Polarization Synthesis of Arbitrary Arrays with Shaped Beam Pattern

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***Abstract*—The joint synthesis of the spatial power pattern and polarization of arbitrary arrays is addressed. Specifically, the element excitations are determined such that the array radiates a shaped power pattern while its polarization is optimized in an angular region. Any state of polarization (elliptical, circular and linear) can be synthesized and there is no restriction regarding the array geometry and element patterns. The synthesis problem is rewritten as a convex optimization problem, that is efficiently solved using readily available software. Various numerical results are presented to validate the proposed method and illustrate its potentialities.**

I. Introduction

The synthesis of shaped beams with antenna arrays is a problem having many applications in radar, remote sensing and communication systems [1], [2]. On top of controlling the spatial power radiated by the array, optimizing the waveform polarization is also of uppermost interest. It enables for instance to improve the performances of active sensing [3] and communication systems [4], [5]. For instance, the generation of a shaped beam with a given polarization is a frequently encountered requirement for spatial applications. Thus, LEO satellite antenna shall provide an isoflux coverage of the earth surface with a circular polarization over a wide angular range [6].

The polarization synthesis of arrays with focused beam patterns has been addressed in [7], [8]. In these papers, the joint synthesis of the focused beam pattern and the polarization is formulated as a convex optimization problem, which ensures the optimality of the obtained solution.

The synthesis of shaped beams is a more difficult problem since it is not convex in the general case. However, an iterative algorithm has been recently proposed in [9] to synthesize shaped beams in which each step is reduced to a simple convex optimization problem.

In this paper, the methods detailed in [8] and [9] are combined to synthesize arrays radiating a shaped beam with an optimized polarization. The proposed algorithm consists in solving a sequence of convex optimization problems. It can thus be readily implemented using freely accessible routines.

The paper is organized as follows. In Section II, the synthesis problem is described and the resolution method is detailed. The proposed method is then applied in Section III to show its interest and efficiency. Conclusions are drawn in Section IV.

II. Array Synthesis Problem

A. Antenna Array

Let us consider an antenna array composed of N elements placed at arbitrary but known locations $\vec{r}_i$ with $i = 1, ..., N$. The problem is described for a one-dimensional pattern synthesis without loss of generality. This synthesis is performed over the polar angle θ in a fixed azimuthal plane $\varphi = \varphi_0$, that is omitted in the notations. The extension to a two-dimensional (2-D) pattern synthesis, i.e. a synthesis over both angular directions θ and φ, is straightforward.

The array factor $\boldsymbol{g}(\theta)$ in the direction θ is:

$$\boldsymbol{g}(\theta)^H = [e^{-jk\vec{r}_1.\hat{r}(\theta)} \cdots e^{-jk\vec{r}_N.\hat{r}(\theta)}], \tag{1}$$

where $.^H$ denotes the Hermitian transposition, k is the free space wave number and $\hat{r}(\theta)$ is the unit vector in the direction θ (and azimuthal plane φ_0).

The element i of the array radiates a vectorial far field pattern which has, in general, both a θ and φ component: $(f_{\theta i}(\theta), f_{\varphi i}(\theta))$. This yields the following vectorial antenna array responses:

$$\begin{cases} \boldsymbol{a}_\theta(\theta)^H = [f_{\theta 1}(\theta)e^{-jk\vec{r}_1.\hat{r}(\theta)} \cdots f_{\theta N}(\theta)e^{-jk\vec{r}_N.\hat{r}(\theta)}] \\ \boldsymbol{a}_\varphi(\theta)^H = [f_{\varphi 1}(\theta)e^{-jk\vec{r}_1.\hat{r}(\theta)} \cdots f_{\varphi N}(\theta)e^{-jk\vec{r}_N.\hat{r}(\theta)}] \end{cases} \tag{2}$$

The electric field radiated by the array is:

$$\boldsymbol{E}(\theta) = \begin{bmatrix} E_\theta(\theta) \\ E_\varphi(\theta) \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_\theta(\theta)^H \boldsymbol{w} \\ \boldsymbol{a}_\varphi(\theta)^H \boldsymbol{w} \end{bmatrix}, \tag{3}$$

where $\boldsymbol{w}$ is the N-dimensional vector of complex (amplitude and phase) element excitations. The excitation vector $\boldsymbol{w}$ is thus composed of N scalars w_i that control the vectorial field $\boldsymbol{E}(\theta)$ radiated by the array. These complex excitations w_i are the unknowns to determine.

B. Array Pattern Synthesis

The array pattern synthesis problem amounts to find the excitations $\boldsymbol{w}$ in (3) to achieve a pattern that has both spatial power and polarization requirements. Specifically, the goal is to synthesize a pattern having:

- a shaped beam, i.e. a power pattern close to a desired shape $d(\theta)$ over an angular region SB,
- sidelobes below a given upper bound $\rho(\theta)$ over SL,
- a wave polarization characterized by (γ_0, δ_0) over POL.

These requirements can be written:

$$\begin{cases} \sup\limits_{\theta\in \text{SB}} \left| |\boldsymbol{E}(\theta)|^2 - d(\theta) \right| \leq \epsilon_{sb}(\theta) \\ |\boldsymbol{E}(\theta)|^2 \leq \rho(\theta), & \forall\ \theta \in \text{SL} \\ \left| E_\varphi(\theta) - \gamma_0 e^{j\delta_0} E_\theta(\theta) \right| \leq \epsilon_{pol}(\theta), & \forall\ \theta \in \text{POL} \end{cases} . \tag{4}$$

In the first constraint, the maximum "distance" between the targeted shape $d(\theta)$ and the power radiated by the array $|\boldsymbol{E}(\theta)|^2$ is bounded by $\epsilon_{sb}(\theta)$. The accuracy of the polarization is guaranteed via the third constraint of (4) which amounts to keep under control the ratio between the two components of the field: $E_\varphi(\theta)$ and $E_\theta(\theta)$. When $(\gamma_0, \delta_0) = (1, \pm\pi/2)$, i.e. when $|E_\varphi(\theta) \pm jE_\theta(\theta)|$ is upper bounded or minimized, a circular polarization is synthesized. More details about this formulation can be found in [8].

Using (3), the synthesis problem can be expressed: find $\boldsymbol{w}$ such as the constraints:

$$\begin{cases} \sup\limits_{\theta\in \text{SB}} \left| \boldsymbol{w}^H [\boldsymbol{a}_\theta(\theta) + \boldsymbol{a}_\varphi(\theta)][\boldsymbol{a}_\theta(\theta) + \boldsymbol{a}_\varphi(\theta)]^H \boldsymbol{w} - d(\theta) \right| \cdots \\ \qquad\qquad \cdots \leq \epsilon_{sb}(\theta) \\ \left| [\boldsymbol{a}_\theta(\theta) + \boldsymbol{a}_\varphi(\theta)]^H \boldsymbol{w} \right| \leq \sqrt{\rho(\theta)}, & \forall\ \theta \in \text{SL} \\ \left| [\boldsymbol{a}_\varphi(\theta) - \gamma_0 e^{j\delta_0} \boldsymbol{a}_\theta(\theta)] \boldsymbol{w} \right| \leq \epsilon_{pol}(\theta), & \forall\ \theta \in \text{POL} \end{cases} . \tag{5}$$

are satisfied.

This method can easily be adapted to optimize the polarization radiated by the array while the shaped beam is upper and lower bounded by $u(\theta)$ and $l(\theta)$ respectively. The synthesis problem becomes:

$$\min_{\boldsymbol{w}} \epsilon_{pol} \quad \text{under} \quad (5),$$

where $\epsilon_{sb}(\theta) = (u(\theta) - l(\theta))/2$ and $d(\theta) = (u(\theta) + l(\theta))/2$.

C. Resolution Method

To solve the synthesis problem (5), the regions SB, SL and POL are sampled in θ_k, θ_m and θ_q directions, respectively. Let us introduce the following notations: $\boldsymbol{a}_{\theta i} = \boldsymbol{a}_\theta(\theta_i)$, $\boldsymbol{a}_{\varphi i} = \boldsymbol{a}_\varphi(\theta_i)$, $d_k = d(\theta_k)$ and $\rho_m = \rho(\theta_m)$. The constraints (5) can then be rewritten:

$$\begin{cases} \max\limits_{k=1,\ldots,K} \left| \boldsymbol{w}^H [\boldsymbol{a}_{\theta k} + \boldsymbol{a}_{\varphi k}][\boldsymbol{a}_{\theta k} + \boldsymbol{a}_{\varphi k}]^H \boldsymbol{w} - d_k \right| \leq \epsilon_{sb}(\theta_k), \\ \left| [\boldsymbol{a}_{\theta m} + \boldsymbol{a}_{\varphi m}]^H \boldsymbol{w} \right| \leq \sqrt{\rho_m}, & \text{for } m = 1, ..., M \\ \left| [\boldsymbol{a}_{\varphi q} - \gamma_0 e^{j\delta_0} \boldsymbol{a}_{\theta q}] \boldsymbol{w} \right| \leq \epsilon_{pol}(\theta_q), & \text{for } q = 1, ..., Q \end{cases} . \tag{6}$$

The first constraint of (6) is equivalent to:

$$\max_{k=1,\ldots,K} \left| \boldsymbol{w}_l^H [\boldsymbol{a}_{\theta k} + \boldsymbol{a}_{\varphi k}][\boldsymbol{a}_{\theta k} + \boldsymbol{a}_{\varphi k}]^H \boldsymbol{w}_r - d_k \right| \leq \epsilon_{sb}(\theta_k), \tag{7}$$

with $\boldsymbol{w}_l = \boldsymbol{w}_r$.

Then, if the vector $\boldsymbol{w}_l$ or $\boldsymbol{w}_r$ in (6) is alternatively fixed, the constraints become linear with respect to the other vector. By fixing $\boldsymbol{w}_l$ for instance, the N dimensional vector $\boldsymbol{c}_k^H = \boldsymbol{w}_l^H [\boldsymbol{a}_{\theta k} + \boldsymbol{a}_{\varphi k}][\boldsymbol{a}_{\theta k} + \boldsymbol{a}_{\varphi k}]^H$ is constant and the problem to solve amounts to find $\boldsymbol{w}_r$ such that:

$$\begin{cases} \max\limits_{k=1,\ldots,K} \left| \boldsymbol{c}_k^H \boldsymbol{w}_r - d_k \right| \leq \epsilon_{sb}(\theta_k), \\ \left| [\boldsymbol{a}_{\theta m} + \boldsymbol{a}_{\varphi m}]^H \boldsymbol{w}_r \right| \leq \sqrt{\rho_m}, & \text{for } m = 1, ..., M \\ \left| [\boldsymbol{a}_{\varphi q} - \gamma_0 e^{j\delta_0} \boldsymbol{a}_{\theta q}] \boldsymbol{w}_r \right| \leq \epsilon_{pol}(\theta_q), & \text{for } q = 1, ..., Q \end{cases} . \tag{8}$$

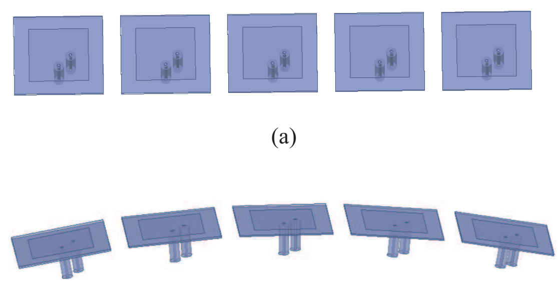
This optimization problem is convex and therefore can be efficiently solved optimally using readily available routine such as Sedumi [10].

To ensure the equivalence between the convex set of constraints (8) and the initial ones (6), $\boldsymbol{w}_r$ must be equal to $\boldsymbol{w}_l$ as written in (7).

To achieve this goal, an iterative scheme inspired from [11] and detailed in [9] is proposed. This algorithm generally converges to a good solution $\boldsymbol{w}$ that satisfies (6).

III. Numerical Results

The synthesis of shaped beams with optimized circular polarization is presented for two configurations: a linear and a conformal array of patches that are represented in Fig. 1. Each square patch is fed by two coaxial probes iX and iY and is therefore dually polarized. The active element pattern method [12] is applied to calculate the pattern of the fully excited array. Each patch is simulated in the array environment with a full wave numerical software (Ansoft HFSS) to provide the array response $\boldsymbol{a}_\theta(\theta)$ and $\boldsymbol{a}_\varphi(\theta)$ of (2). Using this method enables one to take the mutual coupling effects into account.



(a)

(b)

Fig. 1. Geometry of the array composed of five dual polarized patches: (a) top view of the linear array and (b) side view of the conformal array.

A. Linear Array of Patches

The synthesis problem amounts to find the excitations iX and iY (for $i + 1, ..., 5$) of the linear array represented Fig. 1(a) such as it radiates:

- a sectoral shaped beam of ripple ± 0.2 dB over SB=$[-20\,°, 20\,°]$
- sidelobes below $\rho =$-10 dB over SL=$[-180\,°, -50\,°] \cup [50\,°, 180\,°]$
- an optimized circular polarization over POL=$[-30\,°, 30\,°]$.

The optimized far field patterns and axial ratio are plotted in Fig. 2 whereas the corresponding excitations are reported in Table I. The spatial power requirements are satisfied and the axial ratio is very close to 0 dB in the region of interest, which shows the good quality of the circular polarization.

B. Conformal Array of Patches

An array of five patches that are conformed on a cylinder, as represented in Fig. 1(b), is considered. The synthesis

TABLE I. OPTIMIZED EXCITATIONS OF THE LINEAR ARRAY SYNTHESIS PROBLEM

feed	$\|w\|$	$\angle w$ [deg]
1X	1.00	-74
1Y	0.95	-164
2X	0.26	85
2Y	0.21	163
3X	0.17	-29
3Y	0.16	-121
4X	0.25	-97
4Y	0.21	-20
5X	0.15	160
5Y	0.14	-114

TABLE II. OPTIMIZED EXCITATIONS OF THE CONFORMAL ARRAY SYNTHESIS PROBLEM

feed	$\|w\|$	$\angle w$ [deg]
1X	0.32	-165
1Y	0.38	-79
2X	1.00	55
2Y	0.64	171
3X	0.37	-67
3Y	0.45	-1
4X	0.60	175
4Y	0.28	-1
5X	0.30	-18
5Y	0.19	-144

problem consists in finding the excitations iX and iY (for $i+1,...,5$) of this conformal array, such as it radiates:

- a sectoral shaped beam of ripple ± 0.5 dB over SB=$[-40\,°, 10\,°]$
- sidelobes below $\rho =$-10 dB over SL=$[-180\,°, -60\,°] \cup [40\,°, 180\,°]$
- an optimized circular polarization over POL=$[-20\,°, 0\,°]$.

The optimized far field patterns and axial ratio are plotted in Fig. 3 whereas the corresponding excitations are reported in Table II. While the required sectoral beam pattern is achieved, a circular polarization is achieved in the angular range of interest. As shown in Fig. 3(b), the axial ratio is lower than 0.7 dB over POL.

IV. CONCLUSION

A synthesis method to design arrays that radiate a power pattern, that is both upper and lower bounded, with an optimized specified polarization over a given angular range, has been proposed. This joint synthesis (shaped beam and polarization) is a requirement often encountered in many applications.
The advantages of the proposed method are manifold. First, there is no restriction regarding the type of array to be synthesized. Second, any state of polarization and shaped beams can be synthesized. Finally, the proposed algorithm can be easily implemented and it only calls for readily available routines.

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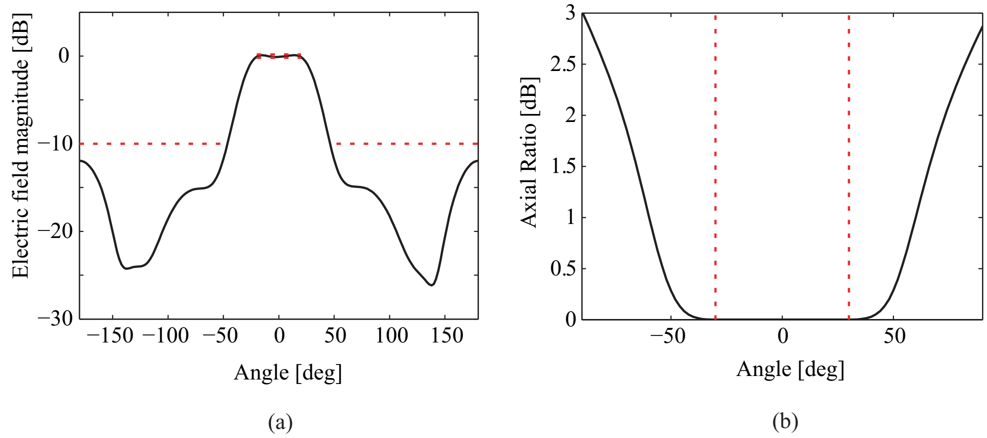

Fig. 2. Synthesis results of the linear array composed of five patches: (a) shaped far field radiation pattern and (b) optimized axial ratio.

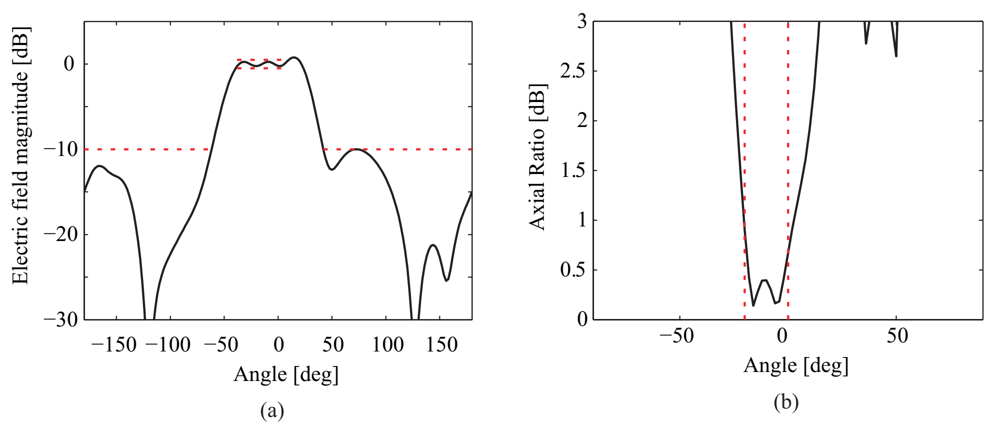

Fig. 3. Synthesis results of the conformal array composed of five patches: (a) shaped far field radiation pattern and (b) optimized axial ratio.