Computation of layered mixed potentials for the accurate and efficient analysis of periodic printed structures

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Computation of Layered Mixed Potentials for the Accurate and Efficient Analysis of Periodic Printed Structures

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Abstract— **Original acceleration procedures are proposed for the efficient calculation of the vertical components of the dyadic and scalar mixed-potential layered-media periodic Green's functions for various types of periodic structures. The extraction of suitable asymptotic terms, i.e., quasi-static images, is performed in order to speed up the convergence of the relevant spectral series. The extracted terms can be expressed as potentials for array of half-plane and half-line sources, depending on the type of the considered periodic Green's function. The relevant numerical results show the remarkable improvements in the efficiency of the approach.**

Index Terms— **Periodic structures, Green's functions, mixed potentials, layered media, metamaterials.**

I. INTRODUCTION

Design techniques for guiding and radiating structures with 1-D or 2-D periodicity in layered media have recently enjoyed an increasing interest. Periodic leaky-wave antennas (LWAs) are one example. Such antenna topologies are usually based on planar layered configurations with suitable periodic inclusions (see Fig. 1). The prediction of the LWA parameters (e.g., phase and leakage constants) is often a difficult task to achieve in an accurate and efficient fashion. Moreover, a general treatment allowing for arbitrary periodic inclusions in a layered medium makes heavy demands on computational resources for realistic modeling [1].

The development of advanced computational tools is usually required, since LWA parameters cannot always be determined rigorously with common commercial software. A number of approximate methods can be developed to overcome these problems [2], but full-wave approaches are needed to characterize the modal properties in all propagation regimes (including stopbands and nonphysical regions). A flexible formulation for the numerical solution of LWAs in layered media can be achieved through integral equation methods. Arbitrarily-shaped objects can be handled by discretizing and testing such integral equations through a spatial-domain method-of-moments (MoM) approach, using Rao-Wilton-Glisson (RWG) subdomain triangular basis functions [3]. Numerical problems due to the spatial singularities of the kernels can be reduced if a mixed-potential formulation integral equation (MPIE) formulation is adopted [4].

Unfortunately, the kernels of the relevant integral equations, given by the periodic Green's functions (PGFs), are expressed through slowly converging series and integrals, thus making impractical any direct numerical solution of the problem. The main difficulty in the development of a flexible MoM for periodic planar stratified LWAs is thus the availability of computationally efficient expressions for the PGFs, which remain valid in the case of improper leaky waves supported by periodic layered structures [5],[6].

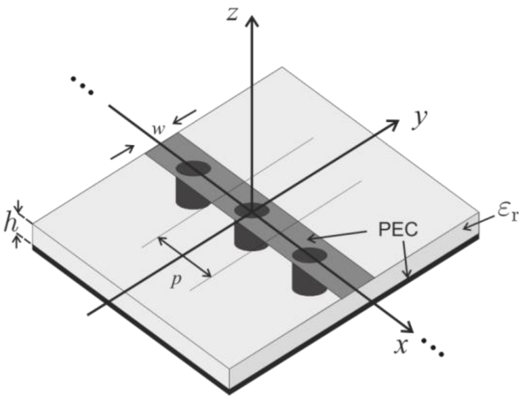

Fig. 1. Example of a layered structure, with inclusions periodic along one direction, which can be studied with the MoM described in this work. Geometrical and physical parameters are highlighted in the figure.

In this context, the present work is focused on the computational aspects of integral-equation approaches to the problem based on an MPIE formulation. By means of specific asymptotic extractions, efficient acceleration techniques for the relevant scalar and dyadic PGFs will be proposed and tested that are suitable for the analysis of periodic printed structures with both planar and vertical elements. These resulting expressions remain valid for complex leaky waves where the fields are improper.

II. FORMULATION OF GREEN'S FUNCTIONS

PGFs are expressed as spectral sum of harmonics, where the actual expressions depend on the kind of periodicity considered. We show here three kinds of spectral series:

$$G(\mathbf{r},\mathbf{r}') = \frac{1}{p}\sum_{n=-\infty}^{+\infty} \tilde{G}\left(k_{x_n}; z, z'\right) e^{-jk_{x_n}\Delta x} \tag{1}$$

$$G(\mathbf{r},\mathbf{r}') = \frac{1}{2\pi p}\sum_{n=-\infty}^{+\infty} e^{-jk_{x_n}\Delta x} \int_{-\infty}^{+\infty} \tilde{G}\left(\mathbf{k}_{\mathrm{t}_n}; z, z'\right) e^{-jk_y\Delta y}\,\mathrm{d}k_y \tag{2}$$

$$G(\mathbf{r},\mathbf{r}') = \frac{1}{A}\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} \tilde{G}\left(\mathbf{k}_{\mathrm{t}_{mn}}; z, z'\right) e^{-j\mathbf{k}_{\mathrm{t}_{mn}}\cdot\Delta\mathbf{r}}\,. \tag{3}$$

where

$$k_{x_n} = k_{x_0} + 2\pi n / p$$

$$\mathbf{k}_{\mathrm{t}_n} = k_{x_n}\mathbf{x}_0 + k_y\mathbf{y}_0$$

$$\mathbf{k}_{\mathrm{t}_{mn}} = \mathbf{k}_{\mathrm{t}_{00}} + (2\pi m)\mathbf{z}_0 \times \mathbf{s}_1 / A + (2\pi n)\mathbf{s}_2 \times \mathbf{z}_0 / A\,.$$

For 1-D periodic arrays along x, p is the spatial period and k_{x_n} is the wavenumber along x of the nth spatial harmonic (Floquet wave). For structures that are periodic along x and invariant in y, the Green's function form of (1) is used. For structures that are periodic in x but finite in y, the Green's function form (2) is used, in which k_y is a continuous spectral integration variable. For 2-D periodic arrays, the form (3) is used, in which A is the area of the unit cell and the lattice vectors that define the unit cell are ($\mathbf{s}_1$, $\mathbf{s}_2$). In all of these expressions, $\tilde{G}$ is the 2-D spectral Green's function for a point source, known in closed form for layered media [3].

The convergence of the series and integrals in (1)-(3) becomes dramatically slow as $z \to z'$ at an interface between media. A common approach to accelerate the convergence of the transverse potentials is the asymptotic extraction of quasi-static images. Using the 2-D periodic array of point sources in (3) as an example, we have

$$G(\mathbf{r},\mathbf{r}') = \frac{1}{A}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{+\infty}\left[\tilde{G}\left(\mathbf{k}_{\mathrm{t}_{mn}}; z, z'\right) - \sum_{i=-1}^{+1} C_i \tilde{g}\left(\mathbf{k}_{\mathrm{t}_{mn}}; \Delta z_i\right)\right] e^{-j\mathbf{k}_{\mathrm{t}_{mn}}\cdot\Delta\mathbf{r}} + \sum_{i=-1}^{+1} C_i g\left(\mathbf{k}_{\mathrm{t}_{00}}; \Delta\mathbf{r}_i\right). \tag{4}$$

The spectral term $\tilde{g}$ and the coefficients C_i are known in closed form for every component of the dyad. They are chosen so that *i*) they cancel the asymptotic contribution of the original terms in the original series (3), and *ii*) the spatial counterpart of $\tilde{g}$, i.e., the potential produced by a 2-D array of sources (g) in a homogeneous medium, is easily computed. This is done using the Ewald method (having a Gaussian rate of convergence, and remaining valid for complex waves).

An example is shown in Fig. 2 (using form (2)), where the improvement in efficiency (without or with extraction) for the computation of all the needed potentials is notable.

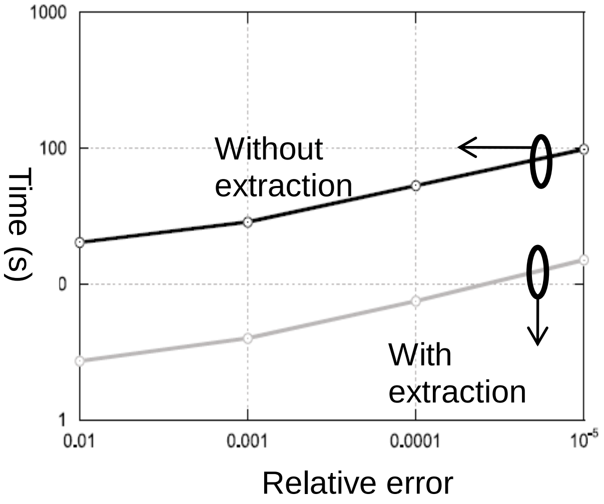

Fig. 2. Time in seconds, as a function of the relative error, to compute 100 equispaced points (without or with extraction) in the unit-cell interval $-p/2 < \Delta x < p/2$, at $\Delta y = p/20$, at the interface between the air and a grounded dielectric slab (relative dielectric constant $\varepsilon_r = 2.2$, thickness $h = 0.508$ mm). Complex wavenumber along the direction of periodicity: $k_{x0}/k = 0.8 - j\,0.02$. The only improper harmonic is $n = 0$.

Further details will be given during the presentation on specific computational aspects of the formulation, together to examples of LWAs successfully analyzed and designed with such an approach.

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