Optimal Reference Trajectories for Walking and Running of a Biped Robot

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Abstract :

The objective of this study is to obtain optimal cyclic gaits for a biped robot without actuated ankles.

Two types of motion are studied: walking and running. For the walking, the gait is composed uniquely of successive single support phases and instantaneous double support phases that are modelled by passive impact equations. The legs swap their roles from one single support phase to the next one. For the running, the gait is composed of stance phases and flight phases. A passive impact with the ground exists at the end of flight.

During each phase the evolution of m joints variables is assumed to be polynomial functions, m is the number of actuators. The evolution of the other variables is deduced from the dynamic model of the biped. The coefficients of the polynomial functions are chosen to optimise criteria and to insure cyclic motion of the biped. The chosen criteria are: maximal advance velocity, minimal torque, and minimal energy.

Furthermore, the optimal gait is defined with respect to given performances of actuators: The torques and velocities at the output of the gear box are bounded. For this study, the physical parameters of a prototype, which is under construction, are used. Optimal walking and running are defined. The running is more efficient for high velocities than the walking with respect to the studied criteria.

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1 Introduction

Much work has been devoted to the anthropomorphic biped robot, like [1-14]. The design of reference trajectories for gait cycles of biped is important and not trivial. Several techniques have been adapted to define reference trajectories. Like many authors [1, 2, 4, 5, 11, 12, 15], we put our interest on low-energy trajectories for biped robots. We search for a periodic trajectory that fulfils a certain objective, in terms of motion velocity, while minimising the input energy needed to produce such a gait. In general, this open and non-trivial problem is solved by finding numerical solutions.

The choice of optimisation parameters is not unique. The torques, the Cartesian coordinates or joint coordinates can be used. In [12], the optimisation parameters are piecewise constant torques. Pontryagin's principle is used to find the torques to design impactless gaits in [14]. The torques are defined by parameterised functions in [15]. However it is necessary, when the torque is an optimised variable, to solve the inverse dynamic problem to find the joint coordinates in order to insure cyclic gaits. Different authors in [1, 2, 4, 5] have used polynomial functions for the Cartesian coordinates of the hip, swing foot and the trunk angle. The coefficients of the polynomials are optimisation parameters. Therefore it is necessary to inverse the geometric robot model to define the joint reference trajectories. To avoid the use of inverse dynamic model and inverse geometric model, we prefer to choose the joint coordinates as optimised variables. In [11], it is also mentioned that the joint coordinates have to be chosen because their dynamics are relatively slow with respect to the actuator dynamics.

We use polynomial functions for the joint leg coordinates in this paper, to limit the number of optimisation parameters. The coefficients of polynomials are the optimisation variables.

Our objective is to obtain cyclic gaits for a biped robot. Two kinds of robots can be distinguished: robots that are under-actuated during single support phase (the robot with non actuated ankle belongs to this family), and robot completely actuated during single support phase (robot with actuated ankle and with all the stance foot sole on the ground). In the first case, the ankle torque is null. In the last case, the size of the feet is generally limited, which induces limits on the torque that can be produced on the ankle. Therefore if the reference trajectory is defined with null torque in the ankle, the torque available in the ankle joint allows to follow correctly the reference trajectory in the presence of disturbances. Thus, for all bipeds, it is interesting to be able to define cyclic optimal motions with null torque at the ankle. This problem is treated in this paper. In this case, the number of configuration variables is greater than the number of actuators during single support or flight phases.

In paper [1], this type of biped robot is also studied for walking motion only. Null velocity of the swing leg tip on the ground is specified at the impact time to avoid impact. However high joint torques values are usually needed to achieve this specification. In our study we will not impose such a condition. An inelastic impact of the swing leg with the ground is taken into account.

Many authors define trajectories with minimal energy but the criteria used can be different [4, 11, 14]. We will consider different criteria here: minimisation of torque norm, minimisation of the energy, maximisation of the motion velocity of the robot. Furthermore, some constraints such as actuator performances and limits on the ground reaction force are taken into account.

Section 2 presents the dynamic model of the biped. Sections 3 and 5 are devoted to the formulation of the optimisation problem for walking and running. In sections 4 and 6 we present the simulation results. Section 7 gives the conclusions and further extensions.

2 Dynamic Model

Our biped walks in a vertical sagittal xy plane. It is composed of a trunk and two identical legs. Each leg is composed of two links articulated with a knee. The knees and hips are one-degree-of-freedom rotational joint (Figure 1).

Let us define vector $\mathbf{X} = (x, y, \theta_1, \theta_2, \theta_3, \theta_4, \Psi)^*$ of seven generalised coordinates for describing the biped in the xy plane. It contains two coordinates for a point of the trunk x, y (this point is located at the hip joints), and five for the orientation of the legs and the trunk $\mathbf{q} = (\theta_1, \theta_2, \theta_3, \theta_4, \Psi)^*$, (Fig. 1). These variables allow to describe stance and flight phases. The vector Θ is defined by $\Theta = (\theta_1, \theta_2, \theta_3, \theta_4)^*$.All links are assumed massive and rigid. The lengths of the thighs and the shins are *0.4 m*. However, their masses are different: *6 Kg*, for the thigh and *4 Kg* for the shin. The length of the trunk is *0.625 m* and its mass is *20 Kg*. The inertia of the links are also taken into account. Let vector $\mathbf{\Gamma} = (\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4)^*$ describe the torques applied in the hip and knee joints (Fig. 1). Let $\mathbf{R}_1$ (R_{1x}, R_{1y}) and $\mathbf{R}_2$ (R_{2x}, R_{2y}) be the forces applied to the leg tips.

In stance phase on leg j (j=1 or 2), the ground exerts the force $\mathbf{R}_j$, so the motion equations of the biped have the following form :

$$\mathbf{A(q)\ddot{X} + H(q, \dot{q}) = D}_G\Gamma + \mathbf{D}_j\mathbf{(q)R}_j \quad (1)$$

where $\mathbf{A}$(7 x 7) is the inertia matrix, $\mathbf{H}$(7 x 1) is the vector of Coriolis, centrifugal and gravity effects, $\mathbf{D}_j$ (7 x 2), and $\mathbf{D}_G$ (7 x 4) allow to take into account the effects of the external force and torques.

The stance leg tip does not move, so its velocity and acceleration are zero:

$$\mathbf{D}_j(\mathbf{q})^* \dot{\mathbf{X}} = \mathbf{0} \qquad \mathbf{D}_j(\mathbf{q})^* \ddot{\mathbf{X}} + \mathbf{H}_j(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{0}. \tag{2}$$

Tacking into account constraint equations (2), there are five degrees of freedom in stance phase and there are four actuators. Thus, a relation between the evolution of the accelerations, velocities and generalised coordinates of the biped that does not depend on the torques, can be written (the index s denotes the stance phase):

$$f_s(q, \dot{q}, \ddot{q}) = 0 \tag{3}$$

A possible way to find this relation is to remark that the kinetic momentum of the robot written around the stance leg tip depends only on the gravity effects (see section 3.3.2).

During flight, no external force acts so the dynamic model is reduced to:

$$\mathbf{A}(\mathbf{q})\ddot{\mathbf{X}} + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{D}_G \Gamma \tag{4}$$

The configuration position is described by seven variables and only four torques exist.

Since only the gravity acts on the robot, the horizontal acceleration of the mass centre of the robot is zero and its vertical acceleration is equal to the gravity acceleration,

$$\ddot{x}_c = 0, \quad \ddot{y}_c = -g \tag{5}$$

where x_c and y_c are the coordinates of the mass centre of the robot, g is the gravity acceleration.

Since there is no external torque applied around the mass centre of the robot, another relation between the evolution of the accelerations, velocities and positions of the configurations variables that does not depend on the torques can also be defined (the index u denotes the flight or unsupported phase).

$$f_u(\mathbf{X}, \dot{\mathbf{X}}, \ddot{\mathbf{X}}) = 0 \tag{6}$$

This relation follows from equation (4) and also from the theorem on the changing of total kinetic momentum around the mass centre of the biped.

3 Gait optimisation method for the walking

The gait studied is composed of stance phases. A passive impact exists at the end of the half step. The legs swap their roles from one half step to the next one.

3.1 Definition of polynomial functions for the legs

The robot is driven by four torques, so the evolution of four independent variables only can be chosen. We choose to define the evolution of the four absolute joint angles of the legs $\mathbf{\Theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$, because this choice allows to define the length and duration of an half step independently of the trunk motion. We choose time polynomial functions to describe the evolution of the orientation of the leg links Θ. The coefficients of these polynomials are used as optimisation parameters. The duration of an half step is denoted T.

It is not possible to choose also the evolution of the absolute rotation of the trunk Ψ as a polynomial function because the reference trajectory must satisfy eq. (3). When $\Theta(t)$ is given, the evolution of $\Psi(t)$ can be calculated as it will be detailed in section 3.3.2.

To insure continuity between two successive steps, the position and velocity of the biped at the beginning and end of each phase must be taken into account by the parameters of the polynomial functions. So, third-order polynomial functions are needed. In some cases, this minimal order would lead to unexpected collision of the swing leg tip with the ground. To avoid this, we define an intermediate configuration for the biped at the time T/2. In conclusion, each of the four joint variables is defined by a fourth-order polynomial function of time,

$$\theta_j(t) = a_{j0} + a_{j1}\, t + a_{j2}\, t^2 + a_{j3}\, t^3 + a_{j4}\, t^4 \qquad \text{with } j=1\ldots4\text{, the number of the joint} \tag{7}$$

these functions allow to connect specified initial and final configurations and velocities and the intermediate configuration ($\Theta_i, \dot{\Theta}_i, \Theta_f, \dot{\Theta}_f$ and Θ_{int}) of the robot legs. The indices i, f and int correspond to the initial (at t=0), final (at t = T) and intermediate (at t=T/2) state of the robot respectively. The polynomial functions θ_j (j=1…4) are uniquely defined using $\Theta_i, \dot{\Theta}_i, \Theta_f, \dot{\Theta}_f, \Theta_{int}$ and T.

In fact, the initial and final configurations for the stance phase are double support configurations. The two leg tips are in contact with the horizontal surface. If the position of the second leg tip with respect to the first one is defined via the hip position, this condition can be written:

$$\begin{cases} 0.4\sin(\theta_{1i}) + 0.4\sin(\theta_{2i}) - 0.4\sin(\theta_{3i}) - 0.4\sin(\theta_{4i}) = 0 \\ 0.4\sin(\theta_{1f}) + 0.4\sin(\theta_{2f}) - 0.4\sin(\theta_{3f}) - 0.4\sin(\theta_{4f}) = 0 \end{cases}$$

Thus only 3 independent variables are necessary to define the initial and final configurations Θ_i and Θ_f of the biped legs.

3.2 Motion during the single support phase

Since the robot is in single support, the coordinates x, y of the trunk and their derivatives can be deduced using the hypotheses on the behaviour of the stance leg tip (2) and the motion of the legs. The dynamic model (especially equation (3)) is then used to define the evolution of the orientation Ψ of the trunk. This point will be detailed in section 3.3.2. The equation (3) allows to define the acceleration of the trunk. Therefore the orientation and angular velocity of the trunk can be deduced, if they are known at a given time. We choose to define their evolution as function of the orientation and angular velocity of the trunk at the end of the stance phase noted: $\Psi_f, \dot{\Psi}_f$.

In summary, if the motions of the legs are defined and $\Psi_f, \dot{\Psi}_f$ are known, the evolution of **X** and its derivatives can be deduced with (2) and (3). Then the torques and reaction forces that allow to produce the motion can be defined by the dynamic model (1). This dynamic model is a system of seven equations and six unknowns: four torques and two components for the stance leg reaction force. However since the evolution of the trunk satisfies (3), this system is compatible and admits one unique solution. The criterion and constraints can be evaluated.

3.3 Definition of periodic motion for the walk

In fact, the desired trajectory has the particularity to be cyclic : two following steps must be identical and, more precisely, the legs will swap their roles from one half step to the next. The condition of periodicity is used to define the trajectory only on one half step and to reduce the number of optimisation parameters.

The ankle of the robot studied is not actuated so the number of configuration variables is higher than the number of actuators. The evolution of the trunk is written as a function of the parameters describing the motion of the legs. To obtain cyclic motion, the trunk itself must have a periodic motion. In section 3.3.2 this condition will be written as an equality constraint. And the parameters describing the motion of the legs must satisfy this equality constraint to produce a cyclic motion of the robot.

3.3.1 Continuity for successive single support phases

Since the position of the robot is constant during the instantaneous passive impact (touch down configuration) and since the legs swap their roles from one half step to the next we have:

$$\theta_{1i} = \theta_{4f},\ \theta_{2i} = \theta_{3f}, \theta_{3i} = \theta_{2f}, \theta_{4i} = \theta_{1f} \tag{9}$$

and $$\Psi_i = \Psi_f \tag{10}$$

We assume that the passive impact phase is such that the leg noted "k" which comes in contact has an inelastic impact and does not slide, and that the leg noted " j" which was previously in contact takes off.

The equation of impact can be obtained by integration of (1) during the instantaneous impact. The impact is passive so no impulsive torque is applied:

$$\mathbf{A(q)}(\dot{\mathbf{X}}^+ - \dot{\mathbf{X}}^-) = \mathbf{D}_k \mathbf{I}_{Rk}, \tag{11}$$

where $\mathbf{I}_{\mathbf{Rk}}$ is the impulsive reaction, $\dot{\mathbf{X}}^-$ and $\dot{\mathbf{X}}^+$ are the velocity vectors before and after the passive impact on the ground.

The leg k has an inelastic impact with no sliding, so the velocity of its leg tip just after impact is zero:

$$\mathbf{D}_k^* \dot{\mathbf{X}}^+ = \mathbf{0}, \ \text{k=1 or 2.} \tag{12}$$

Associated with this set of equations, some constraints must be satisfied, the impact $\mathbf{I}_{Rk}$ must be directed upward and in the friction cone, the velocity of the leg tip j ($j \neq k$) must be directed upward [16].

The knowledge of the velocity of the robot before the impact allows to define the velocity just after the impact using equations (11), and (12). Thus the velocity of the robot after the impact can be defined as function of $\Theta_f, \dot{\Theta}_f, \Psi_f, \dot{\Psi}_f$.

According to the gait chosen, (stance phase with exchange of the legs between two half steps), the knowledge of the velocity before impact, or at the end of the stance phase, allows to define the velocity at the beginning of the stance phase. Therefore, we can write $\dot{\Theta}_i$ and $\dot{\Psi}_i$ as function of $\Theta_f, \dot{\Theta}_f, \Psi_f, \dot{\Psi}_f$:

$$\dot{\Theta}_i = g_\Theta(\Theta_f, \dot{\Theta}_f, \Psi_f, \dot{\Psi}_f) \tag{13}$$

$$\dot{\Psi}_i = g_\Psi(\Theta_f, \dot{\Theta}_f, \Psi_f, \dot{\Psi}_f) \tag{14}$$

Using (9) and (13), the polynomial functions θ_j (j=1…4), can be defined as function of ($\Theta_f, \dot{\Theta}_f, \Theta_{int}, \Psi_f, \dot{\Psi}_f$ and T).

3.3.2 Evolution of the trunk

For a given evolution of the joint variables $\Theta(t)$, the behaviour of the trunk can be defined by a dynamic equation (3), which is independent of the actuator torques: the kinetic momentum of the robot written around the stance leg tip (noted S) depends on the gravity effects only.

Let denote by σ the total kinetic momentum around the stance leg tip S. The kinetic momentum is a linear combination of vector $\dot{\mathbf{X}}$ components and namely of $\dot{\Psi}(t)$ and $\dot{\Theta}(t)$ components with coefficients which are function of $\Psi(t)$ and $\Theta(t)$:

$$\sigma = F_{11}(\Psi(t), \Theta(t))\,\dot{\Psi}(t) + F_{12}(\Psi(t), \Theta(t))\,\dot{\Theta}(t) \tag{15}$$

The external forces are the gravity effects and ground reaction force applied on S. Therefore the theorem on the total kinetic momentum can be written:

$$\dot{\sigma} = -Mg(x_c(\Theta(t), \psi(t)) - x_s) \tag{16}$$

where M is the mass of the biped, x_c its mass centre abscissa and x_s is the abscissa of the stance leg tip S.

The polynomial evolution for the variable $\Theta(t)$ has been defined in the previous section, so the derivation of expression (15) and equation (16) allows to define the acceleration of the trunk as:

$$\ddot{\Psi}(t) = F_1(\Psi(t), \dot{\Psi}(t), \Theta_f, \dot{\Theta}_f, \Theta_{int}, \Psi_f, \dot{\Psi}_f, T, t) \tag{17}$$

This equation can be integrated numerically, if the initial position and velocity of the trunk are known. So from equation (17), the position and velocity of the trunk can be calculated as a function of time t:

$$\dot{\Psi}(t) = F_2(\Psi_i, \dot{\Psi}_i, \Theta_f, \dot{\Theta}_f, \Theta_{int}, \Psi_f, \dot{\Psi}_f, T, t) \tag{18}$$

$$\Psi(t) = F_3(\Psi_i, \dot{\Psi}_i, \Theta_f, \dot{\Theta}_f, \Theta_{int}, \Psi_f, \dot{\Psi}_f, T, t) \tag{19}$$

These two equations can be simplified when Ψ_i and $\dot{\Psi}_i$ are replaced by their respective values given by equations (10) and (14).

The defined trajectory is cyclic, if and only if these two equations, evaluated at t = T, give the expected orientation and angular velocity of the trunk:

$$\dot{\Psi}_f = F_4(\Theta_f, \dot{\Theta}_f, \Theta_{int}, \Psi_f, \dot{\Psi}_f, T) \tag{20}$$

$$\Psi_f = F_5(\Theta_f, \dot{\Theta}_f,\ \Theta_{int}, \Psi_f, \dot{\Psi}_f, T) \tag{21}$$

To conclude, the evolution of the robot is completely defined by the values of $\Theta_f, \dot{\Theta}_f,\ \Theta_{int}, \Psi_f, \dot{\Psi}_f$ and T, but these parameters must be chosen in order to satisfy (20) and (21). This problem is solved numerically.

3.4 Definition of an optimal motion

The context of this study is to define an optimal trajectory for a given robot with given actuators. In particular the constraints considered on the actuators are the maximum torques (**Γmax**) and velocities (**Vmax**).

The limited velocity is the velocity of the actuator, or the relative velocity between two connected links. The effect of the speed reducer, when are exist, must be taken into account. This constraint can be written:

$|\mathbf{S\dot{q}}| < \mathbf{N^{-1}Vmax}$ where **Vmax** is a column vector of manufacturer's data, **N** is the diagonal matrix of gear ratio, $\mathbf{S\dot{q}}$ is the joint velocity and $\mathbf{S} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 \end{pmatrix}$

The constraint on the torque must take into account the effect of the speed reducer. It can be written:

$|\mathbf{\Gamma}| <$ **N Γmax**, where **Γmax** is a column vector of manufacturer's data

Some other constraints must be checked to insure that the optimal trajectory is convenient:

- the reaction force must be directed upward
- the ratio between the horizontal component and vertical component of the ground reaction force must be less than the friction coefficient
- the swing leg tip must not touch the ground before T
- some constraints corresponding to the validity of the impact are also taken into account.

All these constraints can be easily written as inequality conditions.

In the optimisation process, three criteria are considered:

- The motion velocity defined by: $C1 = \dfrac{\text{displacement along x for one step}}{T}$: $C1 = \dfrac{x\,(t=T)\text{-}x(t=0)}{T}$ (22)

- The torque cost is defined as the integral of the norm of the torque [12] for a displacement of one meter:

$$C2 = \frac{1}{x(t=T)-x(t=0)}\left(\int_0^T \Gamma^* \Gamma dt\right) \qquad (23)$$

- The energy cost defined as the integral of the absolute value of the work of external forces [7] for a displacement of one meter: $C3 = \dfrac{1}{x(t=T)-x(t=0)}\displaystyle\int_0^T |\Gamma|^* |S\dot{q}|\, dt$ (24)

For a sufficiently high velocity, the torque and work required to produce a motion increase with the advance velocity. The maximisation of criterion C1 allows to define the fastest displacement of the robot that can be obtained with given characteristics of the robot. In electrical motor, neglecting the friction, and for a cycle of walk, most part of the energy consumption is due to the loss by Joule effect. Criterion C2 is proportional to this loss of energy. It characterises the energy that must be produced by the battery to allow the motion. The criterion C3 is less dependent on the driving actuator. It characterises the variation of mechanical energy of the system. It is defined by assuming that the negative work produced by slow down of an actuator cannot be used by an other one or during the acceleration phase. No brake is used so the negative work must be produced by the actuators. Therefore the absolute value of the work is considered.

Criteria C2 and C3 can be optimised for a given motion velocity of the robot.

To verify that these inequality constraints are satisfied, the torques required to generate the optimal trajectory must be determined. At each time, the configuration of the leg $\mathbf{\Theta}$(t) and its derivative can be evaluated using (7); the evolution of the trunk can be calculated with (18), (19). Since the acceleration of the leg tip is null, the system of equations (2) allows to define $\ddot{x}, \ddot{y}$ and then $\ddot{\mathbf{X}}$(t). Then the torques and reaction forces, $\Gamma(t)$ and $\mathbf{R}_j(t)$ (j is the number of the leg in support) are computed with the dynamic model (1). The dynamic model is a system of seven equations and six unknowns, but since the evolution of the trunk satisfies (3) this system is compatible and admits one unique solution. The criterion and constraints can be evaluated. The algorithm *constr* from the package *Matlab* is used to solve these optimisation problems with the equality constraints given by (20) and (21) and the inequality constraints defined in this section. Some optimal trajectories are presented in the next section for walking. The optimisation process defines fourteen parameters: four for the final single support configuration of the robot (three for the leg which are in double support configuration and one for the trunk), five for the final velocity of the robot, four for the intermediate configuration of the leg, and one for the duration of the motion.

4 Simulation Results

4.1 Constraints and limitations

In the presented study, the four actuators are identical. They can produce a maximal torque of 3.0 Nm and rotate with a maximal velocity of 4000 rev/min. The gear ratio is 50. In fact an actuator cannot produce simultaneously high torque and high velocity. The characteristics given in figure 2 are taken into account in our optimisation tests.

The minimal vertical reaction force is 100 N, and the friction coefficient is 2/3.

The evolution of the free leg tip must be higher than a parabolic function given in figures 3 to 5. The maximum of this parabolic function is 1 cm. The space of research of the optimal parameter is limited: the duration of motion is between 0.2s and 0.7s. The displacement along x for an half step is between 0.2 m and 1 m, the initial orientation of the trunk is between 80° and 90°.

4.2 Walk with minimisation of the torque cost C2

The chosen motion velocity for the biped is 0.75 m/s (2.7 km/h). This motion velocity is a rough estimation for human children gait. The optimal walk has the following characteristics: for one half step, the duration T is 0.65 s, the displacement of the biped along x is 0.48 m. The value of the torque cost criterion C2 is 871 N^2ms.

The figures 3 to 8 present some optimal motions. Each figure regroups (i) the torque after the gear reducer (in N.m) and the components of the ground reaction in the stance leg tip (in N) as function of time (in second), (ii) the stick-diagram of one step of optimal walk (the units are meters), (iii) the corresponding behaviour of the actuator (speed in revolution per minute versus torque in Newton) for the knee and hip actuators, the motion of the swing leg tip (continuous line) and the constraint on this evolution (dashed line) in xy space, the units are meters.

For this walk with the minimisation of C2, the values of the actuator torques are low (Figure 3). Neither the maximal velocity nor the maximal torque of the actuator are used. There is no active constraint.

4.3 Walk with minimisation of the energy criterion C3

The chosen motion velocity for the biped is 0.75 m/s (2.7 km/h). The optimal walk has the following characteristics: for one half step, the duration T is 0.58 s, the displacement of the biped along x is 0.44 m (Figure 4). The value of the energy criterion C3 is 26.38 N.

The motion of the hip joint seems to be horizontal. In the second part of the half step. The motion biped is quasi ballistic (the torque values are near zeros) except for the torque of the stance leg knee. The behaviour of the tip of the free leg is different than in the case of minimisation of C2. The constraint on the leg tip position is active only on the almost middle of the half step. At the beginning of motion the displacement of the free leg tip is almost vertical. The initial torques are higher than in the case of minimisation of the torque. Unlike the previous optimal motion, the ratio between the horizontal component and vertical component of the ground reaction force is an active constraint during the passive impact.

4.4 Walk with fastest velocity

The fastest motion that can be achieved is drawn in figure 5. For an half step, the displacement along x is 0.44 m and the duration is 0.30 s. Thus the velocity is 1.47m/s (5.29 km/h).). The evaluation of criteria C2 and C3 gives C2=1320 N^2ms, and C3= 329 N. The active constraints are the maximum torque (3.0 Nm) for the hip actuator and the condition of no sliding during impact.

5 Gait optimisation method for the running

The running gait is composed of a stance phase and a flight phase. A passive impact exists at the end of the flight. The legs swap their roles from one half step to the next one. The description of motion during the stance phase has been described in section 3.2. The same methodology is used. The notation is slightly different. The index s denotes the stance phase. The index u denotes the flight (or unsupported) phase. The orientation and the angular velocity of the trunk at the end of the stance phase will be noted: $\Psi_{sf}, \dot{\Psi}_{sf}$ respectively.

In the following section we will present the description of the motion during flight phase.

5.1 Definition of polynomial functions for the legs

The robot is driven by four torques, the evolution of four independent variables only can be chosen. We choose to define the evolution of the four absolute joint angles of the legs $\mathbf{\Theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$, because this choice allows us to write easily the condition of continuity with stance phase.

In the case of running, two different polynomial functions will be defined: one during single support phase and one during flight phase. The duration of the single support phase and the flight phase are denoted T_s and T_u respectively.

For the same reason as in the case of walking (§3.1), we choose to define the evolution of the four absolute joint angles of the legs as fourth-order polynomial time functions.

During the single support phase, the four joint variables are defined as:

$$\theta_{js}(t) = a_{js0} + a_{js1}\, t + a_{js2}\, t^2 + a_{js3}\, t^3 + a_{js4}\, t^4 \qquad \text{with } j=1\ldots4\text{, j is the number of the joint} \tag{25}$$

The vectors of initial position Θ_{si} and velocity $\dot{\Theta}_{si}$, the final position Θ_{sf} and velocity $\dot{\Theta}_{sf}$ and an intermediate position $\mathbf{\Theta}_{sint}$ of the legs and the duration T_s are used to define the polynomial functions.

During the flight phase (unsupported phase), the four joint variables are defined as:

$$\theta_{ju}(t) = a_{ju0} + a_{ju1}\, t + a_{ju2}\, t^2 + a_{ju3}\, t^3 + a_{ju4}\, t^4 \qquad \text{with } j=1\ldots4\text{, j is the number of the joint} \tag{26}$$

The vectors of initial position Θ_{ui} and velocity $\dot{\Theta}_{ui}$, the final position Θ_{uf} and velocity $\dot{\Theta}_{uf}$ and an intermediate position $\mathbf{\Theta}_{uint}$ of the legs and the duration T_u are used to define the polynomial functions.

5.2 The motion during flight phase

When the robot is in flight, the coordinates x, y, Ψ of the trunk cannot be deduced uniquely from the motion of the legs. Their evolution must be defined by the dynamic and especially by equations (5) and (6), this point will be detailed in sections 5.3.2 and 5.3.4. Equations (5) and (6) allows to define the acceleration of the trunk. Therefore the position, the orientation, the linear and angular velocity of the trunk can be deduced if they are known at a given time. We choose to define their evolution as function of the orientation, the position and the linear and angular velocity of the trunk at the end of the flight phase noted: $x_{uf}, y_{uf}, \Psi_{uf}, \dot{x}_{uf}, \dot{y}_{uf}, \dot{\Psi}_{uf}$.

In summary, if the evolution of the legs and their derivatives are defined, and if $x_{uf}, y_{uf}, \Psi_{uf}, \dot{x}_{uf}, \dot{y}_{uf}, \dot{\Psi}_{uf}$ are known, the evolution of vector **X** components and their derivations can be deduced with equations (5) and (6).

Then the torques corresponding to the motion can be computed by the dynamic model (1). The dynamic model is a system of seven equations and four unknowns, but since the evolution of the trunk satisfies (5) and (6) this system is compatible and admits one unique solution. The criterion and constraints can be evaluated.

5.3 Definition of a periodic motion for the running

In this study, only cyclic trajectories are desired and, more precisely, the legs will swap their roles from one half step to the next one, like in the case of walking. The trajectory is defined only on one half step composed of a single support followed by a flight. The cyclic evolution of the orientation of the trunk during the stance phase will give an equality constraint as in the case of walking. During the flight phase, the position and orientation of the trunk are not directly assigned but they are defined by the dynamic model. The conditions of periodicity on the Cartesian position of the trunk, which are presented in 5.3.2, will lead to integrated conditions and are used to reduce the number of optimisation parameters. The conditions of periodicity on the orientation of the trunk , which are presented in 5.3.4 will give a new equality constraint.

5.3.1 Continuity for successive phases

Since the motion is continuous between the stance phase and the flight phase, the position and velocity of the biped are the same at the end of the stance phase (noted with index sf) and at the beginning of the flight phase (noted with index ui), they are the take-off configuration and velocity:

$$\Theta_{ui} = \Theta_{sf}\,,\ \dot{\Theta}_{ui} = \dot{\Theta}_{sf}\,, x_{ui} = x_{sf}\,, y_{ui} = y_{sf}\,, \Psi_{ui} = \Psi_{sf}\,,\ \dot{x}_{ui} = \dot{x}_{sf}\,, \dot{y}_{ui} = \dot{y}_{sf}\,, \dot{\Psi}_{ui} = \dot{\Psi}_{sf} \tag{27}$$

Since the position of the robot is constant during impact at the end of the flight phase (touch down configuration) and since the legs swap their roles from one half step to the next one, we can deduce :

$$\theta_{1si} = \theta_{4uf}\,,\ \ \theta_{2si} = \theta_{3uf}\,, \theta_{3si} = \theta_{2uf}\,,\ \theta_{4si} = \theta_{1uf} \tag{28}$$

and $$x_{uf} = x_{si} + d, y_{uf} = y_{si}, \Psi_{si} = \Psi_{uf} \tag{29}$$

where d is the horizontal displacement of the robot for one half-step.

We assume that the passive impact after the flight phase is such that the leg which comes in contact has an inelastic impact and does not slide. The equation of the impact is the same as in the walking gait case (11), and the velocity just after impact must satisfy equation (12). To this set of equations, some associated constraints must be satisfied, the impact I_{Rk} must be directed upward and in the friction cone [15].

The knowledge of the velocity of the robot just before the impact allows to define the velocity just after the impact using (11), and (12). Thus the velocity of the robot just after the impact can be defined as function of $\Theta_{uf}, \dot{\Theta}_{uf}, \Psi_{uf}, \dot{\Psi}_{uf}, \dot{x}_{uf}, \dot{y}_{uf}$

According to the chosen gait (with exchange of the legs between two half steps), the knowledge of the velocity just before impact, or at the end of the flight allows us to define the velocity at the beginning of the stance phase. Therefore, we can write $\dot{\Theta}_{si}$ and $\dot{\Psi}_{si}$ as function of $\Theta_{uf}, \dot{\Theta}_{uf}, \Psi_{uf}, \dot{\Psi}_{uf}, \dot{x}_{uf}, \dot{y}_{uf}$:

$$\dot{\Theta}_{si} = g_{s\Theta}(\Theta_{uf}, \dot{\Theta}_{uf}, \Psi_{uf}, \dot{\Psi}_{uf}, \dot{x}_{uf}, \dot{y}_{uf}) \tag{30}$$

$$\dot{\Psi}_{si} = g_{s\Psi}(\Theta_{uf}, \dot{\Theta}_{uf}, \Psi_{uf}, \dot{\Psi}_{uf}, \dot{x}_{uf}, \dot{y}_{uf}) \tag{31}$$

The continuity conditions have been used to reduce the number of independent parameters that produce a polynomial cyclic trajectory. Using equations (27) (28) and (30), the polynomial functions θ_{sj} (j=1…4), θ_{uj} (j=1…4), can be defined as function of (Θ_{sf}, $\dot{\Theta}_{sf}$, $\Theta_{s\,int}$, T_s, Θ_{uf}, $\dot{\Theta}_{uf}$, $\Theta_{u\,int}$, Ψ_{uf}, $\dot{\Psi}_{uf}$, $\dot{x}_{uf}$, $\dot{y}_{uf}$, T_u). Using equations (27), (29), (31) the evolution of the trunk is described with d, Ψ_{sf}, $\dot{\Psi}_{sf}$ as supplementary parameters.

5.3.2 Evolution of the position the trunk during flight

During the flight, only the gravity forces are exerted on the biped, the global equilibrium of the robot can be written as (5) This equation can be integrated during the flight phase, we obtain the final velocity and position of the biped mass centre as:

$$\dot{x}_{cf} = \text{const} = \dot{x}_{ci} \qquad \dot{y}_{cf} = \dot{y}_{ci} - gT_u \tag{32}$$

$$x_{cf} = x_{ci} + \dot{x}_{ci}T_u \qquad y_{cf} = y_{ci} + \dot{y}_{ci}T_u - \frac{g}{2}T_u^2 \tag{33}$$

The initial and final configurations of the robot in flight are completely defined by $\Theta_{sf}, \Psi_{sf}, \Theta_{uf}, \Psi_{uf}, d$ and equations (27), (29). Then, the initial and final coordinates of the mass centre can be deduced. Equation (33) gives two constraints on the initial velocity of the flight. This velocity is defined by $\dot{\Theta}_{sf}$ (four components) and the scalar $\dot{\Psi}_{sf}$. Since these five components must satisfy the set of two equations (33), only three components are independent. The new set of independent parameters is defined as: two components of $\dot{\Theta}_{sf}$ noted $\dot{\Theta}_{sf}^{N}$ and $\dot{\Psi}_{sf}$.

The velocity of the biped mass centre at the end of the flight is described by $\dot{\Theta}_{uf}$, $\dot{\Psi}_{uf}$, $\dot{x}_{uf}$, $\dot{y}_{uf}$, (32) gives two constraints on these velocities. Taking these constraints into account, an independent set of variables to describe the final velocity of flight can be: $\dot{\Theta}_{uf}$, $\dot{\Psi}_{uf}$

The evolution of the position of the robot during the flight is described by integration of equation (5), which allows to reduce the number of parameters describing the cyclic motion of the robot. The new set of parameters is: Θ_{sf}, $\dot{\Theta}_{sf}^{N}$, Θ_{sint}, Ψ_{sf}, $\dot{\Psi}_{sf}$, T_s, Θ_{uf}, $\dot{\Theta}_{uf}$, Θ_{uint}, Ψ_{uf}, $\dot{\Psi}_{uf}$, T_u and d

5.3.3 Evolution of the trunk during stance

The same approach as in the case of walking (§3.3.2) gives two constraints on the evolution of the orientation of the trunk. The difference with respect to the walking case is that the initial state of the trunk in stance phase depends on the final state of the flight phase.

$$\dot{\Psi}_{sf} = F_{s4}(\Theta_{uf}, \dot{\Theta}_{uf}, \Psi_{uf}, \dot{\Psi}_{uf}, \Theta_{sf}^{N}, \dot{\Theta}_{sf}, \Theta_{sint}, \Psi_{sf}, \dot{\Psi}_{sf}, T_s) \tag{34}$$

$$\Psi_{sf} = F_{s5}(\Theta_{uf}, \dot{\Theta}_{uf}, \Psi_{uf}, \dot{\Psi}_{uf}, \Theta_{sf}^{N}, \dot{\Theta}_{sf}, \Theta_{sint}, \Psi_{sf}, \dot{\Psi}_{sf}, T_s) \tag{35}$$

The evolution of the robot must be chosen in order to satisfy (34) and (35).

5.3.4 Evolution of the orientation of the trunk during flight

The same approach as in the previous section can be used. The difference with respect to the single support case is that the theorem of changing of total kinetic momentum is written around the mass center. The constraint equations can be written:

$$\dot{\Psi}_{uf} = F_{u4}(\Theta_{sf}, \Theta_{sf}^{N}, \Psi_{sf}, \dot{\Psi}_{sf}, \Theta_{uf}, \dot{\Theta}_{uf}, \Theta_{uint}, \Psi_{uf}, \dot{\Psi}_{uf}, T_u) \tag{36}$$

$$\Psi_{uf} = F_{u5}(\Theta_{sf}, \Theta_{sf}^{N}, \Psi_{sf}, \dot{\Psi}_{sf}, \Theta_{uf}, \dot{\Theta}_{uf}, \Theta_{uint}, \Psi_{uf}, \dot{\Psi}_{uf}, T_u) \tag{37}$$

The evolution of the robot must be chosen in order to satisfy equations (36) and (37).

We have seen in this section that a cyclic trajectory of a robot can be defined by $\Theta_{sf}, \dot{\Theta}_{sf}^{N}, \Theta_{sint}, \Psi_{sf}, \dot{\Psi}_{sf}, T_s$, $\Theta_{uf}, \dot{\Theta}_{uf}, \Theta_{uint}, \Psi_{uf}, \dot{\Psi}_{uf}, T_u, d$, and these parameters must satisfy (34), (35), (36) and (37).

6 Simulation Results

6.1 The constraints and limitations

The actuators defined in section 4 are used. The evolution of the swing leg tip during the stance phase must be higher than 5 cm. During the flight phase the leg tips must not touch the ground before the prescribed time. The space of research of the optimal parameters is limited : the duration of motion is between 0.2 s and 1s, the ratio of the flight duration with respect to the period is greater than 0.1. The displacement along x for an half step is between 0.2 m and 2 m, the initial orientation of the trunk for the stance and flight phases is between 45° and 90°.

6.2 Running with the minimisation of the cost torque C2

The chosen motion velocity for the running of the biped is 1.5 m/s or 5.4 km/h. For an half step, the optimal duration ($T_s + T_u$) is 0.54 s and the displacement of the biped along x is 0.81 m. The value of the cost torque criterion C2 is 4271 N^2ms. The ratio of the flight duration with respect to the period is 0.21. The active constraints are the maximum actuator velocities and the condition on the evolution of the swing leg tip during stance phase. Since the torque is minimised, the maximal available torque is not used: the maximal torque required is about 75 N.m. The optimal motion is presented on figure 6.

6.3 Running with minimisation of the energy criterion C3

The chosen motion velocity for the biped is 1.5 m/s or 5.4 km/h. For an half step, the optimal duration ($T_s + T_u$) is 0.42 s and the displacement of the biped along x is 0.64 m. The value of the criterion C3 is 108 N. The ratio of the flight duration with respect to the period is 0.18 . The optimal motion is presented on figure 7. The active constraints are the maximal velocity on the hip, the torque and the condition on the evolution on the swing leg tip during the stance phase. The maximal velocity is not required for the knee. Unlike the previous optimal motion, the ratio between the horizontal component and vertical component of the ground reaction force is an active constraint during passive impact .

6.4 Running with fastest velocity

The fastest motion that can be achieved is drawn in figure 8. For an half step, the optimal duration (T_s +T_u) is 0.29 s and the displacement of the biped along x is 1.02 m. Thus the velocity is 3.46 m/s (12.46 km/h). The ration between the time of the flight and the time of half step is 0.63. The evaluation of criteria C2 and C3 gives C2=12575 N^2ms, and C3= 356 N. So the velocity is much higher than in the case of walk but the energy is not very different. The active constraints are the maximum torque for all joints.

6.5 Comparison between walk and running motion.

Figure 9 to figure 12 present some characteristics of the optimal trajectories as function of the motion velocity of the robot (average velocity of the biped mass centre along the x axis). The criteria C2 and C3 are studied, figures 9 and 11 concern the optimisation of criterion C2 (minimisation of the torque), figures 10 and 12 concern the optimisation of criterion C3 (minimisation of energy).

In the upper parts of figures 9 and 10, the evolution of the criteria cost are drawn as function of the motion velocity for walking and running. It can be observed that from the point of view of C2 and C3 costs, it is better to walk at a low velocity and to run with a high velocity. For a biped motion velocity higher than 1.5 m/s, walking cannot be produced by the chosen actuator, but running is possible. Globally the values of the criteria increase with the velocity motion, it must be recalled that the criteria costs are defined to cover a distance of one meter. The fact that it is better to walk at a low velocity than to run is confirmed by the evolution of the ratio between the flight time and total time for the running. This ratio is drawn in the lower part of the figures 9 and 10. For low velocities, the optimal ratio is 0.1, which is the minimal value for the optimisation process. The ratio increases with the biped motion velocity.

The figures 11 et 12 present the evolution of the duration and displacement along x axis for an half step for walking and running versus the biped motion velocity. Globally the step time decreases, the step length increases then the biped motion velocity increases. However for the walking, the step length for motion velocity between 0.5 m/s to 1.5 m/s is almost constant, around the value 0.45 m. Generally the optimal running corresponds to a longer step than the optimal walking for the same velocity. It can be remarked that the minimal duration for the fastest walking or running is always close to 0.32 s.

The evolution of all the curves corresponding to the optimisation of criterion C2 are smoother than those corresponding to the optimisation of C3. This can be explained by the fact that more constraints are active when C3 is minimised for running. More precisely, in the last case, the maximal torque for the hip is always used. For motion velocity higher than 2 m/s, the maximal torque is also used in the knee. The constraint to avoid the sliding during the impact is active between 1.5 m/s and 2 m/s. For the running, the maximal angular velocity in the hip joint is always used for any motion velocity and any optimised criterion. For the minimisation of criterion C2, the constraint to avoid sliding during impact is active for a velocity motion higher than 2.25 m/s. For the walking, the active constraint is also the constraint to avoid sliding during impact for a motion velocity higher than1m/s.

The gaits defined include impact phase. For the walking, the impact velocity of the swing leg tip varies form 0.8m/s to 4m/s when the motion velocity increases from 0.2m/s to 1.5m/s but after an increase for small motion velocity the impact force modulus is limited around 30N. For the running gaits the impact velocity of the swing leg tip varies around 4m/s. For the minimisation of criteria C2, the impact force modulus decrease from 50N to 30N, but for the minimisation of criteria C3, the impact force modulus is constant: 36N when the motion velocity increase from 0.5m/s to 3.2m/s.

7 Conclusion

Optimal cyclic joint reference trajectories for the walking or running a biped are proposed in this paper. In order to use classical algebraic optimisation techniques, the optimal trajectory is defined by a low number of parameters. The absolute joint evolution of the legs are fourth order polynomial time functions. A cyclic solution is desired. The condition of periodicity is used to reduce the number of parameters describing the motion. But since the case of non actuated ankle is studied, the theorem of changing of kinetic momentum induces periodicity condition which is not integrable. Thus the number of parameters cannot be reduced and an equality constraint must be taken into account in the optimisation process. Some inequality constraints such as the limits on torque and velocity, the condition of no sliding during motion and impact, some limits on the motion of the free leg are taken into account.

The walking gait is composed of single supports and instantaneous double support phase defined by passive impact.

The running gait is composed of stance and flight phases, a passive impact exists at the end of flight phase.

The geometric and dynamic characteristics of the biped model and actuators are those of a prototype under construction

It is shown by numerical results that the optimal trajectories are realistic. It is better for the biped to walk at low velocities and to run at high velocities..

The extension of the method to the case of walk with non-instantaneous double support will be done. But in this case a biped with feet must be considered. The walk with non instantaneous double support of a robot without feet does not give human like motion. This can be understand because during human double support phase the position of the contact between the ground and the feet moves along the soles. We are testing also this method with a quadruped robot.

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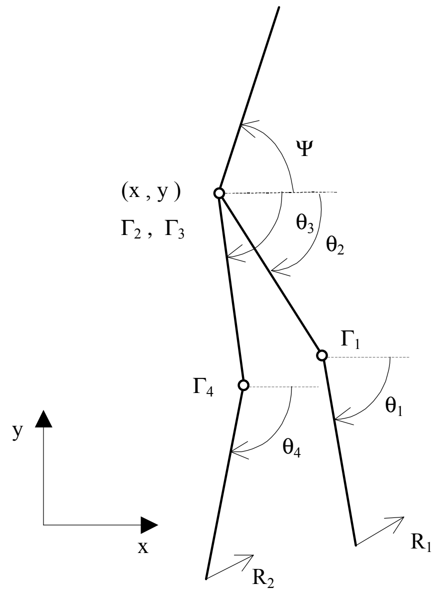


Figure 1 : The studied robot

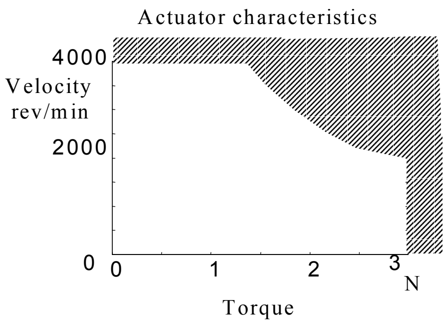


Figure 2 : The actuator characteristics

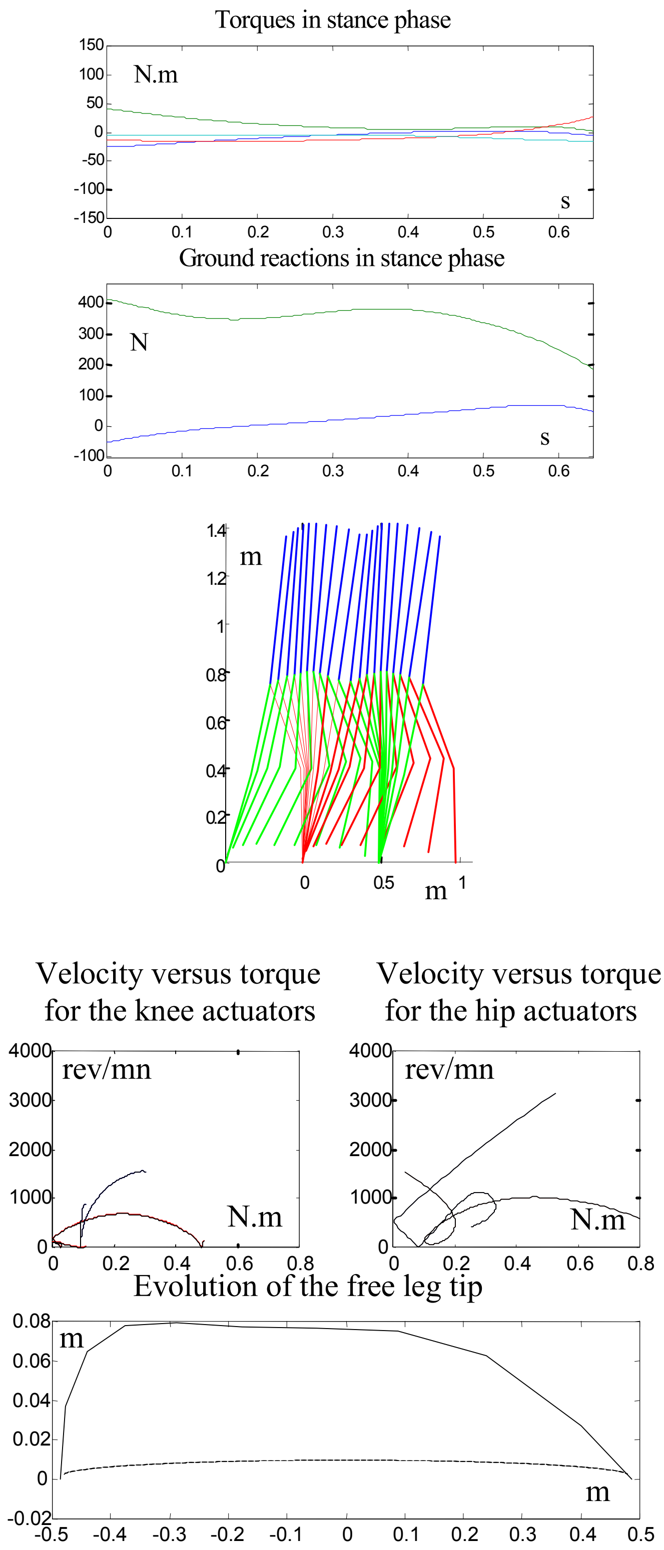


Figure 3: the walk with torque cost criterion C2

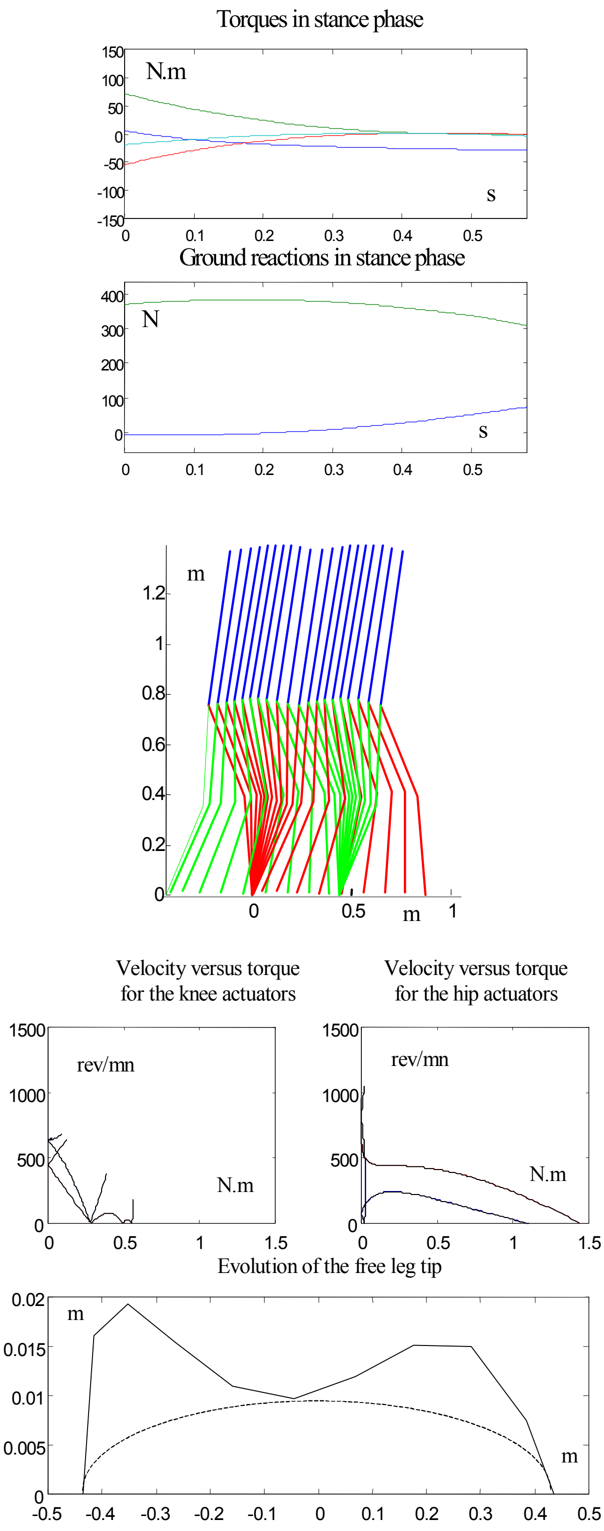


Figure 4: the walk with energy criterion C3

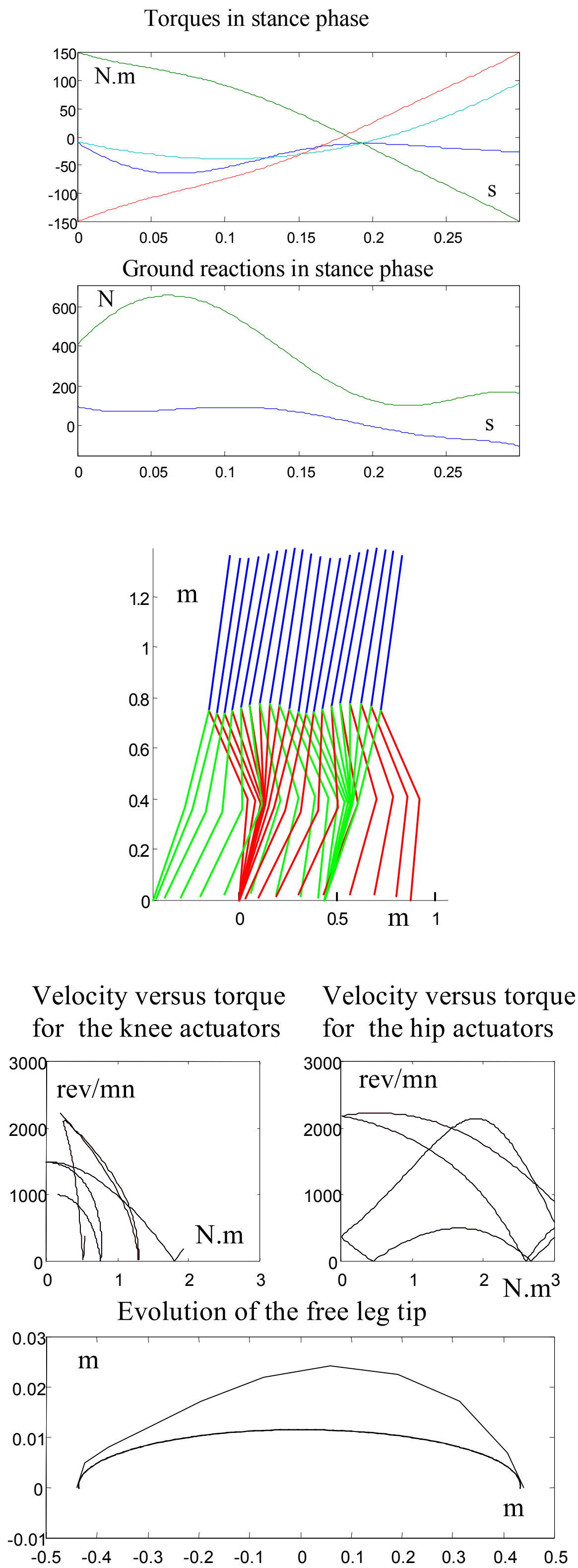


Figure 5: the fastest walk

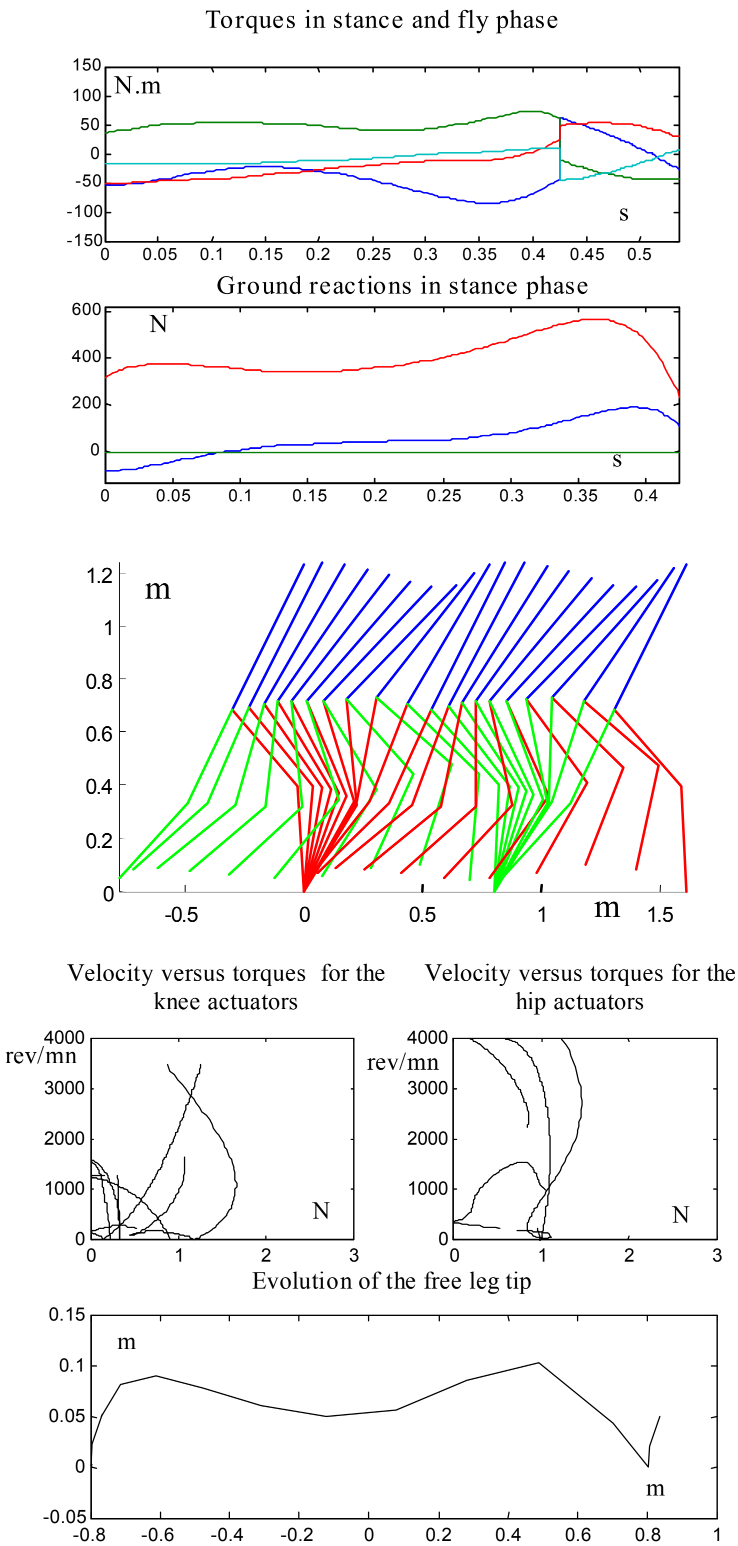
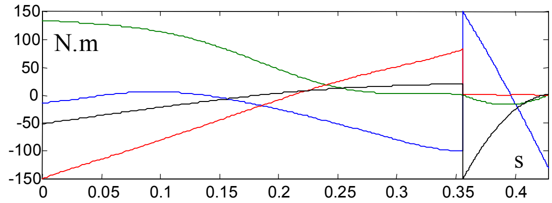
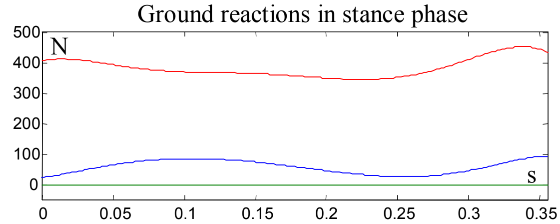


Figure 6: Running with minimisation of C2

Torques in stance and fly



Ground reactions in stance phase



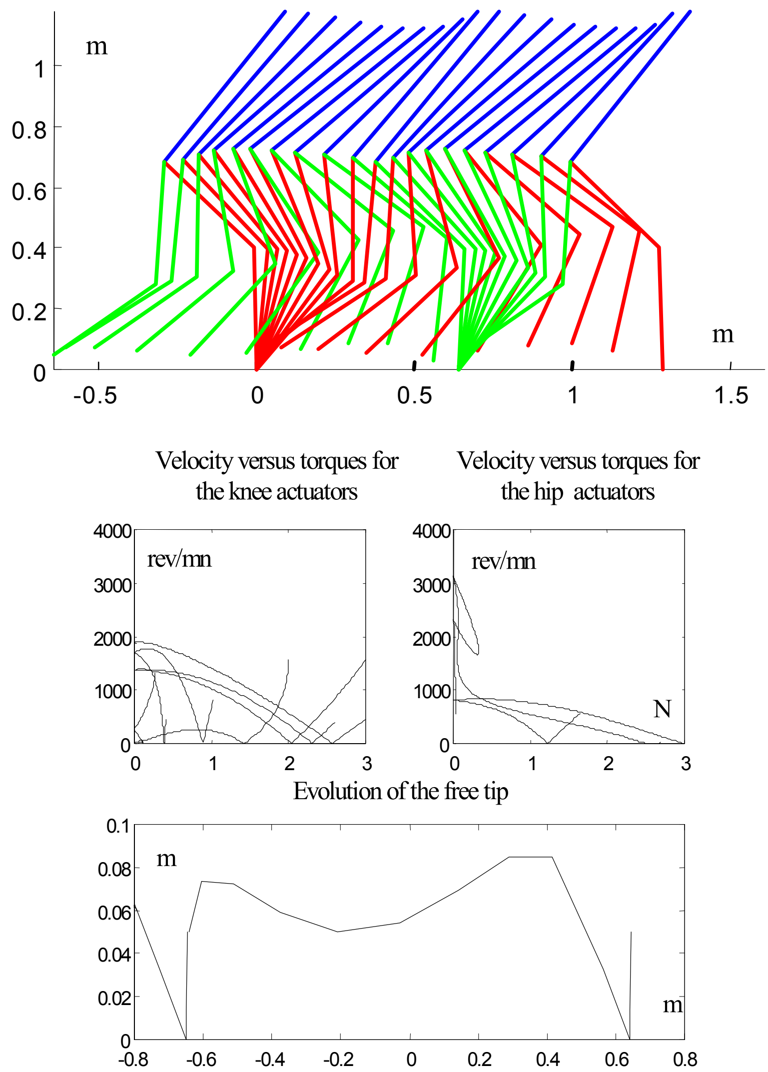


Figure 7: the running with energy criterion C3

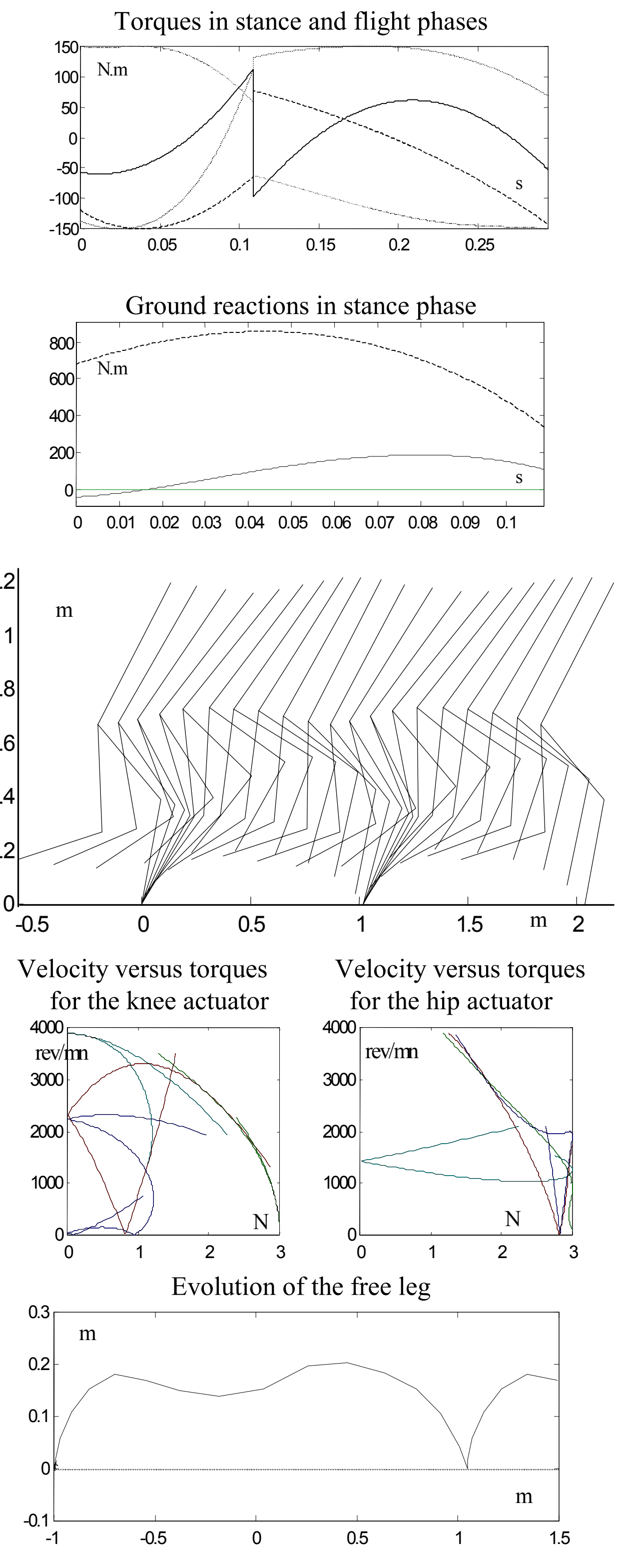


Figure 8: Fastest running

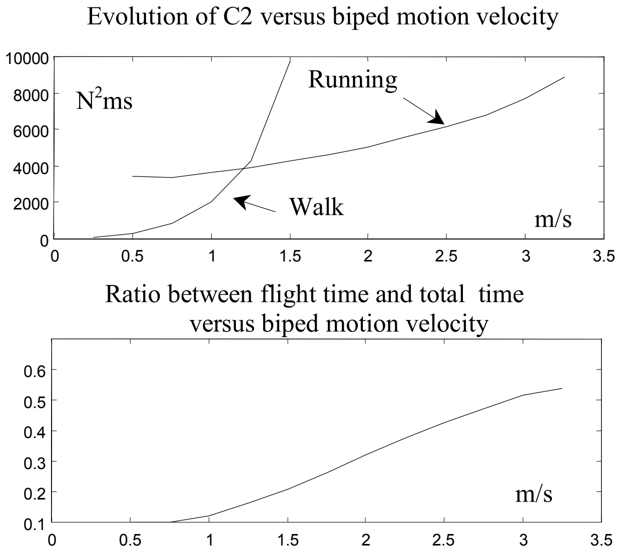


Figure 9: Value of C2 and ratio between flight time and half step for walk and running motion

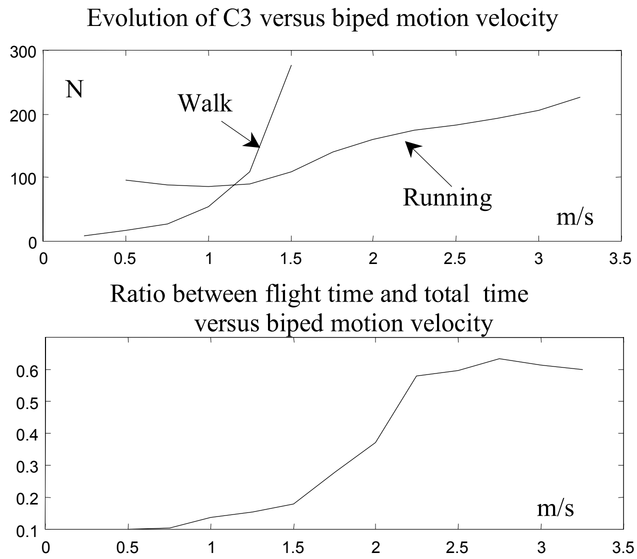


Figure 10: Value of C3 and ratio between flight time and half step for walk and running motion

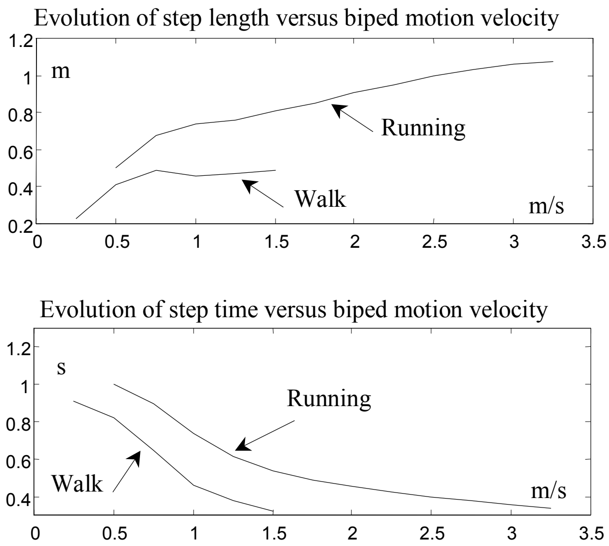


Figure 11: Time and length of a half step for walk and running motion with minimisation of C2

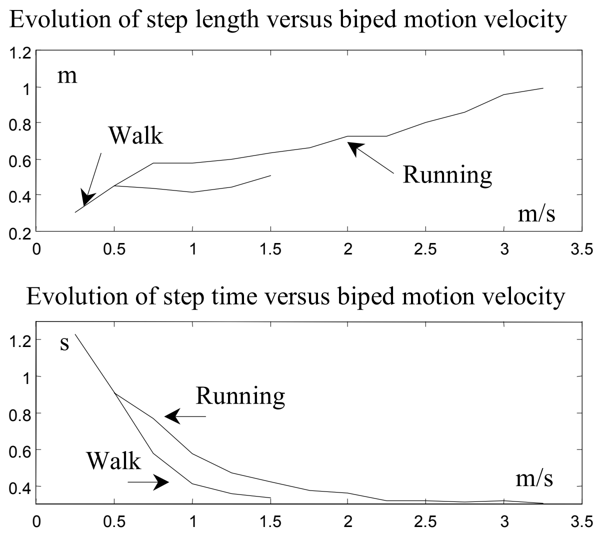


Figure 12: Time and length of a half step for walk and running motion with minimisation of C3