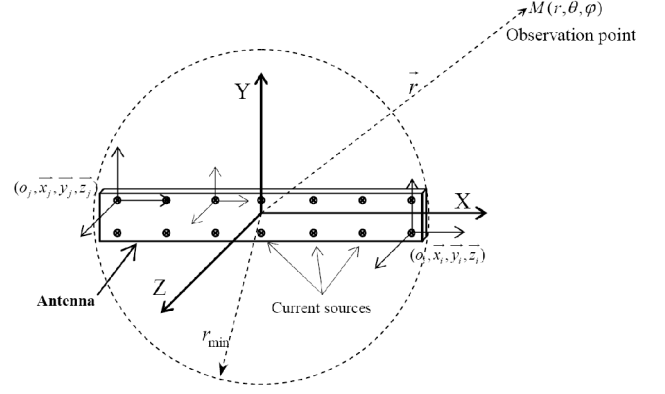

Fig. 1 The antenna minimum sphere and current sources configuration

$$\vec{E}_{mod}(\vec{r}) = \frac{k}{\sqrt{\eta}} \sum_{i=1}^{L_{source}} \sum_{j=1}^{J_{max}} \left(\begin{array}{c} \alpha_i^{e,x} A_{j,i}^{e,x} + \alpha_i^{m,x} A_{j,i}^{m,x} \\ + \alpha_i^{e,y} A_{j,i}^{e,y} + \alpha_i^{m,y} A_{j,i}^{m,y} \end{array} \right) \vec{F}_j^{(3)}(\vec{r})$$

$$= \frac{k}{\sqrt{\eta}} \sum_{j=1}^{J_{max}} \sum_{i=1}^{L_{source}} \left[\mathcal{A}_{j,i} \right] . S_i^{\ T} . \vec{F}_j^{(3)}(\vec{r}) \qquad (3)$$

Identifying $\vec{E}_{AUT}$ with $\vec{E}_{mod}$, and from (1) and (3), we get

$$\sum_{j=1}^{J_{max}} Q_j \vec{F}_j^{(3)}(\vec{r}) = \sum_{j=1}^{J_{max}} \sum_{i=1}^{L_{source}} \left[\mathcal{A}_{j,i} \right] . S_i^{\ T} . \vec{F}_j^{(3)}(\vec{r}) \qquad (4)$$

Due to orthogonal properties of $\vec{F}_j^{(3)}$, the SWC of both sides of (4) have to be equal. Let denote $\mathcal{A} = \left(\mathcal{A}_{j,i} \right)_{\substack{1 \le i \le L_{source} \\ 1 \le j \le J_{max}}}$, $\mathcal{X} = \left[S_1 \ldots \quad S_{L_{source}} \right]^T$ and $\mathcal{Q} = \left[Q_1 \ldots \quad Q_{J_{max}} \right]^T$, where the superscript 'T' expresses the matrix transpose. Equation (4) becomes

$$\mathcal{A}.\mathcal{X} = \mathcal{Q} . \qquad (5)$$

The transition matrix $\mathcal{A}$ expresses linear relations between the SWC of current sources ($\mathcal{X}$) and the SWC $\mathcal{Q}$ of the actual antenna. To solve (5), the least square (LSQR) code of MatLab have been used to determine the $4 \times L_{source}$ unknowns composing the row vector $\mathcal{X}$.

III. RESULTS

In this section, we report a numerical example by simulation of a linear microstrip array of aperture-coupled patches operating at the frequency 5.82 GHz. The antenna dimensions are $D_{AUT}^x = 0.1464m$ (2.84λ) along the x-axis, $D_{AUT}^y = 0.06m$ (1.16λ) along the y-axis, and $D_{AUT}^z = 0.0031m$ along the z-axis with $r_{\min} = 0.085m (1.65\lambda)$ (Fig. 2). The antenna is composed of a linear array of 4 rectangular patches printed on a dielectric support. Each patch is aperture-coupled to the microstrip line. A common ground plane separates the radiating part from the microstrip feeding network. The antenna is issued from Antenna Center of Excellence benchmarking data base, where the detailed description is provided in [10].

The antenna equivalent model construction is based on near field data obtained through the electromagnetic simulation of
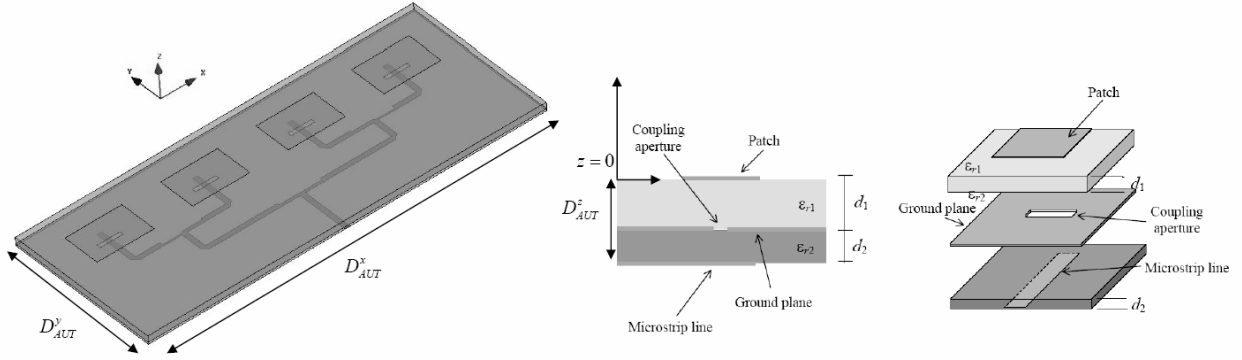
Fig. 2 3D view of the studied antenna array. Dielectric stratification (middle) and layered view of the structure (right).

the antenna in a 3D simulator that uses the finite elements method (FEM). At the distance of $r_{meas} = 15\lambda$ $(\approx 0.75\,m)$, the amplitude and phase of both E-field components E_θ and E_φ over a spherical surface ($0 \le \theta \le \pi$, $0 \le \varphi \le 2\pi$), with the angular resolution of $\Delta\theta = \Delta\varphi \le \pi/N_{tr}$ are recovered. Based on these data, we assess the antenna SWC that are presented in Fig.3. The SWE is truncated at $N_{tr} = \lceil k\,r_{min} \rceil + 10 = 20$, so $J_{max} = 2N_{tr}(N_{tr} + 2) = 880$ SWC Q_j is considered. The antenna being planar, we have chosen the natural way to place the current sources (rectangular grid). The antenna is seen as the rectangular surface

$$S_{equi} = (x, y, z) \in \{-0.073m \le x \le 0.073m, -0.042m \le y \le 0.017m, z = 0\}$$

over which, the equivalent dipoles are uniformly distributed (Fig.4).

The antenna equivalent model is associated with a spatial spacing criterion Δx and Δy that separate equivalent sources. This results in a number of current sources given by $L_{source} = \left[(D^x_{AUT}/\Delta x) + 1\right] \cdot \left[(D^y_{AUT}/\Delta y) + 1\right]$ where $[X]$ means the integer part of X. Based on the chosen Δx and Δy, the spatial positions of equivalent current sources in $(o, \vec{x}, \vec{y}, \vec{z})$ are geometrically determined and consequently the corresponding transition matrix **A** is constructed.

Solving (5) we calculate the excitation of each dipole, and the E-field radiated from the whole dipoles E^{mod} is calculated at different spherical observation domains, near the minimum sphere and in the far field regions. Comparisons with the actual antenna radiation pattern E^{act} are carried using the following error function:

$$RMS_E = \sqrt{\frac{\sum_{\theta,\varphi} \left| E^{act}(\theta,\varphi) - E^{mod}(\theta,\varphi) \right|^2}{\sum_{\theta,\varphi} \left| E^{act}(\theta,\varphi) \right|^2}} \quad, \text{where } E = E_\theta, E_\varphi$$

Comparisons between the actual antenna radiation pattern (Finite Elements Method) and the radiation pattern of the equivalent dipoles in near and far field regions are shown in Fig. 5 for $\Delta x = \Delta y = 0.4\lambda$.

As it can be seen from Fig.5, the equivalent model radiation pattern fits very well with the one resulted from the spherical wave expansion method. In Fig. 6, the error function is
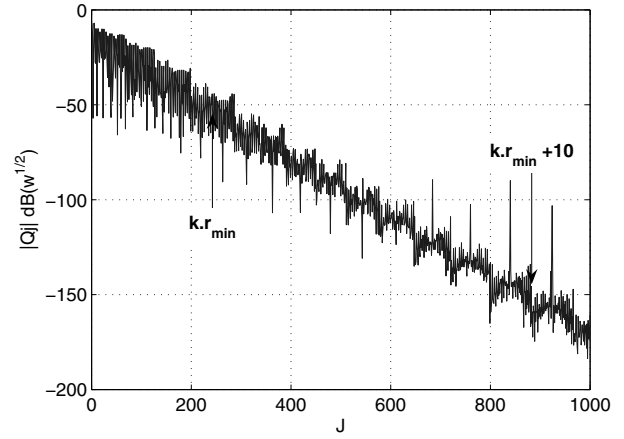

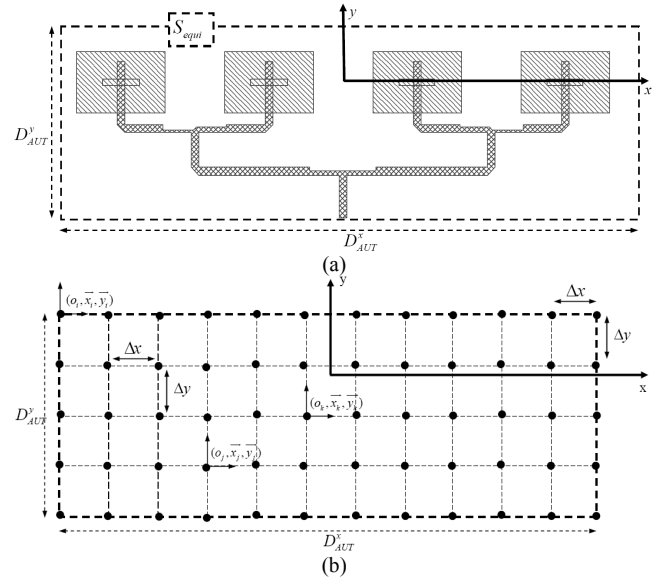Fig. 3. Magnitude of the antenna array spherical wave coefficients

Fig. 4. The description of the equivalent current sources distribution over the antenna surface. (a). The dashed rectangle represents the antenna equivalent surface. (b). The black points represent the equivalent current sources distributed regularly over the antenna equivalent surface respecting the modeling parameter Δx and Δy (spacing between adjacent sources).
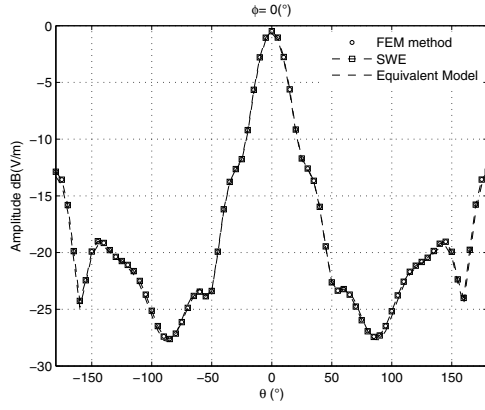
(a)
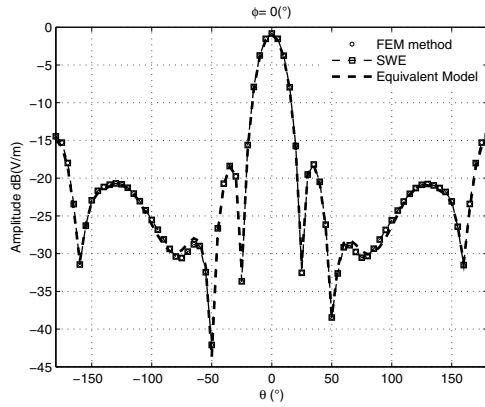
(b)

Fig. 5 At the plane $\phi = 0°$, (a) Near-field observed at the distance of 4λ. (b) Far-field radiation pattern. The actual field resulted from Finite Elements Method (FEM) (o-marker line), the field resulted from the SWE (□-marker line), and the field radiated from the equivalent model

Fig. 6 The error function RMSE versus the distance for $\Delta x=\Delta y=0.25\lambda$ and $0.4\ \lambda$ associated equivalent models.

Fig. 7 The error function RMSE evaluated inside the minimum sphere versus the distance for $\Delta x=\Delta y=0.25\lambda$ and $0.4\ \lambda$ associated equivalent models.

presented as a function of the distance from the antenna. It is seen that the equivalent model corresponding to $\Delta x=\Delta y=0.25\lambda$ results in $0.23\% \leq RMS_{E_\theta} \leq 0.29\%$ and $0.16\% \leq RMS_{E_\varphi} \leq 0.31\%$. Using the criterion $\Delta x=\Delta y=0.4\lambda$, the equivalent model results in $0.77\% \leq RMS_{E_\theta} \leq 0.88\%$ and $0.75\% \leq RMS_{E_\varphi} \leq 1\%$. For a smaller spacing $\Delta x=\Delta y=0.2\lambda$, the accuracy of the antenna equivalent model is improved ($0.04\% \leq RMS_{E_\theta} \leq 0.1\%$, $0.08\% \leq RMS_{E_\varphi} \leq 0.17\%$) but the number of equivalent sources increases ($\Delta x=\Delta y=0.2\lambda$ corresponds to 90 current sources). These equivalent models reproduce accurately the antenna radiation everywhere outside the minimum sphere ($2\lambda \leq distance \leq 40\lambda$).

Inside the minimum sphere, over the plane $\{-0.075m \leq x \leq 0.075m, \ -0.045m \leq y \leq 0.02m,\}$ located at the distance $0.2\lambda \leq z \leq r_{min}$ from the antenna, the E-field component estimated by the 3D simulation software (Finite elements method) and the E-field radiated by the equivalent dipoles are compared. As presented in Fig.7, the equivalent models ($\Delta x=\Delta y=0.25\lambda$ or 0.4λ) provide a good approximation of the field inside the minimum sphere for distances greater than 0.3λ from the antenna. Otherwise, the field in the vicinity of
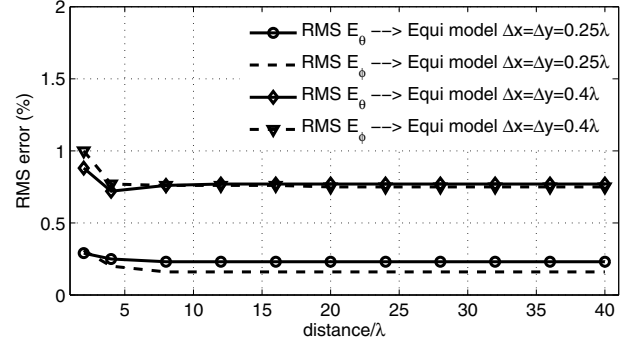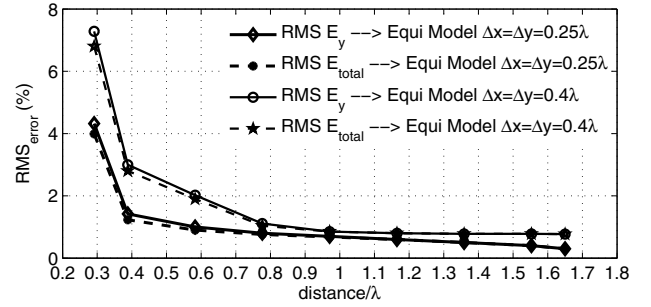
the antenna ($distance \prec 0.2\lambda$) is mainly reactive and the information of the evanescent modes are not available at the measurement distance. Consequently, the equivalent models presents some discrepancies while trying to reproduce the E-field for $distance \prec 0.2\lambda$.

The aim of the examples presented here, is to show the feasibility of the modeling method for the case of a finite open surface (equivalence principle for an open surface). Exploiting the a priori information concerning the antenna geometry, we have defined an equivalent model, which reproduces accurately the antenna radiation pattern. We have established that this modeling technique is able to define different kind of equivalent models where the choice of the spacing between equivalent current sources (complexity) depends on the desired accuracy, so, on the application for which the equivalent model is intended.

Contrarily to the spherical wave expansion, the minimum number of spherical wave coefficients is related to the antenna dimensions. At least we should consider the truncation number $N_{tr} = \lceil k\,r_{min} \rceil = 10$, which leads to the use of $J_{max} = 2N_{tr}(N_{tr}+2) = 240$ transmission coefficients. The SWE is a very accurate way to describe the radiated field and the accuracy of the SWE cannot be questioned. Nonetheless, the SWE is valid outside the minimum sphere and the implementation of the SWE representation of the radiated field in an electromagnetic code is not trivial. Rewriting the antenna SWC in terms of equivalent dipoles, can be a good

way to express the whole information contained in the spherical wave coefficients in a limited number of dipole excitations.

The rearrangement of the SWE into a multiple SWE expressed in different local coordinate systems (using a priori information) is a time consuming procedure (translational and rotational addition theorems). Nevertheless, once the equivalent model is determined, the calculation of the radiation pattern in different distances from the antenna is faster than using the SWE. This is due to the fact that the number of equivalent dipoles is smaller than the number of the transmission coefficients.

IV. Conclusion

A method for antenna modeling has been presented. This method is based on the substitution of the original antenna by a set of equivalent infinitesimal dipoles that reproduce the same radiating field. The use of the spherical wave coefficients to elaborate the antenna equivalent model is interesting. This allows a straightforward matching procedure, where the whole antenna intrinsic information are contained in a finite number of complex values. Once the antenna equivalent model is defined, the electromagnetic radiation of the antenna can be calculated at any point both inside and outside (FF region) the minimum sphere except in the reactive region of the antenna. Also, the antenna a priori information concerning the AUT geometry have been used to express the multiple spherical wave expansion, which is of great interest since it has allowed the rewrite the SWE with a limited and reduced number of dipole excitations.

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