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Deconfinement and Gluon Plasma Dynamics in Improved Holographic QCD

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ABSTRACT: The finite temperature physics of the pure glue sector in the improved holographic QCD model of ArXiv:0707.1324 and ArXiv:0707.1349 is addressed. The thermodynamics of 5D dilaton gravity duals to confining gauge theories is analyzed. We show that they exhibit a first order Hawking-Page type phase transition. In the explicit background of ArXiv:0707.1349, we find $T_c = 235$ MeV. The temperature dependence of various thermodynamic quantities such as the pressure, entropy and speed of sound is calculated. The results show a good agreement with the corresponding lattice data.

Contents

1. Introduction

Despite several decades' efforts, an important part of the dynamics of QCD remains far from analytical control and in several cases numerical techniques have proved too difficult to implement. In particular, recent experiments at RHIC seem to probe dynamical properties of the Quark Gluon Plasma (QGP) phase which are not within the reach of lattice techniques without extra assumptions.

On the other hand large-N_c techniques have promised early-on an alternative approach to the strongly coupled physics of QCD based on an effective string theory description of glue. This route took an interesting twist in 1997 with the advent of the Maldacena conjecture [1], with the unexpected result that the string theory must live in more than four dimensions. In particular there is one extra dimension, known as the holographic dimension, that plays the role of (renormalization group) energy scale of the strongly coupled gauge theory.

Since [1] there has been a flurry of attempts to devise such correspondences for gauge theories with less supersymmetry with the obvious final goal: QCD. Several interesting string duals with a QCD-like low lying spectrum and confining IR physics were proposed [2]. In the simplest $D4$ example flavor can be added via the addition of D_8 branes [3] and its finite temperature phase structure has similarities with QCD [4]. Although such theories reproduced the qualitative features of IR QCD dynamics, they contain Kaluza-Klein modes, not expected in QCD, with KK masses of the same order as the dynamical scale of the gauge theory. Above this scale the theories deviate from QCD. Despite the hostile environment of non-critical theory, several attempts

have been made to understand holographic physics in lower dimensions in order to avoid the KK contamination, based on two-derivative gravitational actions, [5].

A different and more phenomenological approach was in the meantime developed and is now known as AdS/QCD. The original idea described in [6] was successfully applied to the meson sector in [7], and its thermodynamics was analyzed in [8]. The bulk gravitational background consists of a slice of AdS$_5$, and a constant dilaton. There is a UV and an IR cutoff. The confining IR physics is imposed by boundary conditions at the IR boundary. This approach, although crude, has been partly successful in studying meson physics, despite the fact that the dynamics driving chiral symmetry breaking must be imposed by hand via IR boundary conditions. Its shortcomings however include a glueball spectrum that does not fit well the lattice data, the fact that magnetic quarks are confined instead of screened, and asymptotic Regge trajectories for glueballs and mesons are quadratic instead of linear. A phenomenological fix of the last problem was suggested by introducing a soft IR wall, [9]. Although this fixes the asymptotic spectrum, it does not allow a proper treatment of thermodynamics. In particular, neither dilaton nor metric equations of motion are solved. Therefore the "on-shell" action is not really on-shell. The entropy computed from the BH horizon does not match the entropy calculated using standard thermodynamics from the free energy computed from the action, etc. Phenomenological metrics for the deconfined phase were also suggested, [10, 11] capturing some aspects of the expected thermodynamics.

In [12] an improved model for QCD was proposed. It united inputs from both gauge theory and string theory while keeping the simplicity of a two derivative action. It could describe both the region of asymptotic freedom as well as the strong IR dynamics of QCD. It is a 5d theory like AdS/QCD.

In this letter we present the finite temperature dynamics in the pure gauge sector derived from the setup of [12]. We find that this setup describes very well the basic features of large-N_c Yang Mills at finite temperature. It exhibits a first order deconfining phase transition. The equation of state and speed of sound of the high temperature phase are remarkably similar to the corresponding lattice results. Moreover, using the zero temperature potential and without adding any extra parameter, we obtain a value for the the critical temperature in very good agreement with the one computed from the lattice. A detailed derivation of the results will appear elsewhere, [13].

2. Improved Holographic QCD at T=0

The holographic model introduced in [12] is five-dimensional. The basic fields that are non-trivial in the vacuum solution, and describe the pure gauge dynamics, are the 5d metric $g_{\mu\nu}$, a scalar Φ (the dilaton) that controls the 't Hooft coupling λ_t of QCD, and an axion a, that is dual to to the QCD θ angle. Moreover, as the kinetic

term of the axion is suppressed by $1/N_c^2$, it does not play any role neither in the geometry, nor in the evolution of the 't Hooft coupling. It has however a non-trivial profile in the vacuum, implying an IR running of the effective θ-angle, [12]. Quarks can be added to the pure gauge theory by adding space-filling $D_4 - \bar{D}_4$ brane pairs in the background gauge theory solution. The $D_4 - \bar{D}_4$ tachyon condensation then induces chiral symmetry breaking, [15, 12].

The action for the 5D Einstein-dilaton theory reads,

$$S_5 = M_p^3 N_c^2 \left(- \int d^5 x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial \lambda)^2}{\lambda^2} + V(\lambda) \right] + 2 \int_{\partial M} d^4 x \sqrt{h} \, K \right) \qquad (2.1)$$

where M_p is the Planck mass [1] and we use the conventions of [14]. The second term in the action is the Gibbons-Hawking with K being the extrinsic curvature on the boundary.

The only nontrivial input in the two-derivative action of the graviton and the dilaton is the dilaton potential $V(\lambda)$, where $\lambda = e^\Phi$. λ is proportional to the 't Hooft coupling of the gauge theory, $\lambda = \kappa \lambda_t$. The constant of proportionality κ cannot be calculated at present from first principles but as we discuss below all of the physical observables turn out to be independent of κ. The potential is directly related to the gauge theory β-function once a holographic definition of energy is chosen. Although the shape of $V(\lambda)$ is not fixed without knowledge of the exact gauge theory β-function, its UV and IR asymptotics can be determined.

In the UV, the input comes from perturbative QCD. We demand asymptotic freedom with logarithmic running. This implies in particular that the asymptotic UV geometry is that of AdS_5 with logarithmic corrections. It requires a (weak-coupling) expansion of $V(\lambda)$ of the form $V(\lambda) = 12/\ell^2 (1 + v_1 \lambda + v_2 \lambda^2 + \cdots)$. Here ℓ is the AdS radius and v_i are dimensionless parameters of the potential directly related to the perturbative β-function coefficients of QCD, [12]. In conformal coordinates, close to the AdS_5 boundary at $r = 0$, the metric and dilaton behave as [2]:

$$ds_0^2 = \frac{\ell^2}{r^2} \left(1 + \frac{8}{9} \frac{1}{\log r\Lambda} + \cdots \right) \left(dr^2 + dx_4^2 \right), \qquad (2.2)$$

$$\lambda_0 = -\frac{1}{\log r\Lambda} + \cdots$$

where the ellipsis represent higher order corrections that arise from second and higher-order terms in the β-function. The mass scale Λ is an initial condition for the dilaton equation and corresponds to Λ_{QCD}.

Demanding confinement of the color charges restricts the large-λ asymptotics of $V(\lambda)$. In [12] we focused on potentials such that, as $\lambda \to \infty$, $V(\lambda) \sim \lambda^{\frac{4}{3}} (\log \lambda)^{(\alpha-1)/\alpha}$

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[1]The physical Planck mass that governs the interactions is $M_p N^{\frac{2}{3}}$. We will however call M_p the Planck mass for simplicity.

[2]We will use a "zero" subscript to indicate quantities evaluated at zero temperature.

where α is a positive parameter. The IR asymptotics of the solution in the Einstein frame are:

$$ds_0^2 \to e^{-C\left(\frac{\tau}{\ell}\right)^\alpha}\left(dr^2 + dx_4^2\right), \quad \lambda_0 \to e^{3C/2\left(\frac{\tau}{\ell}\right)^\alpha}\left(\frac{r}{\ell}\right)^{\frac{3}{4}(\alpha-1)} \tag{2.3}$$

where the constant C is a positive constant related to Λ in (2.2). Confinement requires $\alpha \geq 1$. The parameter α characterizes the large excitation asymptotics of the glueball spectrum, $m_n^2 \sim n^{2(\alpha-1)/\alpha}$. For linear confinement, we choose $\alpha = 2$.

The parameters of the holographic model a priori are: the Planck mass M_p, which governs the scale of interactions between the glueballs in the theory, κ that relates λ and the 't Hooft coupling, the parameters v_i that specify the shape of the potential, the scale Λ that plays the role of Λ_{QCD} and the AdS scale ℓ. The latter is not a physical parameter but only a choice of scale: only $\Lambda\ell$ enters into the computation of physical observables. A specific choice for $V(\lambda)$ was made in [12] with the appropriate asymptotic properties, that only depended on a single parameter which can be taken as v_1, hence fixing all v_i for $i > 1$. Furthermore, one can show that all of the physical observables both at zero T and finite T are left invariant under a rescaling of λ. More concretely, given a potential $V(\lambda)$ and a dilaton profile that follows from this potential with an integration constant Λ, there exists another profile with a different integration constant Λ_η which follows from a rescaled potential $V_\eta(\lambda) = V(\eta\lambda)$ and the two solutions yield the same glueball spectra and the same thermodynamic observables. This symmetry allows one to scale away the parameter κ. Finally, v_1 and Λ are fixed by matching to the lattice data for the first two 0^{++} glueball masses. Once Λ is fixed, all other interesting scales, like the fundamental string scale ℓ_s and the effective QCD string tension σ are also fixed.

This determines all the parameters of the theory except the Planck mass $M_p\ell$. We shall show below that M_p can be indirectly inferred from the large temperature behavior.

3. The deconfinement transition

At finite temperature there exist two distinct types of solutions to the action (2.1) with AdS asymptotics, (2.2):

i. The thermal graviton gas, obtained by compactifying the Euclidean time in the zero temperature solution with $\tau \sim \tau + 1/T$:

$$ds^2 = b_0^2(r)\left(dr^2 + d\tau^2 + dx_3^2\right), \quad \lambda = \lambda_0(r). \tag{3.1}$$

This solution exists for all $T \geq 0$ and corresponds to a confined phase, if the gauge theory at zero T confines.

ii. The black hole (BH) solutions (in Euclidean time) of the form:

$$ds^2 = b^2(r)\left(\frac{dr^2}{f(r)} + f(r)d\tau^2 + dx_3^2\right), \quad \lambda = \lambda(r). \qquad (3.2)$$

The function $f(r)$ approaches unity close to the boundary at $r = 0$. There exists a singularity in the interior at $r = \infty$ that is now hidden by a regular horizon at $r = r_h$ where f vanishes. Such solutions correspond to a deconfined phase.

As we discuss below, in confining theories the BH solutions exist only above a certain minimum temperature, $T > T_{min}$.

The thermal gas solution has two parameters: T and Λ. The black hole solution should also have a similar set of parameters: the equations of motion are second order for λ and f, and first order for b [13]. Thus, *a priori* there are 5 integration constants to be specified. A combination of two integration constants of b and λ determines Λ. (The other combination can be removed by reparametrization invariance in r). The condition $f \to 1$ on the boundary removes one integration constant and demanding regularity at the horizon, $r = r_h$, in the form $f \to f_h(r_h - r)$, removes another. The remaining integration constant can be taken as f_h, related to the temperature by $4\pi T = f_h$. From Einstein's equations one can show [13]:

$$4\pi\, T = b^{-3}(r_h)\left(\int_0^{r_h} \frac{du}{b(u)^3}\right)^{-1}. \qquad (3.3)$$

In the large N_c limit, the saddle point of the action is dominated by one of the two types of solutions. In order to determine the one with minimum free energy, we need to compare the actions evaluated on solutions i. and ii. with equal temperature.

We introduce a cutoff boundary at $r/\ell = \epsilon$ in order to regulate the infinite volume. The difference of the two scale factors is given near the boundary as [13]:

$$b(\epsilon) - b_0(\epsilon) = \mathcal{C}(T)\epsilon^3 + \cdots \qquad (3.4)$$

By the standard rules of AdS/CFT we can relate $\mathcal{C}(T)$ to the difference of VEVs of the gluon condensate: $\mathcal{C}(T) \propto \langle \mathrm{Tr}F^2 \rangle_T - \langle \mathrm{Tr}F^2 \rangle_0$.

The free energy difference is given by [13]:

$$\begin{aligned}
\mathcal{F} &= M_p^3 N_c^2 V_3 \left(15\mathcal{C}(T)\ell^{-1} - \pi T b^3(r_h)\right) \\
&= 15\mathcal{C}(T)\, M_p^3 N_c^2 V_3 \ell^{-1} - \frac{TS}{4},
\end{aligned} \qquad (3.5)$$

where, in the last equality, we used the fact that the entropy is given by the area of the horizon. It is clear that the existence of a non-trivial deconfinement phase transition is driven by a non-zero value for the thermal gluon condensate $\mathcal{C}(T)$.

For a general potential we can prove the following statements, that only require the validity of the laws of black hole thermodynamics:

i. *There exists a phase transition at finite T, if and only if the zero-T theory confines.*

ii. *This transition is of the **first order** for **all** of the confining geometries, with a single exception described in iii:*

iii. *In the limit confining geometry $b_0(r) \to \exp(-Cr)$ (as $r \to \infty$), the phase transition is of the **second order** and happens at $T = 3C/4\pi$.*

iv. *All of the non-confining geometries at zero T are always in the black hole phase at finite T. They exhibit a second order phase transition at $T = 0^+$.*

We now sketch a heuristic argument, limited to asymptotics of the type (2.3). A general, coordinate independent proof will appear in [13].

The existence of a minimum black hole temperature T_{min} in confining theories follows from the small and large r_h behavior of the geometries. On one hand, the black-hole approaches an AdS-Schwarzschild geometry near the boundary, which obeys $T = 1/\pi r_h$. On the other hand, as the horizon approaches the deep interior *i.e.* $r_h \to \infty$, the mass of the black-hole vanishes and the black hole solution approaches the zero-T geometry in this limit. In passing, we note that this implies vanishing of $\mathcal{F}$ in this limit. Using the large r_h limit in (3.3), we find the following asymptotics for T:

$$T \to \frac{3C\alpha}{4\pi} r_h^{\alpha-1}, \; r_h \to \infty; \quad T \to \frac{1}{\pi r_h}, \; r_h \to 0. \tag{3.6}$$

The large r_h behavior in eq. (3.6) is valid under the assumption that the zero-T solution, with IR asymptotics (2.3), can be continuously deformed into a black hole with arbitrarily small mass and arbitrarily large value of r_h. This assumption indeed holds, as we will show elsewhere [13] for a more general class of confining backgrounds.

Eq. (3.6) shows that for $\alpha \geq 1$, that there exists a minimum temperature $T_{min} > 0$ above which the black-hole solutions exist. Here, for simplicity, we assume a single extremum of the function $T(r_h)$. We illustrate the function $T(r_h)$ schematically in figure 1. The simple convex shapes in (a) are due to our assumption of a single minimum. In general the function $T(r_h)$ may exhibit multiple extrema. Our demonstration here can be generalized to these cases [13]. In the confining geometries $\alpha > 1$, for a given $T > T_{min}$, there exist a big and a small black hole solution, given by $r_h < r_{min}$ and $r_h > r_{min}$ respectively, see fig.1. The big BH has positive specific heat hence it is thermodynamically stable, whereas the small BH is unstable. In the borderline confining geometry $\alpha = 1$, there is a single BH solution.

Existence of a critical temperature $T_c \geq T_{min}$ for $\alpha \geq 1$ follows from the physical requirement of positive entropy. From the first law of thermodynamics, it follows that
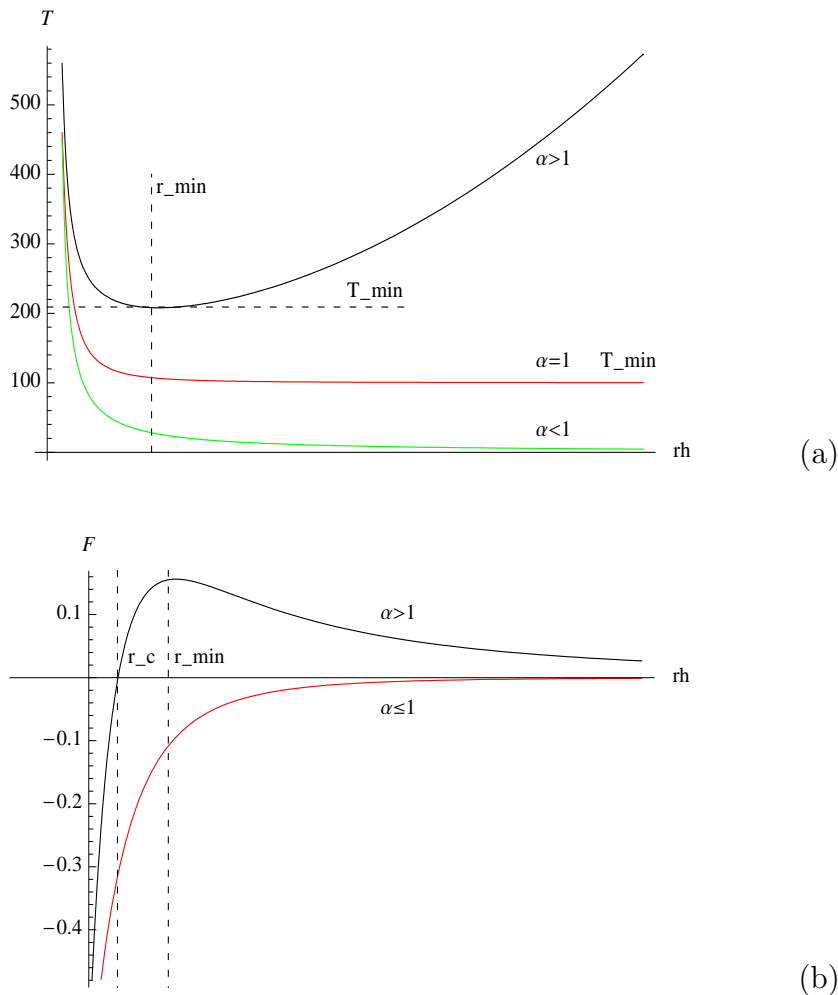
Figure 1: Schematic behavior of temperature (a) and free energy density (b) as a function of r_h, for the infinite-r geometries of the type (2.3), for different values of α.

$d\mathcal{F}/dr_h = -S\,dT/dr_h$. Then, as $S > 0$ for any physical system, extrema of $\mathcal{F}(r_h)$ should coincide with the extrema of $T(r_h)$. Using also the fact that $\mathcal{F}(r_h) \to -\infty$ for $r_h \to 0$ and $\mathcal{F}(r_h) \to 0$ near $r_h \to \infty$, we arrive at conclusion (ii) described above: *There is a first order transition for all of the confining geometries.*

An interesting case is the borderline confining geometry, where T_c coincides with T_{min} and located at $r_h = \infty$. The entropy vanishes there because the geometry shrinks to zero size. The free energy also vanishes because this point coincides with T_c. Therefore the latent heat also vanishes and one has a *second order transition*. Although this geometry is not interesting for the gauge theory, it is of some interest for GR. We recall [12], that it corresponds to an asymptotically AdS geometry that becomes a linear dilaton background in the deep interior. We have shown that such a geometry exhibits a second order Hawking-Page transition into a black-hole solution. By similar arguments, point iv of the proposition above can also be demonstrated

without difficulty.

Finally, the small r_h asymptotics also allows us to fix the value of the Planck mass in (2.1). Small r_h corresponds to high T. This geometry corresponds to an ideal gas of gluons with a free energy density $\mathcal{F} \to (\pi^2/45)N_c^2 V_3 T^4$. On the other hand, as the geometry becomes AdS, eq. (3.5) implies[3] that: $\mathcal{F} \to \pi^4(M_p\ell)^3 N_c^2 T^4 V_3$. Hence we conclude that,

$$M_p\ell = \left(45\pi^2\right)^{-\frac{1}{3}} . \tag{3.7}$$

Using the value of ℓ in [12], we obtain $M_p \approx 2.32$ GeV.

4. Numerical Results

In [12] an explicit form of the scalar potential with the correct asymptotics was proposed. The resulting background, that corresponds to the choice $\alpha = 2$ in (2.3), exhibits asymptotic freedom, linear confinement, and a glueball spectrum in very good quantitative agreement with the lattice data. Here we present a numerical computation of the relevant thermodynamic quantities in the same theory.

The potential chosen in [12] was fixed such that the UV expansion reproduces the Yang-Mills beta-function up to two loops and has the large-λ asymptotics $V(\lambda) \sim \lambda^{4/3}(\log \lambda)^{1/2}$. It depends on two parameters: the first is the overall normalization (that fixes the AdS length ℓ and the energy units); the second is b_0, that is equivalent to the coefficient of linear term in the small λ expansion, i.e. v_1. These parameters were fit to reproduce the lattice results for the two lowest scalar glueball masses.

Our general analysis shows that this theory has black hole solutions above a temperature T_{min} and exhibits a first order phase transition at some $T_c > T_{min}$

To analyze the behavior of the theory at finite temperature, we have solved numerically Einstein's equations for the metric and dilaton. The integration constants were fixed as explained earlier. We find a minimum temperature for the existence of black hole solutions, $T_{min} = 210$ MeV.

Next, we compute the free energy difference between the black hole and thermal gas solutions, as a function of temperature. As shown in eq. (3.5), there are two competing contributions, which must be dealt with separately:

1. The term $\pi T b^3(r_h)$ can be obtained directly by evaluating the numerical solution at the horizon.

2. The term $15\mathcal{C}(T)\ell^{-1}$ must be extracted by fitting the coefficient of the cubic term in the black hole scale factor close to the boundary, $b(r) - b_0(r) \sim \mathcal{C}(T)r^3$. This is a large source of error in our numerics, since it is a tiny quantity arising as a difference of $O(1)$ quantities.

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[3]It can be shown that the first term in (3.5) is subleading in the high T limit.

The resulting free energy as a function of the temperature is shown in figure 2, which clearly shows the existence of a minimum temperature, and a first order phase transition at $T = T_c$, where $\mathcal{F}(T_c) = 0$. For $T < T_c$, the thermal gas dominates, and the system is in the confined phase. For $T > T_c$, the (large) black hole dominates, corresponding to a deconfined phase. The small black hole branch is thermodynamically disfavored at all temperatures.
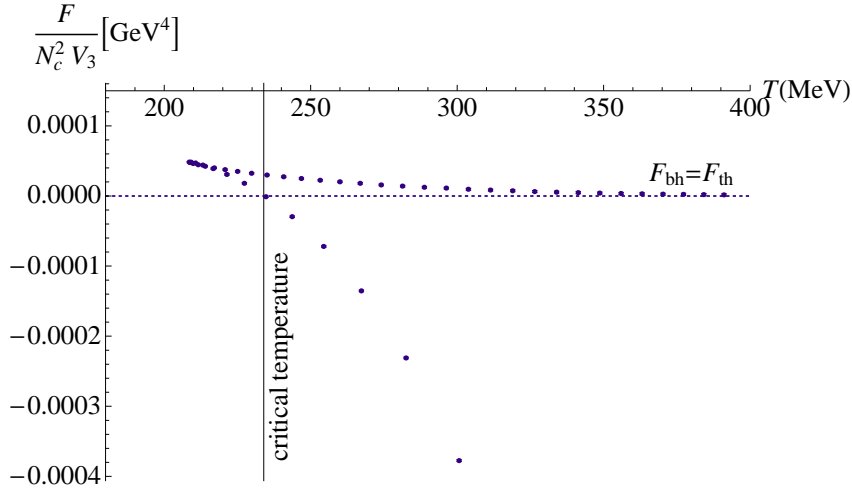


Figure 2: Black hole free energy

The value we obtain for the critical temperature, $T_c = \mathbf{235 \pm 15}$ MeV, is close to the value obtained for large-N Yang-Mills [16], which with our normalization of the lightest glueball would be 260 ± 11 MeV [4]. It should be emphasized that, we did not have to adjust any new parameter with respect to the zero-temperature theory in order to obtain this result.

From the free energy we can determine all other quantities by thermodynamic identities. However, for numerical precision it is preferable to derive the entropy directly as the black hole area, rather than as a derivative of the free energy. The latter suffers from the uncertainty in the determination of $\mathcal{C}(T)$. Also, due to the linear dependence of all thermodynamic quantities on V_3, it is convenient to use densities. The pressure, and the energy and entropy densities of the deconfined phase are given by:

$$p = -\mathcal{F}/V_3, \quad s = 4\pi M_p^3 N_c^2 b_T^3(r_h), \quad \epsilon = p + Ts. \tag{4.1}$$

Next, we present some of the thermodynamic quantities that are compared with the lattice results. It is useful to compare dimensionless quantities, so that the ℓ-dependence drops out.

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[4]The physical units are obtained by fixing $m_{0^{++}} = 1475$ MeV as in [12]. The value 260 ± 11 MeV is obtained combining the results in [16] and [17]
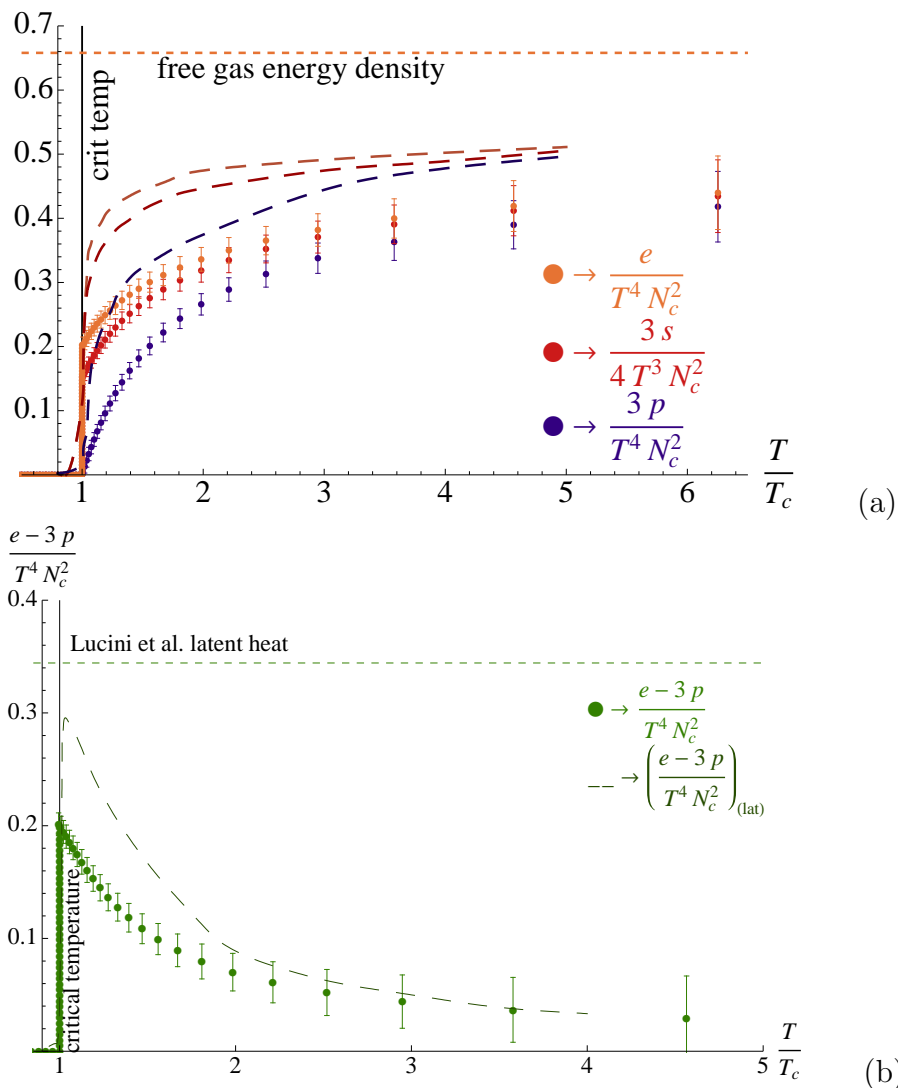
Figure 3: (a) Dimensionless thermodynamic functions and (b) interaction measure. The dashed curves correspond to the lattice data of [18]

Latent Heat The latent heat per unit volume is defined as the jump in the energy at the phase transition, $L_h = T_c \Delta s(T_c)$, and it is expected to scale as N_c^2 in the large N_c limit [16]. From eq. (4.1) we note that this expectation is reproduced in our theory. Quantitatively, we find $L_h^{1/4}/T_c \simeq 0.65\sqrt{N_c}$. This is to be compared with the value 0.77 reported in [16].

Equation of state and the interaction measure. A useful indication about the thermodynamics of a system is given by the relations between the quantities ϵ/T^4, $3(p/T^4)$, $3/4(s/T^3)$ (the normalizations are chosen so that they all equal the same constant in the case of a free relativistic gas). In figure 3 (a) we compare our

results for these quantities with the corresponding lattice results, reported in [18][5]. In the low temperature phase, the thermodynamic functions vanish to the leading order in N_c^2 and the jump in ϵ and s at T_c reflects the first order phase transition.

The *interaction measure*, $(\epsilon - 3p)/T^4$ (proportional to the trace anomaly), is plotted in figure 3 (b), together with the lattice result from [18]. From eq. (3.5), $\epsilon - 3p \propto \mathcal{C}(T)$, consistent with our interpretation of $\mathcal{C}(T)$ as the gluon condensate.

Speed of sound. This quantity is defined as $c_s^2 = (\partial p/\partial \epsilon)_S = s/c_v$. It is expected to be small at the phase transition, and to reach the conformal value $c_s^2 = 1/3$ at high temperatures. In figure 4 we compare our results with the lattice data, finding good agreement.
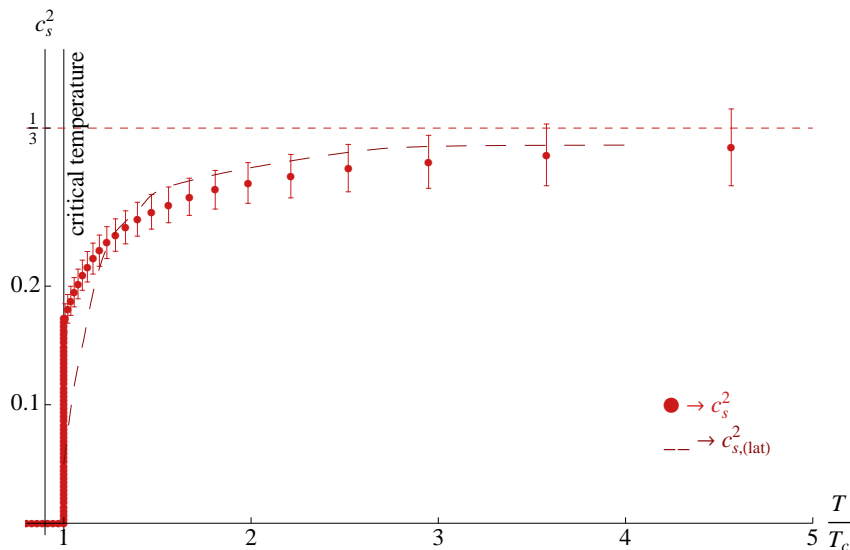
Figure 4: Comparison between the speed of sound in our model and the lattice result of [18] (dashed curves)

Shear viscosity. In agreement with the general results of [19], the ratio between shear viscosity and entropy density is $\eta/s = (4\pi)^{-1}$.

5. Discussion

The model presented here describes well the basic features of large-N_c Yang Mills at finite temperature: it exhibits a first order deconfining phase transition, and the temperature dependence of the pressure, entropy, energy density, interaction measure and speed of sound in the high temperature phase behave similarly to the corresponding lattice results. Without adding any extra parameter, one obtains a value for the critical temperature 10 % off the lattice value.

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[5]These results are for $N_c = 3$; we are unaware of similar plots obtained in the large N_c limit.

On the other hand the model can be improved in many ways. The latent heat L_h/T_c^4 is 40% off the lattice value. Also, our comparison shows that (see e.g. fig 3a) approach to the free field limit at high T is slower than the lattice data. This may be traced back to the relative smallness of the latent heat in our potential. Although the UV and the IR asymptotics of the dilaton potential are fixed by general requirements from the field theory, the intermediate region is free to modify. The reason is that the low-level glueball spectrum and the thermodynamics near the phase transition are not controlled by the same regions of the potential. With a suitable deformation one hopes to obtain better agreement with the lattice data. In particular, it is possible to obtain a fit to quantities in figs. 3 and 4, well within the errors of the lattice data in a temperature range $T_c < T < 5T_c$ [13]. Retrofitting the potential is an interesting challenge that we plan to address in [13].

Acknowledgments

Note added While this paper was being written, the work [20] appeared, discussing related issues in a similar setup.

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