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Abstract—**Power efficiency and linearity are key parameters of amplification systems but they cannot be achieved simultaneously. A perfect linearity is observed when the power efficiency is low and vice versa. In this paper, we first analyze through some theoretical expressions, the power efficiency and the linearity measured by the Error Vector Magnitude (EVM) metric. Then we propose an analytical trade-off that ensures a good linearity with reasonable efficiency by combining Peak-to-Average Power Ratio (PAPR) reduction and linearization. This analysis is carried out based on Solid State Power Amplifiers (SSPA) and Predistortion (PD) as linearization technique. We show that a trade-off can be achieved for a high distortionless PAPR reduction gain followed by an effective predistortion. Last but not least the most important is to avoid the amplifier saturation by setting the PAPR of the signal after PAPR reduction technique identical to the input back-off (IBO).**

I. INTRODUCTION

Wireless communication standards impose stringent requirements on linearity performance of power amplifiers (PA). In addition, since the PA consumes most of the energy in telecommunication equipments, its power efficiency becomes a primary concern. In fact, with the meteoric growth in voice and data communication usage, a high efficiency will contribute to reduce the energy consumption at mobile devices level which result in long batteries lifetime and at base stations level with power savings and environmental pollution reduction. However, these two requirements, linearity and power efficiency, tend to be mutually exclusive in traditional PA design, so that any increase of the PA linearity by amplifying in linear region is usually achieved at the expense of the efficiency and conversely. This PA design problem is aggravated by multi-carrier modulations like Orthogonal Frequency Division Multiplexing (OFDM) with high Peak-to-Average Power Ratio (PAPR) what requires to consider an extremely low power efficiency [14].

In such conditions, techniques including linearization and PAPR reduction have been proposed separately in the literature [2], [10] to improve the performance of the transmitters, including PA. The linearization ensures high linearity of the PA in order to avoid carrier inter-modulations and to respect the power mask. In wireless communication systems, the most promising and cost-effective linearization's method is predistortion which guarantees an acceptable linearity level of the PA over its intended power range [14]. Regarding the PAPR reduction, it consists in reducing the dynamic range of the signal. This allows the PA to operate closer to the saturation power so more efficiently [6], [8]. PAPR reduction includes some techniques like clipping and filtering [19], coding techniques [4], Tone Reservation [7], etc [10]. Since PAPR reduction is associated to linearization and improves its effectiveness [8], [16], the methodology of PA design focuses on a trade-off between linearity and power efficiency represented by linearization and PAPR reduction respectively.

Several papers in literature [3]–[5], [7]–[9], [13], [15], [17] consider the association of PAPR reduction technique and digital predistortion as linearization method aiming to improve the performance of the PA in term of linearity and efficiency. They can be classified in two approaches. The first approach consists in combining a PAPR reduction technique followed by predistortion [4], [13]; the second one takes into consideration the mutual effects of PAPR reduction and predistortion in order to propose a optimal combination [5], [7], [9], [15], [17]. Obviously, the second approach will achieve better performance and a good trade-off between the PA linearity and the power efficiency. Few papers investigate the PA design problem by the second approach. In [15] for example, a way to jointly optimize PAPR reduction by Tone Reservation scheme and linearization by Digital predistortion is investigated considering IEEE 802.11 standard. The authors propose to tune the targeted clipping level of the PAPR reduction block according to the new saturation information of the combined AM/AM response of the predistorter followed by the PA. The authors of [9] noticed that by bypassing the PAPR reduction block during the characterization step and by applying it concurrently with the Digital predistortion in the linearization step, the performance of the PA is enhanced.

This paper is based on the second approach. Its objective is to analyze theoretically the PA linearity and its power efficiency in order to find out an analytical trade-off that maximizes the PA performance. All the derivations are done in baseband domain in scenario where PAPR reduction tech-

$x(t) = r(t)e^{j\theta_0(t)}$ $\tilde{x}_1(t) = \tilde{r}_1(t)e^{j\theta_1(t)}$ $\tilde{x}_2(t) = \tilde{r}_2(t)e^{j\theta_1(t)}$ $y(t) = \tilde{r}_3(t)e^{j\theta_1(t)}$

PAPR reduction | Predistortion | POWER AMPLIFIER

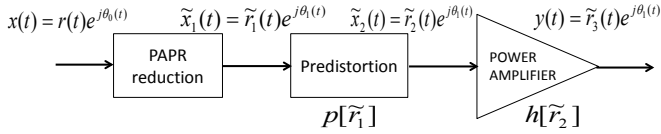$p[\tilde{r}_1]$ $h[\tilde{r}_2]$

Fig. 1. Simplified transmission scheme

nique is followed by predistortion and a memoryless PA. Error Vector Magnitude (EVM) metric is used to evaluate the predistortion linearity performance. In the rest of this paper, in Section II, an analytical calculation of the EVM metric is presented depending on the common performance of PAPR reduction and predistortion. The EVM expression is based on the predistortion error and its moments defined in [1]. Then in Section III, the same study is done for the power efficiency. In Section IV, these two relations, EVM and power efficiency, are simultaneously analyzed and a trade-off is proposed. Some simulations are carried out to confirm our analysis. Conclusions are drawn in Section V.

II. AMPLIFIED SIGNAL LINEARITY CONSIDERING PAPR REDUCTION AND PREDISTORTION

Nonlinear effects of the PA are generally evaluated using metrics like Adjacent Channel Power Ratio (ACPR) or Error Vector Magnitude (EVM). In this paper, we are focusing on the calculation of EVM as linearity metric used by the regulatory authorities. Specifications of IEEE 802.11b fix a maximum EVM level at 35% but it can be more strict like for IEEE 802.11a,g 6% or for 3G HSPA 12%. Let's first define the predistortion error and its second order moment [1].

A. The predistortion error

We consider the simplified transmission chain presented in Fig. 1. The OFDM signal $x(t)$ becomes $\tilde{x}_1(t)$ after PAPR reduction technique and $\tilde{x}_2(t)$ after predistortion. The amplified signal is $y(t)$. The PA is a memoryless Solid State Power Amplifier (SSPA). Under the assumption that the power amplifier has no phase distortion, signals $\tilde{x}_1(t)$, $\tilde{x}_2(t)$ and $y(t)$ have the same phase $\theta_1(t)$. Consequently AM/AM and AM/PM power amplifier characteristics are given by (1),

$$h(r) = \frac{r}{\left(1 + \left(\frac{r}{A}\right)^{2b}\right)^{\frac{1}{2b}}}, \quad (1)$$

$$\phi(r) = 0,$$

where r is the input signal amplitude and A the maximum output amplitude at the saturation. b is the "knee factor" that controls the transition smoothness from the linear region to the saturation. predistortion consists in applying to the input signal, a non-linearity that is the inverse of PA characteristics. So, the concatenation of the two will ideally be equivalent to a linear function. The predistortion function $p(r)$ corresponding

to the SSPA is equal to

$$p(r) = \frac{r}{\left(1 - \left(\frac{r}{A}\right)^{2a}\right)^{\frac{1}{2a}}}, r \in [0, A[, \quad (2)$$

with a the predistorter "knee factor". When $a = b$, a perfect linearization is performed.

We define a predistortion error $\epsilon(\tilde{r}_1)$ [1] that compares the amplified signal $y(t)$ to the signal before predistortion $\tilde{x}_1(t)$. This error quantifies the performance of predistortion and is expressed by

$$\begin{aligned}\epsilon(\tilde{r}_1) &= |\tilde{r}_1 - \tilde{r}_3|, \quad (3)\\ &= |\tilde{r}_1 - h(p(\tilde{r}_1))|, \quad \tilde{r}_1 \in [0, A[, \\ &= \left|\tilde{r}_1 - \tilde{r}_1\left(\left(1 - \left(\frac{\tilde{r}_1}{A}\right)^{2a}\right)^{\frac{b}{a}} + \left(\frac{\tilde{r}_1}{A}\right)^{2b}\right)^{-\frac{1}{2b}}\right|.\end{aligned}$$

When the predistortion is optimal, $a = b$, and the error is null independently from the PAPR reduction technique but this situation is difficult to achieve. Then in practice the error is not null and in the following, we discuss its variations based on the distribution of the signal $\tilde{x}_1(t)$ after PAPR reduction technique. However, the error in (3) is not expressed in a closed form but it is possible to provide an approximation. One way is to upper bound it. By definition $\tilde{r}_1 \in [0, A[$ so after some calculations, we show that:

$$\epsilon(\tilde{r}_1) \le \tilde{r}_1 \left|1 - 2^{\frac{b-a}{2ab}}\right|. \quad (4)$$

In the rest of this section, the approximation done in (4) will be considered to determine the second order moment of ϵ depending on the distribution of the signal after the PAPR reduction technique.

B. Expression of the second order moment of the predistortion error

We assume that the OFDM signal $x(t)$ is characterized by a complex stationary Gaussian process [18]. Therefore, its amplitude $r(t)$ converges to a Rayleigh distribution with σ_r variance and expressed by

$$p_r(r) = \frac{2r}{P_r}e^{\frac{-r^2}{P_r}}, \quad (5)$$

with $P_r = \sigma_r^2$ the mean power of the OFDM signal. Our objective is to study the distribution of the signal after PAPR reduction in order to calculate the second order moment of the predistortion error firstly for probabilistic PAPR reduction methods and secondly for amplitude clipping methods.

1) For probabilistic PAPR reduction methods: Probabilistic methods basically multiply the data symbols by a discrete vector that is transmitted as side information. For that reason, in the case where the PAPR reduction technique uses a probabilistic method [2], [10] like Selective Mapping (SLM) techniques, Random Phasor (RP) technique, etc. the distribution of the signal $\tilde{x}_1(t)$ remains Gaussian with mean power denoted $P_{\tilde{r}_1}$ [1]. In practice, OFDM signals have a minimum

and maximum values, so the second order moment $m_2^{(prob)}$ of ϵ can be written as followed,

$$m_2^{(prob)} \triangleq E\left[\left|\epsilon\left(\tilde{r}_1\right)\right|^2\right] = \int_{r_{min}}^{r_{max}} \left|\epsilon\left(r\right)\right|^2 p_r\left(r\right) dr, \quad (6)$$

where r_{min} and r_{max} are respectively the minimum and the maximum values of the reduced PAPR signal amplitude $\tilde{r}_1$. Thanks to (4), $m_2^{(prob)}$ can be upper bounded. Let's take $\rho = \frac{\tilde{r}_1^2}{P_{\tilde{r}_1}}$ with $\rho \in [0, \frac{A^2}{P_{\tilde{r}_1}}[$; the upper bound is given after some maths [1],

$$m_{2\_max}^{(prob)} = P_{\tilde{r}_1}\left(1 - 2^{\frac{b-a}{2ab}}\right)^2 \left[(\rho+1)\,e^{-\rho}\right]_{\rho=\rho_{max}}^{\rho=\rho_{min}}, \quad (7)$$

where ρ_{min} and ρ_{max} are respectively the minimum and the maximum values of ρ, $\rho_{min} = \frac{r_{min}^2}{P_{\tilde{r}_1}}$ and $\rho_{max} = \frac{r_{max}^2}{P_{\tilde{r}_1}}$. Without loss of generality, the minimum amplitude r_{min} is approximatively zero. Consequently we can take $\rho_{min} = 0$ and we get (8):

$$\tilde{m}_{2\_max}^{(prob)} = P_{\tilde{r}_1}\left(1 - 2^{\frac{b-a}{2ab}}\right)^2 \left(1 - (PAPR_{\tilde{r}_1}+1)\,e^{-PAPR_{\tilde{r}_1}}\right). \quad (8)$$

The PAPR of the signal $\tilde{x}_1(t)$ is denoted by $PAPR_{\tilde{r}_1}$ and is equal to the maximum value of ρ, $PAPR_{\tilde{r}_1} = \rho_{max}$.

2) For amplitude clipping: Amplitude clipping is one of the most used methods for PAPR reduction due to its simplicity and its reduction gains. The study of the clipped signal without filtering shows that its distribution $\upsilon\left(r\right)$ is given by [18]:

$$\upsilon\left(r\right) = p_r\left(r\right) 1_{r \le A_{clip}} + Pr\{r > A\}\delta\left(r - A_{clip}\right), \quad (9)$$

where $p_r(r)$ is the Probability Density Function (PDF) of the OFDM signal amplitude given by (5) and $\delta(r)$ the Dirac impulse. $Pr\{r > A_{clip}\}$ represents the probability that the OFDM signal amplitude r is larger than the clipping threshold A_{clip} and its expression is

$$Pr\{r > A_{clip}\} = \int_{A_{clip}}^{+\infty} p_r\left(r\right) dr = e^{\frac{-A_{clip}^2}{P_r}}. \quad (10)$$

When amplitude clipping technique is considered as PAPR reduction technique, the second order moment $m_2^{(clip)}$ and its upper bound $m_{2\,max}^{(clip)}$ of the predistortion error are calculated as follows:

$$m_2^{(clip)} \triangleq E\left[\left|\epsilon\left(\tilde{r}_1\right)\right|^2\right] = \int_{r_{min}}^{r_{max}} \left|\epsilon\left(r\right)\right|^2 \upsilon\left(r\right) dr. \quad (11)$$

After some maths [1],

$$m_{2\_max}^{(clip)} = \frac{P_{\tilde{r}_1}}{\gamma}\left(1 - 2^{\frac{b-a}{2ab}}\right)^2 \left[(\gamma\rho+1)\,e^{-\gamma\rho}\right]_{\rho=\rho_{clip}}^{\rho=\rho_{min}} + |\epsilon\left(A_{clip}\right)|^2 e^{-\gamma\rho_{clip}}, \quad (12)$$

with $\rho_{clip} = \frac{A_{clip}^2}{P_{\tilde{r}_1}} = PAPR_{\tilde{r}_1}$. The parameter $\gamma = \frac{P_{\tilde{r}_1}}{P_r} = 1 - e^{\frac{-A_{clip}^2}{P_r}}$ is the ratio between the mean power $P_{\tilde{r}_1}$ of the

signal after amplitude clipping and the mean power P_r of the OFDM signal. In particularly, if $\rho_{min} = 0$, (12) becomes

$$\tilde{m}_{2\_max}^{(clip)} = \frac{P_{\tilde{r}_1}}{\gamma}\chi\left(1 - (\gamma PAPR_{\tilde{r}_1}+1)\,e^{-\gamma PAPR_{\tilde{r}_1}}\right) + |\epsilon\left(A_{clip}\right)|^2 e^{-\gamma PAPR_{\tilde{r}_1}}, \quad (13)$$

with $\chi = \left(1 - 2^{\frac{b-a}{2ab}}\right)^2$.
Upper bounding $\epsilon(A_{clip})$ using (4), we get

$$\hat{m}_{2\_max}^{(clip)} = \frac{P_{\tilde{r}_1}}{\gamma}\left(1 - 2^{\frac{b-a}{2ab}}\right)^2 \left(1 - e^{-\gamma PAPR_{\tilde{r}_1}}\right), \quad (14)$$

After calculation of $m_2^{(prob)}$ and $m_2^{(clip)}$ of the predistortion error given in (8) and (14) corresponding respectively to the probabilistic PAPR reduction methods and amplitude clipping, the EVM expression is deducted.

C. Estimation of the amplified signal quality thanks to the EVM metric considering PAPR reduction and linearization

The EVM of the amplified signal is defined as the ratio of the Root Mean Square (RMS) of the predistorsion error to the root of the mean power of the signal after PAPR reduction. The EVM of $y(t)$ is given by

$$EVM = \sqrt{\frac{E\left[\left|\epsilon(\tilde{r}_1)\right|^2\right]}{E\left[\left|\tilde{x}_1(t)\right|^2\right]}} \quad (15)$$
$$= \sqrt{\frac{m_2}{P_{\tilde{r}_1}}}.$$

We see from (15) that the EVM expression is directly proportional to the second order moment m_2 of the predistortion error.

Let's first consider a probabilistic PAPR reduction method. Using (15) anf (8), EVM can be upper bounded by

$$EVM_{max}^{(prob)} = \left|1 - 2^{\frac{b-a}{2ab}}\right| \sqrt{1 - (PAPR_{\tilde{r}_1}+1)\,e^{-PAPR_{\tilde{r}_1}}}. \quad (16)$$

When amplitude clipping is considered as PAPR reduction technique, the EVM upper bounded is:

$$EVM_{max}^{(clip)} = \left|1 - 2^{\frac{b-a}{2ab}}\right| \sqrt{\left(\frac{1}{\gamma}\right)\left(1 - e^{-\gamma PAPR_{\tilde{r}_1}}\right)}. \quad (17)$$

From (16) and (17), we notice that the EVM is mainly influenced by the performance of the predistortion. Nevertheless a powerful PAPR reduction can slightly increase the linearity by decreasing the EVM value. When the "knee factors" are same ($a = b$, consequently to an ideal predistortion), a perfect linearity (the EVM is null) is ensured independently from the PAPR reduction technique. In practice, the predistortion is not ideal (a is different from b). In such conditions we notice the importance of the PAPR reduction as it improves the effectiveness of the predistortion in terms of EVM. In the literature [7]–[9], [15], [17], some simulations have already pointed out this but none in an analytical way.
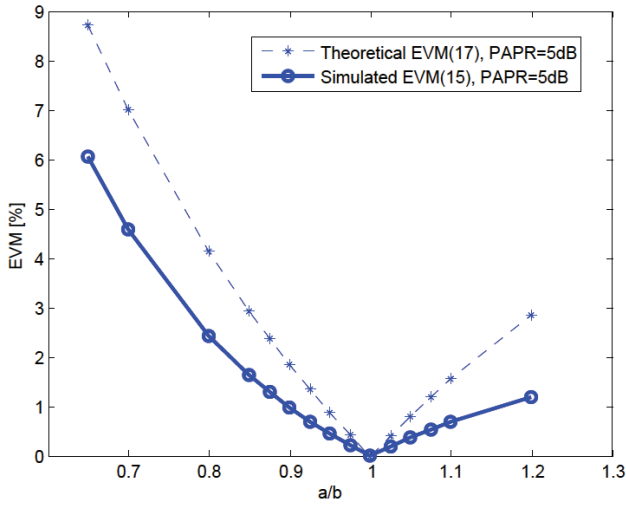
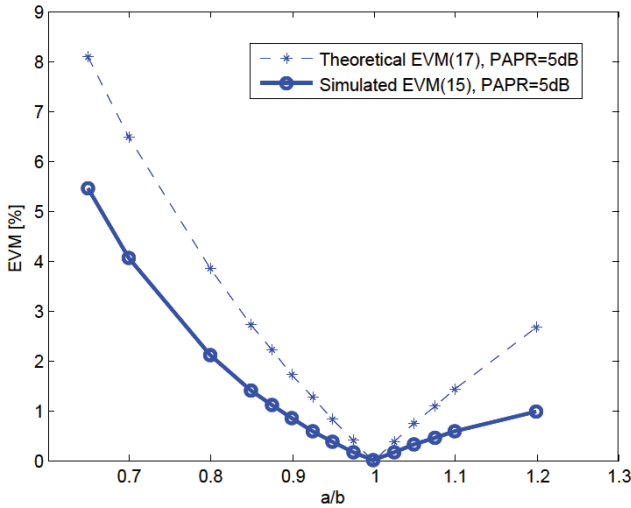Fig. 2. EVM metric depending of the "knee factors" ratio a/b when amplitude clipping is used as PAPR reduction technique

Fig. 4. EVM metric depending of $PAPR_{\tilde{r}_1}$ for $a/b = 0.65$, 0.875 and 0.975 when Amplitude Clipping is used

Fig. 3. EVM metric depending of the "knee factors" ratio a/b when SLM is used as PAPR reduction technique
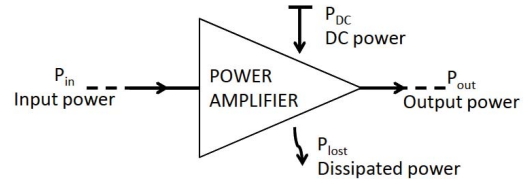
Fig. 5. Power budget of a power amplifier

Fig. 2, 3 and 4 illustrate (16) and (17). Each simulation considers 5.10^3 randomly generated OFDM symbols with 64 sub-carriers each 16-QAM modulated. Fig. 2 and 3 depict the theoretical EVM upper bound and its simulated values depending on the "knee factor" ratio $\frac{a}{b}$ respectively for Amplitude Clipping and SLM. The PAPR is reduced from 8dB to 5dB with both PAPR reduction technique. When $\frac{a}{b}$ tends to 1, EVM converges towards zero (high linearity). On the other hand, to show that the PAPR reduction has slight influence on EVM, let's take three values of $\frac{a}{b}$: 0.65, 0.875 and 0.975. Fig. 4 shows EVM depending on PAPR reduction performance. For $\frac{a}{b} = 0.875$ and 0.975, to decrease the EVM of 1%, it requires more than 5dB PAPR reduction gain. We can notice that when $\frac{a}{b}$ is close to 1, theoretical EVM is close to simulation but the more a is different from b, the more (16) and (17) are far from simulation, accordingly to the upper bound (4).
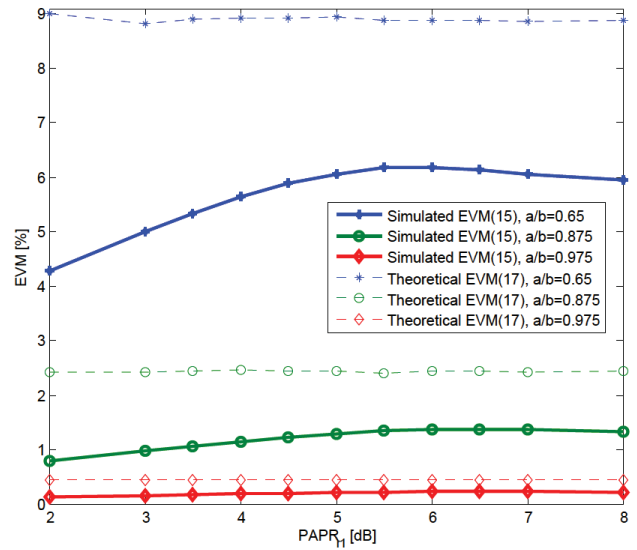
III. PA EFFICIENCY CONSIDERING PAPR REDUCTION AND PREDISTORTION

Environmental or "green" issues, such as the need to reduce both direct and indirect CO_2 emissions, and equipment size, are nowadays up-to-date concerns. Several operators have pledged to work with suppliers to increase the energy efficiency of their networks but this is mainly feasible if PA efficiency is greatly improved. Hence, an improvement in PA efficiency will boost the batteries lifetime of mobiles. Improving PA efficiency for high PAPR systems like OFDM is particularly challenging due to the need to use linear PAs to meet the critical RF performance criteria.

In the previous section, we have analytically shown that, the PA's linearity measured by the EVM metric depends on the performance of predistortion and PAPR reduction. In this section, we characterize likewise the efficiency of the PA depending on the performance of the PAPR reduction and also on predistortion.

A. Definition of PA efficiency

PA is a key component of wireless RF transmitters and consumes the major part of the total power. The main characteristic of the PA is that it dissipates power whatever the amplitude of the input signal. Referring to Fig. 5, the power efficiency is defined as the ratio of the output power to the

TABLE I
PARAMETERS OF POWER EFFICIENCY EXPRESSIONS

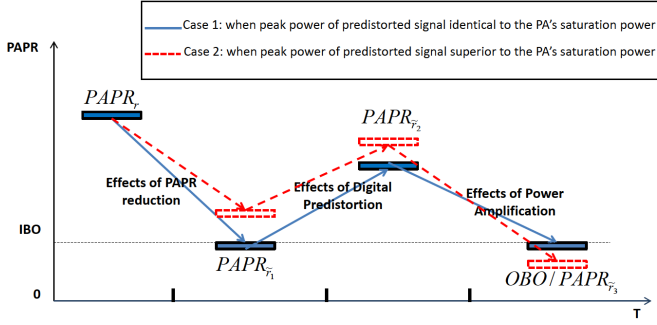| Class	G [%]	g
A	58.7	0.1247
B	90.7	0.1202

Fig. 6. Evolutions of signal's PAPR in OFDM transmitter

Fig. 7. Power efficiency depending of $PAPR_{\tilde{r}_1}$ for classes A and B PA

DC power as in (18):

$$\eta_{DC} = \frac{P_{out}}{P_{DC}}. \tag{18}$$

The maximum efficiency is achieved at the maximum amplitude of the linear output signal. This efficiency mainly depends on the input signal PAPR as shown in the next subsection but also on the PA class. η_{DC} cannot exceed 50% and 25% for class A and B power amplifiers respectively when operating in linear conditions.

B. Expression of the power efficiency considering PAPR reduction and predistortion

Let's assume that a PAPR reduction technique is used followed by a predistortion before the PA as in Fig. 1. The power efficiency is conversely proportional to the PAPR of the PA input signal whose relationship depends on the class of the PA and on its particular design. Authors in [11], [12] have investigated this relationship for linear PA. A theoretical efficiency upper bound for classes A and B is given by:

$$\eta_{DC} = G \cdot exp\left(-g \cdot PAPR_{\tilde{r}_1}\right), \tag{19}$$

where the efficiency η_{DC} is in % and $PAPR_{\tilde{r}_1}$ in dB. The values of G and g are given in Table I. It it's assumed that for class B PA non-linear distortions due to cross-over are negligibility small.

Fig. 6 shows the evolution of the signal's PAPR through the transmission chain depicted in Fig. 1. For inherent reasons, the OFDM signal has a high PAPR named $PAPR_r$. Thus a PAPR reduction technique is required to increase the efficiency. The signal after the PAPR reduction technique has a lower PAPR namely $PAPR_{\tilde{r}_1}$ and $\Delta = PAPR_r - PAPR_{\tilde{r}_1}$ is the PAPR reduction gain of the used technique. In [3], the authors have proved that predistortion increases the PAPR of the signal. Assuming that $PAPR_{\tilde{r}_2}$ is the PAPR of the signal after predistortion, $PAPR_{\tilde{r}_2} > PAPR_{\tilde{r}_1}$. The PA can not be used above the saturation's power, even considering
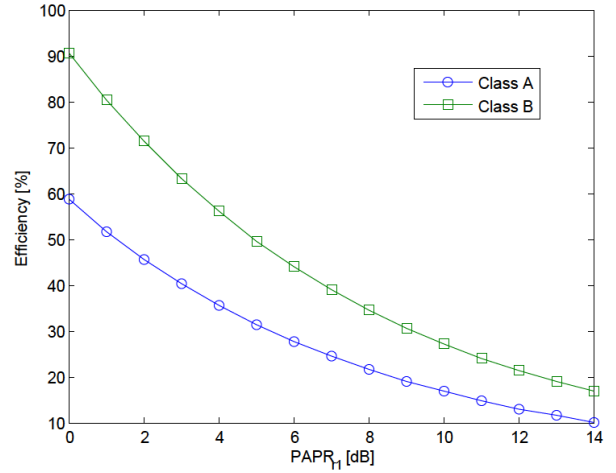
predistortion. Consequently, we need to consider two cases. In case 1, represented by full-lines in Fig. 6, the peak power of the predistorted signal is equal to the saturation power of the PA. The signal is then amplified with high linearity and the amplified signal is similar to the signal after PAPR reduction, so $PAPR_{\tilde{r}_3} = PAPR_{\tilde{r}_1}$. In these conditions, the efficiency is maximum.

Conversely in case 2, represented by dashed-lines in Fig. 6, the peak power of the predistorted signal is larger that the saturation power of the PA. This can be due to a non efficient PAPR reduction method or a bad PA design. In these conditions, the amplifier saturates and acts as a limiter, so $PAPR_{\tilde{r}_3} \leq PAPR_{\tilde{r}_1}$. The amplified signal is thus distorted and is no more similar to the signal after PAPR reduction.

Fig. 7 illustrates (19) for classes A and B amplifiers. We can observe for example that for both amplifier class, increasing the efficiency from 20% to 30% requires a 3dB PAPR reduction gain.

From Fig. 6 and 7, we can noticed that power efficiency mainly depends on the performance of the PAPR reduction technique, but it is important to avoid PA saturation in order to remain linear. The maximum possible power efficiency avoiding PA saturation is achieved when peak power of the amplified signal coincides with the saturation power. In other words, the Input Back-Off (IBO) of the PA must be set identical to the PAPR of the signal before predistortion in order to keep the maximum power efficiency gained by the PAPR reduction.

IV. JOINT COMBINATION OF PAPR REDUCTION AND PREDISTORTION

As explained before, PA's linearity and its efficiency are mutually exclusive and cannot be achieved simultaneously. So the main challenge of designers is to achieve an optimal trade-off by applying PAPR reduction and linearization techniques. In literature [2], [10], predistortion has been proposed to improve PA's linearity while PAPR reduction increases its efficiency.Nevertheless these two techniques are studied and

applied independently. In the previous section, we have shown analytically their interdependency; the linearity (measured in this paper by EVM) and the efficiency are both influenced by PAPR reduction and predistortion. Our objective in this section is to highlight the conditions of an optimal combination of these two techniques to find a trade-off between the PA linearity and the efficiency.

A. Discussion and analysis

From (16) and (17), the maximum linearity corresponding to minimum EVM value is achieved by a perfect predistortion, $a = b$ or a high gain PAPR reduction if $a \neq b$.

From (19), the maximum efficiency of the PA is achieved under two conditions. The first one is to ensure a linear amplification by fixing the PAPR of the signal before predistortion equal to the IBO of the PA. The second condition is to use an effective PAPR reduction technique.

Substituting (19) in (16) and (17), we can carried out a theoretical relationship between the EVM and the PA efficiency depending on the performance of the predistortion. When SLM is considered for example, this relation is :

$$EVM_{max}^{(prob)} = \left| 1 - 2^{\frac{b-a}{2ab}} \right| \sqrt{1 - \left(\frac{\eta_{DC}}{G} \right)^{\frac{1}{g}} \left(1 - \frac{1}{g} ln \left(\frac{\eta_{DC}}{G} \right) \right)}, \tag{20}$$

and illustrated on Fig. 8. We can notice the mutual influence between the linearity and the efficiency. When the performance of the predistortion is improved, for example from $\frac{a}{b} = 0.65$ to 0.975, the linearity is likewise improved and EVM decrease from 9% to 0.5%. At the same time, for a given "knee factors" ratio $\frac{a}{b}$, the increasing of the power efficiency, from 0% to 50%, has a slight influence on the linearity as the EVM decreases of 3.5% for $\frac{a}{b} = 0.65$ and less than 1% for $\frac{a}{b} = 0.875$ and 0.975. The reason is that an increase of efficiency requires a large PAPR reduction gain. Regardless of the signal degradation caused by PAPR reduction, the PA linearity is improved as well as EVM that is measured here between the amplified signal and the signal after PAPR reduction.

In all cases, we can notice that an effective PAPR reduction is required. Most of time, a high PAPR reduction gain induces a complex technique, additional distortions or a need of side information transmission. The PAPR reduction technique could have minimum drawbacks. Many PAPR reduction techniques exist in literature [2], [10], [19], a classification can be found in [2]. However, when signal's coding or modulations parameters cannot be modified and when a downward compatibility is targeted, adding signal techniques can be considered.

The main goal of our trade-off analysis is to ensure good linearity with reasonable efficiency in order to satisfy specifications. From above, we can conclude that an optimal combination of PAPR reduction and predistortion is achieved under the following conditions, namely:

- the PAPR reduction technique must have high reduction gain and minimum drawbacks. We suggest signal adding
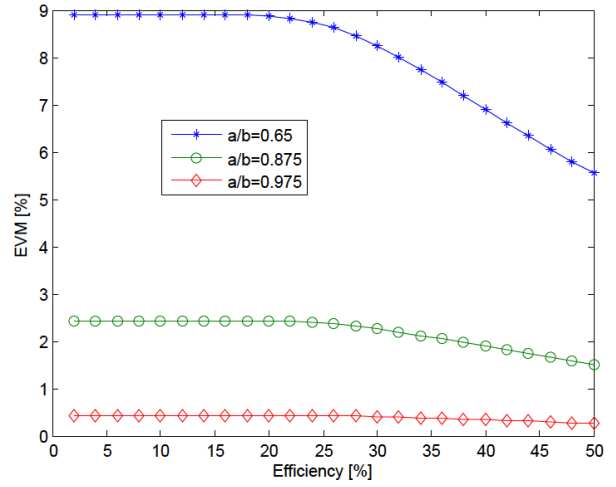

Fig. 8. EVM metric depending on the power efficiency of class A PA for different values of the "knee factors" ratio a/b when SLM is considered as PAPR reduction technique

technique that are promising with high PAPR reduction gain and no BER degradation [2];
- predistortion must be effective and designed by taking into account PAPR reduction technique. Its parameters must be calculate accordingly;
- fix the IBO of the PA equal to the PAPR of the signal before predistortion. That means that the average PAPR of the signal after PAPR reduction must be known or upper bounded in order to avoid PA saturation and ensure a maximal efficiency;
- Adaptive predistortion can be considered by setting a feedback from the PA.

B. Simulations and Results

We have investigated the IEEE802.11 system with the class A SSPA power amplifier whose "knee factor" $b = 2$. The perfect linearity is ensured by predistortion with "knee factor" $a = b$. According to our trade-off analysis, the maximum possible efficiency is achieved by setting the IBO equal to the PAPR of the signal after the PAPR reduction technique. Therefore, besides some few PAPR reduction techniques like the coding techniques based on Reed-Muller codes [10] or clipping, the PAPR of the signal after PAPR reduction is not fixed. So we propose to fix a targeted PAPR and iterate the PAPR reduction technique until satisfaction. The IBO is then set to this targeted PAPR. For our simulations, we consider two PAPR reductions techniques: Amplitude Clipping [19] and Selective Mapping (SLM) [10]. As the mean PAPR of the OFDM signal is around 8dB before PAPR reduction, we fix the targeted PAPR to $PAPR_{target} = 5$dB after PAPR reduction. Like above, each simulation considers 5.10^3 randomly generated OFDM symbols with 64 sub-carriers 16-QAM modulated.

Fig. 9 shows the evolution of the EVM metric measured between the amplified signal and the PAPR reduced signal for different IBO when predistorter is optimal, $a = 2$. We
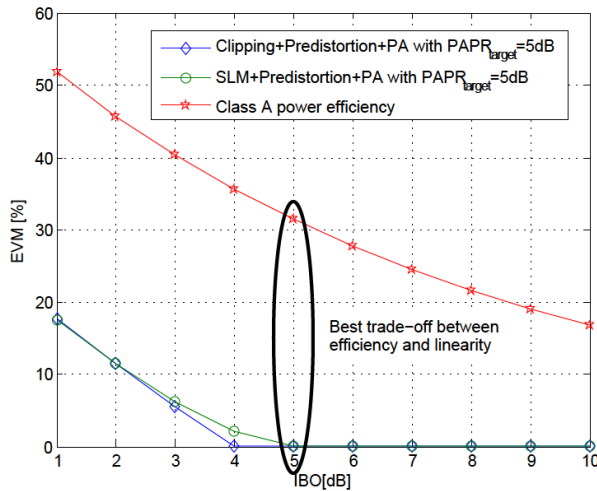
Fig. 9. Optimal trade-off between linearity and power efficiency when amplitude clipping and SLM are used with predistortion and SSPA Class A power amplifier.

observe that the $IBO = 5$dB, corresponding to the targeted PAPR, could be a good trade-off between the linearity and the power efficiency. When $IBO < 5$dB, the EVM increases from 0 to 17% for both SLM and Amplitude Clipping. This means that the linearity is degraded while the power efficiency is improved. When $IBO > 5dB$, the EVM is minimal, 0%, what means that the highest linearity is achieved while the efficiency is decreasing.

We precise that a higher efficiency can be achieved by fixing lower targeted PAPR. This implies a higher complexity in PAPR reduction but with the up growing of the calculation power of handsets or base stations using DSP (Digital Signal Processor) and FPGA (Field-Programmable Gate Array), this solution can be seriously envisaged.

V. CONCLUSION

This paper has explored a theoretical analysis of the trade-off between linearity measured by EVM metric and the efficiency in OFDM context. Analytical expressions of the EVM and the power efficiency for a memoryless SSPA have been formulated based on systems where PAPR reduction technique is followed by a predistortion before the non-linear PA. The validity of the theoretical expressions has been shown through simulations for Amplitude Clipping and SLM techniques. Some others simulation results present the performance improvement of the PA by our proposed trade-off. Thus the performance of an OFDM system with non-linear PA where a PAPR reduction technique is followed by a predistortion can be estimated theoretically without the need to perform extensive simulations. Some other configurations can be envisaged taking into account the memory effects of the PA or considering other linearity metrics like Adjacent Channel Power Ratio (ACPR). This will be the subject of our future work.

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