A Tensor-Based Algorithm for the Optimal Model Reduction of High Dimensional Problems

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Abstract. *We propose a method for the approximation of the solution of high-dimensional problems formulated in tensor spaces using low-rank approximation formats. The method can be seen as a perturbation of an ideal minimal residual method with a residual norm corresponding to the error in a solution norm of interest. We introduce and analyze an algorithm for the approximation of the best approximation in a given low-rank tensor subset. A weak greedy algorithm based on this ideal minimal residual formulation is introduced and its convergence is proven under some conditions. The robustness of the method is illustrated on numerical examples in uncertainty propagation.*

Keywords: Model reduction; tensor approximation; minimial residual; ideal norm

Due to the need of more realistic numerical simulations, models presenting uncertainties or either numerous parameters are receiving a growing interest. To solve such high dimensional problems, one has to circumvent the so called *curse of dimensionality* when using classical numerical approaches. To overcome such an issue, model reduction techniques based on low-rank approximations have became popular these last years.

This presentation is concerned with the solution of high dimensional linear equations by means of approximations in low-rank tensor subsets [4, 3]. To compute the optimal approximation of the solution in a tensor subset, an ideal best approximation problem which consists in minimizing the distance to the exact solution for a given norm $||\cdot||$ is introduced. Since the exact solution is not available, such a problem cannot be solved directly. However, it can be replaced by the computation of a low rank tensor approximation of the solution that minimizes the residual of the equation (which is computable) measured with another norm $||\cdot||_*$. Nevertheless, if $||\cdot||_*$ is chosen in a usual way, the resulting approximation may be far from the one expected by solving the initial best approximation problem with respect to the solution norm $||\cdot||$.

Here, we present an *ideal minimal residual method*, inspired from [1, 2], that relies on an ideal choice for $||\cdot||_*$ and that can apply to high dimensional weakly coercive problems. $||\cdot||_*$ is chosen to ensure the equivalence between the best approximation problem for $||\cdot||$ and the residual minimization problem with $||\cdot||_*$. Yet, the computation of the residual norm with $||\cdot||_*$ is not affordable in practice. Here, the residual norm is not exactly computed but estimated with a controlled precision δ. We thus propose a perturbed minimization algorithm that provides an approximation of the optimal approximation of the solution with an error depending on δ.

We also introduce a progressive construction of the low-rank approximate solution of the initial problem by means of greedy corrections [5] computed with the proposed iterative algorithm. The resulting weak greedy algorithm is proven to be convergent under some assumptions on δ. The proposed method is applied to the numerical solution of stochastic partial differential equations.

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