Accurate Array Diagnosis from Near-Field Measurements Using ℓ_1 Reweighted Minimization

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***Abstract*—In this contribution the use of ℓ_1 weighted minimization for the diagnosis of arrays from a reduced set of near-field data is investigated. Numerical results show that reweighed method gives a higher probability of an accurate estimation of the failures compared to the classic ℓ_1 minimization proposed in the past literature.**

I. Introduction

Near-field measurements are largely used in arrays diagnosis. In these measurement systems the data acquisition time is an important factor, and investigation of algorithms able to reduce the number of measured data is of interest [1], [2].

Recently, efficient sparse recovery technique was proposed in the framework of antenna measurements in order to reduce the number of measured data [4] - [6]. In particular, in [5] an ℓ_1 minimization technique was proposed to identify the fault elements in large arrays from a highly reduced set of measurements.

On the other hand, the reweighed ℓ_1 minimization algorithm proposed by Candes, Wakin and Boyd [7] was succesfully applied in the framework of sparse array synthesis [8], [9], showing better performance compared to the ℓ_1 standard minimization algorithm. Starting from these results, the application of the reweighed ℓ_1 minimization algorithm in array diagnosis from near-field measurements is currently under investigation.

This contribution presents some preliminary results, that confirm the effectiveness of the reweighed ℓ_1 minimization in antenna diagnosis.

II. The Model

Let us consider an Array Under Test (AUT) consisting of N radiating elements located in known positions $\mathbf{r_n}$. Let x_n and $\mathbf{f_n}(\theta, \phi)$ be the excitation coefficient and the electric-field radiation pattern of the n-th radiating element, respectively. A probe having effective height $\mathbf{h}(\theta, \phi)$ is placed in M spatial points $\mathbf{r_m}$, $m = 1, ..., M$. The voltage at the probe output can be expressed by the linear system

$$\mathbf{Ax} = \mathbf{y} \tag{1}$$

wherein $\mathbf{y} = (\mathbf{y_1}, \mathbf{y_2}, ..., \mathbf{y_n})^\mathbf{T} \in \mathbf{C^M}$, y_m being the probe voltage measured at point $\mathbf{r_m}$, $\mathbf{x} = (\mathbf{x_1}, ..., \mathbf{x_N})^\mathbf{T} \in \mathbf{C^N}$, $\mathbf{A} \in \mathbf{C^{M\times N}}$ is a matrix whose element (m, n) is equal to $\exp(-j\beta r_{m,n})/(4\pi r_{m,n})\mathbf{f}(\theta_\mathbf{m,n}, \phi_\mathbf{m,n}) \cdot \mathbf{h}(\theta'_\mathbf{m,n}, \phi'_\mathbf{m,n})$, $r_{m,n} = |\mathbf{r_m} - \mathbf{r_n}|$, $\theta_{m,n}$ and $\phi_{m,n}$ are the relative angles between the m-th measurement point and the n-th element position in a reference system centered on the n-th array radiating element.

In array diagnostic the goal is to to identify the fault elements.

Following the approach proposed in [5], we suppose that a reference failure-free array is available. As first step, we characterize this array of reference, obtaining the vector $\mathbf{x}^r \in C^N$ containing the (failure-free) excitation coefficients, and the vector $\mathbf{y}^r \in C^M$ containing the value of the probe voltage in the measurement points.

Then the field radiated by the AUT is measured Let $\mathbf{x}^d = \{x_1, ..., x_N\}^T$ be the vector of excitations of the AUT and $\mathbf{y}^d = \{y_1, ..., y_M\}^T$ the vector collecting the far-field measured data.

Now, let us consider the system

$$\mathbf{Ax} = \mathbf{y} \tag{2}$$

wherein $\mathbf{x} = \mathbf{x}^d - \mathbf{x}^r$ and $\mathbf{y} = \mathbf{y}^d - \mathbf{y}^r$.

If the number of fault elements S is small (as usually happens) compared to the total number of elements N, i.e. if $S << N$, we have an equivalent problem involving a highly sparse array. It means that the $\mathbf{x}$ we are looking for is sparse.

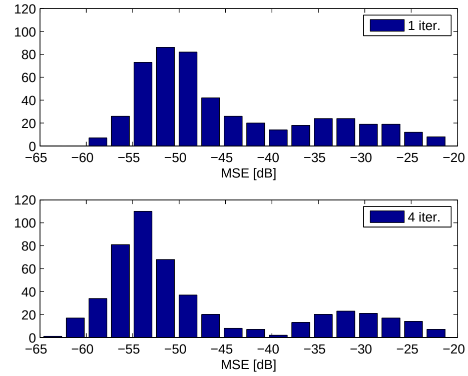

Fig. 1. Linear array, N=111, M=25, S=7, SNR=50 dB; occurrence of the MSE, 500 trials; upper histogram: standard ℓ_1 minimization algorithm; lower histogram: weighted ℓ_1 minimization algorithm (4 iterations).

The above technique basically allows to decrease the amount of information required to retrieve the unknown vector by introducing a-priori information on the nominal excitation in the model [10].

In [5] the sparse data were retrieved using the ℓ_1 minimization proposed in sparse recovery/compressed sensing literature. Recently, a reweighted version of this algorithm was proposed by Candes, Wakin and Boyd [7].

Basically, the use of weighted ℓ_1 norm allows to avoid to penalize the highest entries of $\mathbf{x}$, solving the following iterative procedure:

$$\arg\min \sum_{i=1}^{N} w_i^k |x_i^k| \text{ subject to } \|\mathbf{y} - \mathbf{A}\mathbf{x}^k\|_2 < \epsilon \qquad (3)$$

wherein k is the iteration index, ϵ is fixed by the noise level affecting the vector $\mathbf{y}$ of the measured data, $w_i^k = \frac{1}{|x_i^{k-1}|+\eta}$ and η is a small quantity greater than 0 to ensure the numerical stability of the algorithm.

Note that at the first step the reweighed ℓ_1 minimization gives the same result of the standard ℓ_1 minimization procedure.

III. Numerical results and Conclusions

The AUT is a linear array of $N = 111$ isotropic radiating elements. The nominal excitation is given by Chebyshev coefficients giving a -30 dB equiripple far-field pattern. A number of $S = 7$ failures, represented by zero amplitude excitation coefficients, are randomly selected among the 111 coefficients. The radiated fields are measured in $M = 25$ points, placed on a uniform 12λ linear grid placed at $d = 20\lambda$ distance from the AUT. A Gaussian random noise is added to the measured data. A number of 500 trails, considering random failure positions and measurement positions, were carried out for a given SNR.

The histogram of the occurrence of the Mean Square Error of the excitations is plotted in Fig. 1 in case of standard ℓ_1 minimization algorithm (upper histogram) and weighed

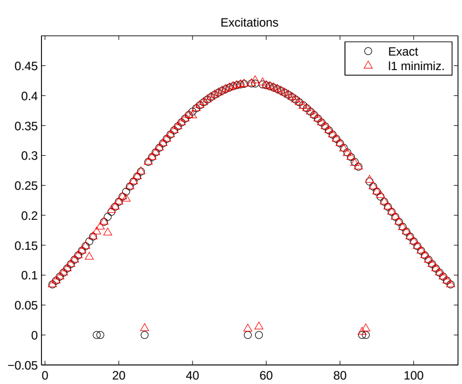

Fig. 2. Linear array, N=111, M=25, S=7,; black points=exact excitation coefficients; red points: excitation coefficients estimated using ℓ_1 minimization; MSE= -32 dB.

ℓ_1 minimization after 4 iterations (lower histogram). The histograms show a bi-modal behavior, with two maxima respectively around -32 dB and -53 dB. In order to have a qualitative indication of the effectiveness of the excitation estimation as function of the MSE, the excitation in case of MSE equal to -32 dB is plotted in Fig. 2. The plot shows that, even if the most part of the failures are recognizable, the solution is 'non accurate' since the procedure is not able to clearly identify all the failures associated to small excitation coefficients. Numerical simulations indicated that an error lower than about -38 dB makes highly likely to identify all the failues.

Coming back to Fig. 1, the histograms show that the weighed ℓ_1 minimization is able to improve the accuracy of the solutions assoaciated to an error lower than about -38 dB in the ℓ_1 minimization. However, the number of trials that give a MSE lower that -38 dB does not significantly change with the number of iterations. As a consequence, the figures suggest that the use of the reweighted ℓ_1 algorithm is advantageous provided that a preliminary study on the number of measurements required to reduce the occurrence of 'non-accurate' reconstructions to a negligible value is carried out.

It is worth stressing that the results presented in this paper are preliminary, and further studies are required to understand the performance of reweighted ℓ_1 minimization in the framework of near-field antenna measurements.

Acknowledgement

This work was partially supported by MIUR under grant #20093CJEJ5_002

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