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A New Differential Space-Time Modulation Scheme based on Weyl Group

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Abstract—In this paper, a new differential space-time modulation (DSTM) scheme for 4×4 multiple input multiple output (MIMO) systems is proposed. This scheme is used for MIMO systems where the channel coefficients are not available at both the transmitters and the receivers. The transmission matrix used in this scheme is based on the Weyl group. Simulation results show that this new scheme with four transmit antennas outperforms the well-known Tarokh's differential space-time block coding (DSTBC) scheme. The spectral efficiency of this scheme can be up to 3 bps/Hz.

Keywords—MIMO, DSTM, DSTBC, non-coherent.

I. Introduction

In the late 1990s, due to the large demands of wireless spectral resource, researchers resort to multiple antennas to enlarge the capacity and robustness of wireless communication systems. Huge amounts of studies have been carried out on this domain. Reference [1], [2] analyzed the capacity and the error exponents of such systems in the presence of Gaussian noise and got some fundamental theoretical results. Generally, people design space-time coding/modulation schemes to get the requirements. At the beginning, when the throughput was not so high, people could estimate the channel coefficients easily. Several schemes with known channel state information (CSI) have been widely analyzed.

However, the estimation of CSI consumes much time or frequency resource, especially when the number of antennas is large or the propagation channel changes rapidly. As the number of transmit antennas grows, the training time to probe the channel grows proportionately [3], which in turn reduces the overall system throughput. Therefore, solutions that do not require channel information are very interesting. Marzetta and Hochwald [4] analyzed the capacity of the MIMO systems in such cases. Based on the results, they proposed the unitary space-time modulation scheme (USTM) [5]. Analogous to single antenna DPSK (differential phase-shift keying) systems, Hochwald and Sweldens proposed the differential space-time modulation scheme (DUSTM) [6]. These two schemes are difficult to design, and furthermore there are not general design criteria for these two schemes. At the same time, Tarokh and Jafarkhani proposed a differential space-time block coding scheme [7] which is based on Alamouti's diversity scheme [8], and soon they expanded this scheme to systems with 4 transmit antennas [9]. This scheme is just suitable for MIMO systems with less than 4 transmit antennas and the spectral efficiency is limited. In [10], [11], the authors proposed a new kind of non-coherent space-time modulation scheme–matrix coded modulation (MCM) which is suitable for 2×2 MIMO systems. In the study of the MCM scheme, we found that the group used in this scheme can be applied in differential MIMO schemes.

In this paper, we propose a new differential space-time modulation scheme which is suitable for MIMO systems with 4 transmit antennas. The simulation results demonstrate the advantages of this new scheme over Tarokh's DSTBC scheme. Furthermore, the modulation of Tarokh's scheme with 4 transmit antennas can only be BPSK [9], which prevents the scheme to expand to higher spectral efficiency. Our scheme with four transmit antennas can be extended to 3 bps/Hz. And the bit error rate performance can be improved with the method of searching the best distance spectrum.

The following notations are used through the paper: $\text{Tr}\{A\}$ denotes the trace of the matrix A and A^H denotes the conjugate transpose of A. $\|A\|$ means the Frobenius norm of A, i.e., $\|A\| = \sqrt{\sum_{i,j} |a_{ij}|^2} = \sqrt{\text{Tr}\{A^H A\}}$. $\text{Re}\{z\}$ is the real part of the complex number z. The sign $\lfloor K \rfloor$ denotes the nearest integer number less than K, and $\otimes$ denotes the Kronecker product. The zero-mean, unit-variance, circularly symmetric, complex Gaussian distribution is denoted as $CN(0,1)$.

II. Differential MIMO System Model

The MIMO system model in the matrix form is:

$$Y_\tau = H_\tau X_\tau + W_\tau \tag{1}$$

where τ is the time index. H_τ is the channel coefficient matrix at time τ and its size is $M \times N$, where M is the number of transmit antennas and N is the number of receive antennas. The element h_{nm} is the path gain of the quasi-static channel from the transmit antenna m to the receive antenna n, and follows complex Gaussian distribution with zero-mean and unit-variance, i.e., $h_{nm} \sim CN(0,1)$. X_τ is the $M \times T$ transmission matrix, where T denotes the normalized symbol durations of each matrix, i.e., on each transmit antenna, T symbols are transmitted. Y_τ is the received matrix. W_τ is the additive white Gaussian noise matrix, $w_{nt} \sim CN(0,\sigma^2)$ and σ^2 is the power of the noise. We denote the spectral efficiency of the system as R expressed in bps/Hz.

In this paper, we assume that the channel coefficients are constant during L symbols. Therefore, we ignore the index τ

of the matrix H_τ and the channel matrix is written to be H.

Furthermore, we assume a normalized power over M transmit antennas:

$$\sum_{m=1}^{M} |x_{mt}|^2 = 1, t = 1, \ldots, T. \tag{2}$$

As it is proved in [4], for non-coherent MIMO systems, for any block length T, any number of receive antennas N and any SNR (signal to noise ratio), the capacities obtained with $M > T$ and $M = T$ are equal. Therefore, we choose $M = T$ in our study.

A. Differential space-time modulation

In the differential space-time modulation systems, one block of information bits is mapped onto a matrix in the candidate set P. For example, the transmitter send a reference matrix $X_0 = V_0$ at time τ_0. The first block of the information bits is mapped to the matrix V_{τ_1} and the second block is mapped to V_{τ_2} etc. The fundamental differential transmission relation is:

$$X_{\tau+1} = X_\tau V_{i_{\tau+1}}, \tau = 0, 1, \ldots \tag{3}$$

Therefore, the sequence of transmitted matrices is:

$$X_0 = V_0$$
$$X_1 = X_0 V_{i_1} = V_0 V_{i_1}$$
$$\cdots$$
$$X_\tau = X_{\tau-1} V_{i_\tau} = V_0 V_{i_1} \ldots V_{i_\tau}$$
$$\cdots$$

At the receiver side, the antennas received a matrix stream $Y_0, \ldots, Y_\tau, Y_{\tau+1}, \ldots$. We know that

$$Y_\tau = HX_\tau + W_\tau$$
$$Y_{\tau+1} = HX_{\tau+1} + W_{\tau+1} \tag{4}$$

Based on the differential transmission equation (3), we get

$$\begin{aligned} Y_{\tau+1} &= HX_{\tau+1} + W_{\tau+1} \\ &= Y_\tau V_{i_{\tau+1}} + W_{\tau+1} - W_\tau V_{i_{\tau+1}} \\ &= Y_\tau V_{i_{\tau+1}} + W'_{\tau+1} \end{aligned} \tag{5}$$

where $W'_{\tau+1} = W_{\tau+1} - W_\tau V_{i_{\tau+1}}$.

Therefore, to estimate the information matrix, the maximum likelihood demodulator is

$$\begin{aligned} \hat{V}_{i_{\tau+1}} &= \arg\min_{V \in P} \|Y_{\tau+1} - Y_\tau V\| \\ &= \arg\min_{V \in P} \text{Tr}\{(Y_{\tau+1} - Y_\tau V)^H (Y_{\tau+1} - Y_\tau V)\} \\ &= \arg\min_{V \in P} \text{Tr}\{\text{Re}(Y_{\tau+1}^H Y_\tau V)\} \end{aligned} \tag{6}$$

III. NEW DIFFERENTIAL SPACE-TIME MODULATION SCHEME

A. The constellation of the new scheme

In our scheme, the transmitted matrices are based on the Weyl group used in [10], [11]. The Weyl group G_w is a set that contains 12 cosets $(C_0, C_1, \ldots, C_{11})$. Each coset contains 16 invertible matrices. The first coset is defined as:

$$C_0 = \left\{ \alpha \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \alpha \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \alpha \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \alpha \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\} \tag{7}$$

with $\alpha \in \{1, -1, i, -i\}$. The 12 cosets of G_w are derived from C_0 as follows:

$$C_k = A_k C_0, \forall k = 0, 1, \ldots, 11 \tag{8}$$

where the matrices $A_k, k = 0, 1, \ldots, 5$ are respectively:

$$A_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}, A_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$
$$A_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}, A_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix}, A_5 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix},$$

and the matrices $A_k, k = 6, 7, \ldots, 11$ are given by:

$$A_{k+6} = \eta A_k, with \quad \eta = (1+i)/\sqrt{2}, \forall k = 0, 1, \ldots, 5 \tag{9}$$

Remind that, there are 192 matrices in this group, and we number the matrices as $M_0, M_1, \ldots, M_{191}$. Furthermore, they are all *unitary* matrices, i.e., the inverse of the matrix is equal to the conjugate transpose of the matrix.

We define the distance between two matrices M_a and M_b as:

$$D_{a,b} = \|M_a - M_b\| \tag{10}$$

We can see that $D_{a,b} = D_{b,a}$. Therefore, there are $191 \times 192/2 = 18336$ values $D_{a,b}$ with $0 \leq a < b \leq 191$. For any value a, the distribution of the 191 values $D_{a,b}$ with $b \neq a$ is the same, as shown in Fig. 1(a). For C_0, this distribution is given in Fig. 1(b). As in [12]-[14], these results are used to optimize the BER performance.

B. Differential transmission and reception

Consider a MIMO system with $M = 2$ transmit antennas and $N = 2$ receive antennas. Each transmit matrix is sent during $T = 2$ symbol durations. The number of receive antennas is arbitrary, i.e., we can set $N = 1, 2, 3, \ldots$. We use the coset C_0 as the candidate mapping matrices set. There are 16 matrices in the coset C_0, say $M_0, M_1, \ldots, M_{15}$, so it is possible to map 4 bits to a given matrix of the coset C_0. Thus, 4 bits are sent during 2 symbol durations. Hence, the spectral efficiency R is 2 bps/Hz.
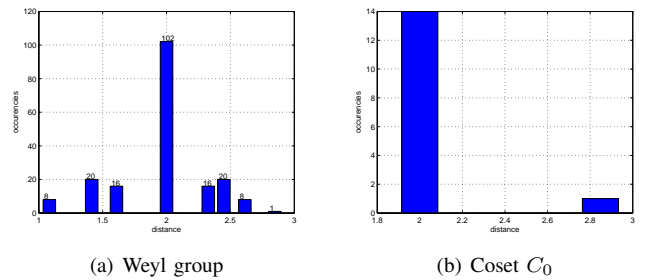

(a) Weyl group   (b) Coset C_0

Fig. 1.   Distance spectrum

At time $\tau = 0$, we transmit a reference matrix $X_0 = M_0 = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$.

Suppose that at time τ, X_τ is transmitted. At time $\tau + 1$, a block of 4 bits arrives. These bits are mapped onto one of the matrices $M_{i_{\tau+1}} = M_a$ of the coset C_0, and then

$$X_{\tau+1} = X_\tau M_{i_{\tau+1}} \qquad (11)$$

is transmitted.

The demodulation procedure is shown in the Section II.

As there are $K = 192$ matrices in the Weyl group G_w, for MIMO systems with 2 transmit antennas, the maximum spectral efficiency we can get is $R = \frac{1}{M}\lfloor \log_2 K \rfloor = 3.5$ bps/Hz.

C. MIMO systems with 4 transmit antennas

To design a MIMO system with 4 transmit antennas, the *Kronecker product* is used to expand the Weyl group.

With the assumption $M = T$ defined in section II, for MIMO systems with 4 transmit antennas, 4×4 transmit matrices are used.

Using the Kronecker product between each couple of 2×2 matrices of the Weyl group, 4×4 matrices are obtained. Only 4608 matrices are distinct. They are denoted $N_0, N_1, \ldots, N_{4607}$. The set of these matrices is also a group denoted G_{w4}.

In our first proposal which is used for $R = 1$ bps/Hz, we make the Kronecker products between the first matrix $M_0 = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$ of C_0 and all the matrices in C_0 to get a set C_{00}.

$$C_{00} = M_0 \otimes C_0 \qquad (12)$$

$$C_{00} = \left\{ \alpha \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \alpha \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \right.$$
$$\left. \alpha \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \alpha \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \right\}$$

where $\alpha \in \{1, -1, i, -i\}$.

Four information bits are viewed as a block. The block is mapped onto one of the 16 matrices of C_{00} as an information matrix. Once the matrix V_{i_τ} is obtained, it is used to compute the current transmit matrix $X_\tau = X_{\tau-1} V_{i_\tau}$. There are 4 information bits transmitted during 4 symbol-durations, thus the spectral efficiency is $R = 1$ bps/Hz.

The constellation of the modulation of this scheme (i.e., the possible value of the matrices' elements) is $\{\pm 1, \pm i, 0\}$ which corresponds to $QPSK \cup 0$, and the spectral efficiency is 1bps/Hz. For comparison, the 4×4 DSTBC scheme [12] with modulation BPSK has the same spectral efficiency. However, this new scheme has better BER performance. Furthermore, there are $K = 4608$ distinct matrices in the group G_{w4}. The maximum spectral efficiency we can get is

$R = \frac{1}{M}\lfloor \log_2 K \rfloor = \frac{1}{4}\lfloor \log_2 4608 \rfloor = 3$ bps/Hz. The simulation results with different spectral efficiencies are shown in Fig.4.

For the spectral efficiency $R = 2$, 8 bits should be transmitted in 4 symbol-durations. The information bits are mapped into one of the $2^8 = 256$ matrices. We select the first 256 matrices from G_{w4} as the candidate transmission set. For $R = 3$, we should transmit 12 bits in 4 symbol-durations. Similarly, we select the first 4096 matrices from G_{w4} as the candidate transmission set. The selection of the matrices is arbitrary.

As the method used for 2 transmit antennas, we can also resort to the distance spectrum to improve the BER performance.

IV. SIMULATION RESULTS

In this section, we present several simulation results of the new differential space-time modulation scheme compared to some existing differential space-time modulation schemes. We assume that the channel is quasi-static, i.e., the channel coefficients do not change during one frame of transmission matrices. In our simulation, the length of the frame is $L = 100$ (i.e., the channel coefficients keep constant during 100 symbol durations and change to new independent realizations for the next frame).

We find that for MIMO systems with 2 transmit antennas, our new scheme performs worse than the Tarokh's DSTBC scheme [7], as shown in Fig. 2. This is because the decoding method of our scheme is a general maximum likelihood decoding without any pre-process, while the variable used to decode in [7] is linearly scaled by the channel coefficients due to some pre-process. However, this new scheme performs better than the corresponding DUSTM scheme [6] when SNR is less than 14 dB. This is because the DUSTM is designed for large SNR environments.

Using the distance spectrum of the Weyl group, a new set $S = C_0^r \cup A_3 C_0^r$ is considered as an alternative to C_0. The set C_0^r contains the 8 real matrices of C_0 and $A_3 C_0^r$ is the set obtained by multiplying A_3 with the matrices of C_0^r. As the set C_0, the set S contains 8 couples (M_a, M_b) with $D_{a,b} = ||M_a - M_b|| = 2.8284$, the greatest distance between 2 matrices of G_W. If we consider 2 couples (M_a, M_b) and (M_c, M_d) of C_0, with $D_{a,b} = D_{c,d} = 2.8284$, we have $D_{a,c} = D_{a,d} = D_{b,c} = D_{b,d} = 2$, while for the set S, if $D_{a,b} = D_{c,d} = 2.8284$, then $[D_{a,c}\ D_{a,d}\ D_{b,c}\ D_{a,d}] = [1.4142\ 1.4142\ 2.4495\ 2.4495]$. As shown in Fig.2, the results obtained for C_0 and S are similar.

Fig. 3 shows that, for 4-transmit-antenna systems, this new system performs better than the original Tarokh's differential STBC [9]. On the bit error rate level of 10^{-4}, it's about 0.57 dB better than [9]. And this new scheme performs a little worse than the DUSTM scheme for large SNR.

Fig. 4 shows the BER performance of the new scheme with different spectral efficiency. It shows that with the spectral efficiency up to R = 3 bps/Hz, the new scheme performs well enough. However, Tarokh's DSTBC scheme can not be expanded to 2^b-PSK ($M \geq 4$) modulations, which prevent it
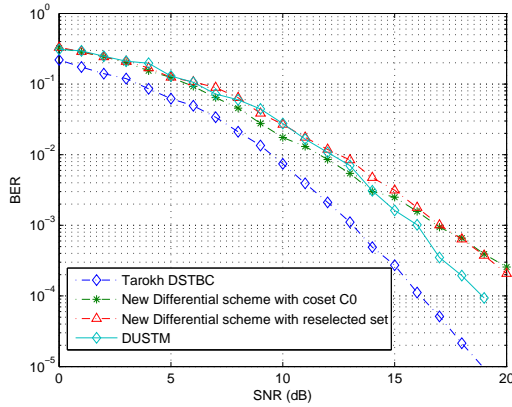
Fig. 2. Simulation results of DSTBC [7], [9], DUSTM [6] and our new differential space-time modulation scheme (M=2, N=2, R=2).

Fig. 3. Simulation results of DSTBC [7], [9], DUSTM [6] and our new differential space-time modulation scheme (M=4, N=4, R=1).

Fig. 4. Simulation results of the new differential space-time scheme for 4 transmi antennas and 4 receive antennas with spectral efficiency 1, 2 and 3 bps/Hz respectively.

to achieve higher spectral efficiency. Furthermore, considering the distance spectrum, we can get a better set for this scheme. As shown in Fig. 4, the new set selected for R = 2 scheme has a better distance spectrum. In fact, we select the first 16 matrices from every successive 192 matrices of the group G_{w4}, i.e., the set is

$$\{N_0, N_1, \ldots, N_{15},$$
$$N_{192}, N_{193}, \ldots, N_{207},$$
$$N_{384}, N_{385}, \ldots, N_{399},$$
$$\ldots, N_{2880}, \ldots, N_{2895}\}.$$

V. Conclusion

In this paper, we proposed a new differential space time modulation scheme based on the Weyl group for 4×4 MIMO systems. Simulation results show that this scheme outperforms the well-known Tarokh's differential STBC scheme. The spectral efficiency of this scheme can be up to 3 bps/Hz, so higher than 1 bps/Hz of Tarokh's DSTBC scheme with four transmit antennas. Furthermore, considering the distance spectrum, the BER performance of our scheme can be improved. The new scheme can be extended to MIMO systems with more transmit antennas, say 8 or 16.

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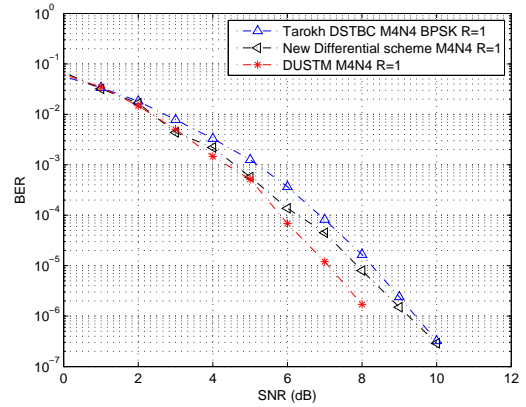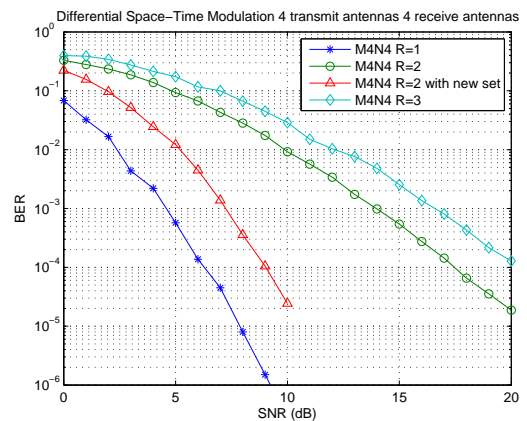
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