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The Sagnac effect and the role of simultaneity in relativity theory

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ABSTRACT
We thoroughly examine the role of absolute and relative simultaneity in the interpretation of the Sagnac effect, using an approach that allows for determining the local speed along the light path. If the local speed of light is assumed to be *c* over the whole closed contour, there is no agreement with the observed result. There is agreement if the local speed is *c* along an open section of the contour only. Thus a rigorous and coherent interpretation of the Sagnac effect favours absolute over relative simultaneity. The implications for the Lorentz transformations and synchronization by means of the Global Positioning System are considered.

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1. Introduction

The purpose of our paper is to review rigorously the interpretation of the Sagnac effect [1] using an approach that assumes the non-equivalence of absolute and relative simultaneity and takes into account the decisive role of clock synchronization, disregarded in almost all the interpretations over more than a century. Before considering the controversy about the Sagnac effect, we recall briefly the properties of the Lorentz transformations in relation to two relevant clock synchronization procedures, Einstein or absolute, that have been applied in the literature within the scenario of relativistic theories [2–6]. The following coordinate transformations may apply between the inertial frame S and the frame S' in motion with velocity $\mathbf{u} = \hat{\mathbf{i}}u$ relative to S,

$$\text{LT } t' = \gamma(t - ux/c^2);\ x' = \gamma(x - ut); \quad y = y; \quad z' = z$$
$$\text{LTA } t' = t/\gamma; \quad x' = \gamma(x - ut);\ y = y; \quad z' = z. \tag{1}$$

In (1), LT stands for the standard Lorentz transformations based on Einstein synchronization, and LTA (known also as the Tangherlini or the Selleri transformations [7]) for the Lorentz transformations with absolute synchronization (other transformations conserving simultaneity have been considered by Lundberg [8] and Field [9].

Before Einstein, the absolute time of Newton transforms as $t' = t$. With Einstein synchronization and the LT, time becomes relative and depends on space. As shown in (1), relative time transforms as $t' = \gamma(t - ux/c^2)$, where t' is now a function of the relative velocity u (through the factor $\gamma = (1 - u^2/c^2)^{-1/2}$) and the spatial position x.

Thus, with Einstein's time paradigm and the LT, simultaneity is not conserved and is relative. If instead we adopt absolute synchronization, with the relation $t' = t/\gamma$ in the LTA, time is still relative through the factor γ that accounts for tested relativistic effects such as time dilation, but simultaneity is conserved because t is space-independent. Obviously, absolute simultaneity, achieved with $t' = t/\gamma$ in the LTA, is also a characteristic of the simpler time transform $t' = t$.

Absolute synchronization is achieved as follows. Let us suppose that there is an inertial frame S where space is homogeneous and isotropic and the one-way speed of light is c. In the case of absolute simultaneity, such a frame corresponds to the unique preferred frame of the relativistic theories where the time transform is given by $t' = t/\gamma$. In frame S we have a set of stationary clocks placed along the x axis. These spatially separated clocks can be synchronized – or 'coordinated', using the term suggested by Lundberg [8] – adopting Einstein synchronization procedure by means of the speed of light. Clocks in S will progressively display the time readings t as time evolves from the past to the future (from $t < 0$ to $t > 0$). In the relatively moving frame S' we have a different set of stationary spatially separated clocks placed along the

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x' axis. These clocks will be absolutely synchronized (or 'externally' synchronized [2]) with clocks stationary in S, if set at $t' = 0$ when facing the corresponding clocks of frame S displaying $t = 0$. The clock of S', facing at $t' = t = 0$ the clock at the origin O of S, will determine the origin O′ of frame S'.

As shown in the literature [2–5,10,11], relativistic theories based on the LTA and absolute synchronization are capable of interpreting all the known experiments supporting *standard* SR based on the LT with Einstein synchronization. Authors adhering to the conventionalist thesis [2,4,10,12–15] assume that the LT and LTA are physically equivalent. However, the equivalence of Einstein and absolute synchronization implies the equivalence of relative and absolute simultaneity, while the two are conceptually incompatible. Moreover, a recent work [5] has shown that relative and absolute simultaneity (and the LT and LTA) can be discriminated experimentally with the implication that the one-way speed of light is measurable in principle and, therefore, the conventionality of the speed of light no longer holds. There are already numerous examples of the use of the Lorentz transformations adopting absolute clock synchronization procedure, in lieu of the LT with standard Einstein synchronization. One reason for the spreading use of the LTA, for instance in [12], is for interpreting coherently all the experiments supporting special relativity, including the famous optical experiment of Sagnac where the use of the LT with Einstein synchronization seems to lead to inconsistencies [1,3,6,8,9,14,16–18].

In the literature, the term 'synchronization' has not always been used in accordance with its actual, original meaning. When in an inertial reference frame S we use the world line $s = s(t)$ to denote the position s of a photon or an event in space-time, we implicitly assume that, throughout frame S, we have a set of synchronized clocks that mark simultaneously the same common time t. If superluminal, quasi-infinite speed signals were available, there would be no problem in synchronizing spatially separated clocks. However, in the absence of such signals, if space is homogeneous and isotropic in S and the one-way speed of light is c, clocks can be synchronized by means of the Einstein procedure equally well as if synchronized by means of an infinite speed signal. In this case, two synchronized clocks, spatially separated by the distance L, will mark simultaneously the same time t when either a light signal, sent to both clocks from the middle position $L/2$ reaches them, or an infinite speed signal is sent from one clock to the other. From an ideal perspective, although an infinite speed signal does not exist (except possibly within the context of quantum entanglement), the concept is still useful in principle for clarifying and understanding the concept of simultaneity.

When we pass from S to a frame S' in relative motion, absolute or relative simultaneity comes into play, as the theory may be based on absolute synchronization, as for the case of Newtonian physics and modern relativistic theories adopting the LTA, or Einstein synchronization with standard SR. The problem with Einstein synchronization, based on the average two-way speed c and making use of a mirror, is that we have no way of knowing a priori whether or not space is isotropic and the one-way speed is c in any relatively moving frame. In that regard, Lundberg [8] refers to the word 'synchronization' in SR as being misused, suggesting that the term 'coordinating' better reflects Einstein synchronization procedure. In any event, if the concept of simultaneity is assumed to hold in frame S where clocks are synchronized, the one-way speed of light is physically meaningful and can be tested [5], restoring its original physical meaning to special relativity. Thus in this paper we assume that conventionalism does not hold [5] and that relative simultaneity and the one-way speed, can be tested.

Only by means of experiments can we corroborate the invariance of c and discriminate absolute versus relative simultaneity.

In 1913 Georges Sagnac [1], after performing his famous optical test, claimed that the result disproved Einstein's special relativity.

In the original experiment performed by Sagnac, light trajectory was a quadrangle. In more recent experiments [19], light trajectory is circular (Figure 1a), where two counter-propagating light rays are emitted from an interferometer (or a clock, the measuring apparatus) on the rim of a uniformly rotating disk of radius r and co-moving with it at the tangential speed $v = \omega r$.

The experimental result was interpreted by Sagnac as showing that the two light rays, moving on the disk circumference, have different local speeds ($c + v$ and $c - v$) relative to the clock. Then, the time of flights,

$$T_+ \simeq \frac{2\pi r}{c+v} \qquad T_- \simeq \frac{2\pi r}{c-v} \tag{2}$$

(of first order in v/c), for the two light rays covering the same round trip path $2\pi r$ are different and consistent with the time delay $\Delta T \simeq T_- - T_+$ detected at their arrival back to the clock. Among the subsequent many papers on the Sagnac effect, there are several supporting Sagnac's point of view. Most influential is the position of Selleri [3] who elaborated a paradox showing that the result of Sagnac's experiment is not compatible with Einstein's second postulate on the invariance of the speed of light c. Since the Sagnac effect is of first order in v/c, if

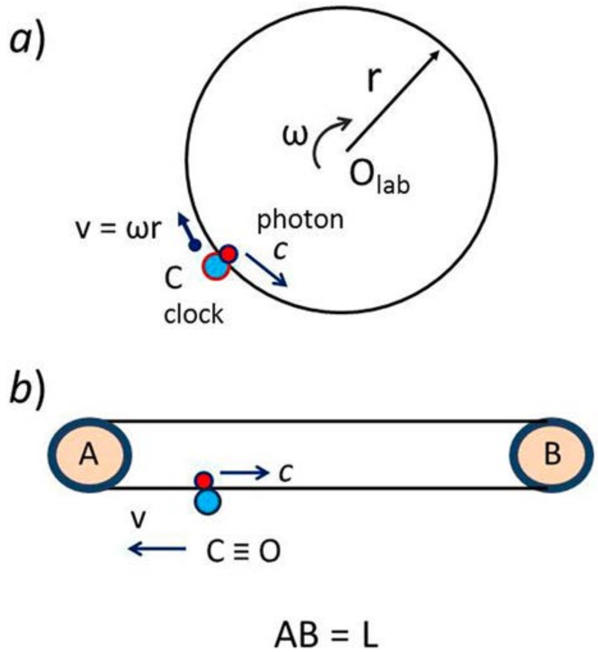

Figure 1. We show in (a) a Sagnac-like thought experimental arrangement for representing the standard circular Sagnac effect, where counter-propagating light pulses in the air are sent from clock C on the platform, which is rotating with constant angular velocity. The 'conveyor belt' linear equivalent of the circular Sagnac arrangement is shown in (b), where the clock C is fixed to the belt in motion with velocity v relative to the pulleys A and B driving the clockwise movement of the belt. A light pulse, emitted from clock C, is counter-propagating with speed c relative to the clock.

the experiment is described in the laboratory or any other relatively moving inertial frame where the speed of light is assumed to be constant and equal to c, the result (2) of the Sagnac experiment can be predicted equally well, to first order, by both Newtonian physics and Einstein's special relativity. Because of this, disregarding the arguments of Einstein's detractors, most physicists adhere to the current time paradigm arguing that, by predicting the correct result, relativity theory coherently interprets the Sagnac effect. However, the controversial point is not to foresee the result of the Sagnac effect, a result that nobody questions because it is easily foreseen by both Newtonian and relativistic physics. Indeed, according to Sagnac and several others [1,3,6,8,9,14,16–18], the real problem is to interpret the result of the experiment when seen by an observer co-moving with the measuring apparatus. In this case, as also pointed out particularly by Landau and Lifshiz [20], there are problems with Einstein synchronization applied to the closed contour and, therefore, the claim is that a constant *local* light speed c is not consistent with Sagnac's result.

For determining the value of the local light speed, in this paper we use a novel approach that emphasizes the essential roles of absolute and relative simultaneity and the related clock synchronization procedures in the interpretation of the Sagnac effect. Then, taking rigorously into account clock synchronization, our approach reveals the inconsistencies that emerge by requiring the local light speed to be c in both the linear and circular Sagnac effect, here considered in the context of Relativity Theory. Since most of the papers dealing with the Sagnac effect (see as a typical example [19]) do not specifically address simultaneity and clock synchronization, the results can not indicate whether or not the local light speed is c along every part of the closed contour. In the next section, for convenience of the reader, we provide a résumé of the linear Sagnac effect in the framework of special relativity (SR) and consider two examples. In the subsequent section, we discuss the interpretation of the circular Sagnac effect in the rotating frame of the measuring apparatus. We find that, if the LT are used, the inconsistencies pointed out by Sagnac [1], Sellери [3], and many others [6,8,9,14,16–18] are confirmed, while no inconsistencies are found by adopting absolute simultaneity.

2. The linear Sagnac effect, special relativity and clock synchronization

Some of the difficulties in interpreting the circular Sagnac effect arise because the frame of the measuring apparatus, in uniform circular motion, is not an inertial frame. Fortunately, the original 'circular' Sagnac effect has been recently tested and verified in its kinematically equivalent 'linear' version [21]. In essence the linear version can be conceptualized as a conveyor belt system (Figure 1b) where the belt is moving at the velocity v relative to the frame of the pulleys while carrying the clock C.

By assuming that the conveyor arm $AB = L$ is much larger than the finite radius r_p of the pulley, the circular Sagnac effect of Figure 1(a) is linearized by the kinematically equivalent system of Figure 1(b). The equivalence is restricted to the fact that the theoretical predictions (2), confirmed by observation, are the same (with $2\pi r = 2L$) and independent of the clock acceleration to first order in v/c. Moreover, the equivalence is merely kinematical because in the circular case the clock possesses a uniform acceleration throughout the round-trip time T of propagation of the light ray (or photon), while in the linear case the clock undergoes an acceleration when turning around the pulley for the time interval η that can be much shorter than T. Because of the difference in the clock accelerations, the two effects have to be treated separately if the effect of the acceleration has to be calculated. We consider here two examples (or thought-experiments), in the first the clock is in uniform motion save when it is accelerated for the short finite time interval η while moving around the pulley. In the second example, the clock does not turn around the pulley and, thus, is not accelerated and always in uniform motion during the photon round-trip time T.

The results obtained are the same for the two examples. In order to simplify the calculations in our thought-experiments we make some approximations, derived in [6], which are $\eta < 2vL/c^2$ and $L >> r_p c^2/v^2$, implying that the finite time interval η can be neglected relative to T if we choose L very large relative to the radius r_p of the pulley. With our approximations, both the time interval η and the clock acceleration are finite.

Moreover, since the radius r_p of the pulley is much smaller than L, the time $t_{r_p} \simeq cr_p$, taken by light to traverse the arc length r_p when travelling around the pulley, is much smaller than the time $T_L \simeq cL \simeq T$, taken by light to traverse the distance L corresponding to the length of the conveyor belt.

Therefore, in calculating the time of flight of the photon on the closed contour, we may neglect, for simplicity, the small time interval t_{r_p}.

About the acceleration of the clock in the first example, there are two aspects to be considered. One aspect is the modification of the time rate of the clock while being accelerated during the time interval η. The corresponding modification can be determined by means of the Equivalence Principle and the result is that, if the variation of the velocity produced by the acceleration is $u = 2v$, the time rate of the clock during the interval η changes by terms of the second order in u/c [6,22] and, thus, if we can neglect η relative to T, we can also neglect $\eta(1 \pm u^2/c^2)$.

The other aspect of acceleration is that, relative to the initial frame S'_+, co-moving with the lower part of the belt where the clock is at rest before accelerating, the velocity of the clock has changed when later at rest in the final frame S'_-, co-moving with the upper part of the belt, after having been accelerated. Since the relative velocity $u = 2v$ of S'_- with respect to S'_+ represents the velocity change achieved by the clock through acceleration, if Einstein synchronization is adopted and the first order time relation $t'_- \simeq t'_+ - ux'_+/c^2$ is used, in agreement with (1), we find that two events spatially separated by $\Delta x'_+$ and simultaneous in S'_+, are separated by the time difference $\Delta t'_- \simeq -u\Delta x'_+/c^2$ in S'_-. This time difference has to be taken into account in the kinematic description of light propagation by means of Einstein synchronization. However, the mentioned time difference does not occur if absolute synchronization is adopted and the time relation $t'_- \simeq t'_+$ is used. It is true that acceleration is what brings the clock from the initial S'_+ to the final S'_-, but what the clock is met with when at rest in S'_- (i.e. either $\Delta t'_- \simeq -u\Delta x'_+/c^2$ or $\Delta t'_- = 0$) depends directly on the synchronization adopted, and not on the acceleration of the clock. Then, in our approach to the linear Sagnac effect the problem to be solved is to determine which one of the two synchronizations provides a coherent interpretation.

If the belt consists of an optical fibre inside which the photon (or light pulse) propagates, the belt optical fibre represents the 'ground' where the photon is propagating. Inside the optical fibre the local speed is $u_0 = c/n < c$, where n is the medium refractive index. Then, for a photon u_0 is denoted as being the local 'ground' speed relative to the inertial frame (co-moving with the belt, or optical fibre) where synchronized clocks are at rest. For an inertial frame in relative motion with respect to the belt ground frame, the local speed of the photon is no longer the ground speed u_0, but is given by the velocity composition that depends on synchronization. For simplicity, in the following we assume absence of a medium and $u_0 = c$.

2.1. First example

We consider here the case where a photon is emitted from the clock in the counter-propagating direction (the result for a co-propagating photon is easily obtained by changing the sign of v). In the first example, the sequence of path sections, covered by the light pulse in its round trip, starts from clock C emitting the photon while co-moving with the frame S'_+ on the lower part of the belt, as shown in Figure 1(b). The photon, which is made to move along the belt ground path of length $2L$, returns to the clock in the measured round-trip time of flight $T = 2L/(c + v)$, valid to first order in v/c. Our basic assumptions are that, in the outward trip, in frame S'_+ the (ground) local light speed is c and the photon reaches B when, simultaneously, pulley A reaches clock C. In this case, as shown in Figure 2, the section CB has length L and the time interval taken by the photon to cover it is $T_{CB} = T_{out} = L/c$ as measured by C in frame S'_+. After the clock has moved to the upper section of the belt and starts co-moving with frame S'_-, the photon eventually reaches it in its return trip. The measured round trip proper time T must correspond to the sum of the proper time interval measured by C when on the lower part of the conveyor belt, plus the proper time interval measured by it when on the upper part. Thus, in the photon return trip back to C on the upper part of the belt, the measured time is $T_{ret} = T - T_{out} = L/(c + 2v) = L(1 - 2v/c)/c$, where the equality $L/(c + 2v) = L(1 - 2v/c)/c$ is valid to the first order in v/c.

Absolute simultaneity.

Since the average light speed over the closed ground path $2L$ is superluminal and given by $c + v$, in the case

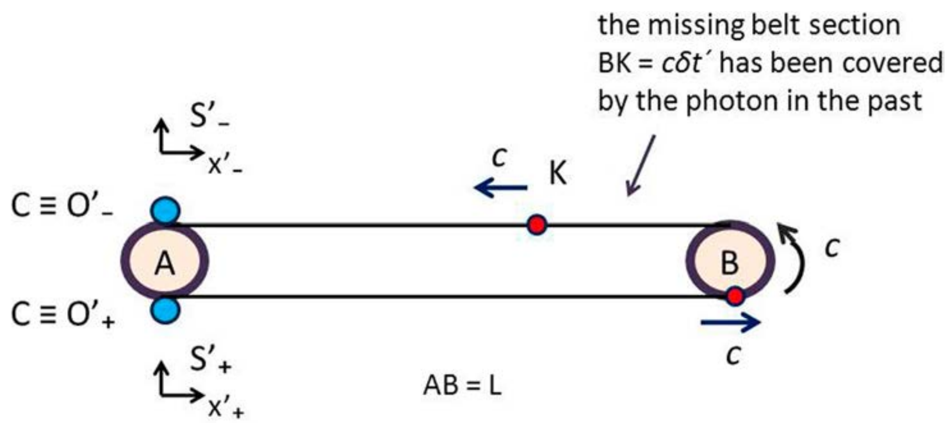

Figure 2. In this figure, we show the photon trajectory when Einstein synchronization and the LT are used. The two events 'photon at B' and 'clock at A', are simultaneous in the frame S'_+ co-moving with the belt lower section, but are not simultaneous in the frame S'_- co-moving with the belt upper section, where the event 'photon at B' occurs at the earlier time $-\delta t'$. Thus, at the instant $t'_+ = t'_- = 0$, when the clock is at A in the upper part of the belt, the photon has already moved by $c\delta t'$ from B to point K closer to the clock. The photon, already at K, 'seems' to have skipped section BK as a consequence of non-conservation of simultaneity.

of absolute synchronization we have [6] that, as initially assumed, the local light speed is c along the lower section of the belt CB. Since in the return trip $T_{ret} = L/c' = L/(c+2v)$, we have the ground speed $c' = c + 2v$ along the belt upper section BC. The ground path covered in the outward trip is $L_{out} = cT_{out} = L$, while in the return trip, $L_{ret} = c'T_{ret} = L$. Thus the total ground path covered is closed and given by $L_{out} + L_{ret} = 2L$, in agreement with the length of the closed contour.

Relative simultaneity.

If we instead apply Einstein synchronization and the LT are used, the two events 'photon at B' and 'clock at A', simultaneous in the frame S'_+ co-moving with the belt lower section, are no longer simultaneous in the frame S'_- co-moving with the belt upper section. For our conveyor belt system, the exact LT (in the x'_+, x'_- direction) are [6,18],

$$x'_- = \gamma_w(x'_+ - wt'_+); \quad t'_- = \gamma_w(t'_+ - wx'_+/c^2), \quad (3)$$

where $w \simeq u = 2v$. When in S'_- the clock is at A, the event 'photon at B' turns out to have occurred at the earlier time, $t'_- = -\delta t' \simeq -wL/c^2$, so that in the subsequent time interval $\delta t'$ the photon has moved by $c\delta t'$ from B to the point K closer to the clock. Thus, as shown in Figure 2, at the instant $t'_+ = t'_- = 0$, when the clock is at A in the upper part of the belt, in the frame S'_- the photon is already at K, 'seeming' to have skipped section BK as a consequence of non-conservation of simultaneity. To first order in v/c, the round trip proper time T is expressed as,

relative simultaneity (open contour)

$$T = T_{out} + T_{ret} = T_{CB} + T_{KC} = \frac{CB}{c} + \frac{KC}{c}$$
$$= \frac{L}{c} + \frac{L(1-2v/c)}{c} = \frac{2L(1-v/c)}{c} = \frac{2L}{c+v}, \quad (4)$$

where the term $T_{CB} = L/c$ represents the proper time interval measured by clock C before turning around the pulley A. However, when at $t'_+ = t'_- = 0$ clock C starts co-moving with the upper belt section, the pulse is already at point K, at the distance $L(1-2v/c)$ from C. Then, if the pulse travels at the local speed c in the frame S'_- where C is now at rest, the term $T_{KC} = L(1-2v/c)/c$ in (4) measured by clock C, represents the proper time taken by the light pulse to cover the path section KC. With relative simultaneity, in the round trip time T of expression (4), we have the partial time delays corresponding to the sections CB and KC, but the time delay taken by the light pulse to cover the section BK is missing! The ground path covered in the outward trip is $L_{out} = cT_{CB} = L =$ CB, while the ground path covered in the return trip is $L_{ret} = cT_{KC} = L(1-2v/c) =$ KC. The total ground path covered is $L_{out} + L_{ret} = 2L - 2(v/c)L < 2L$, implying that the closed contour CB + BC is reduced to the open contour CB + KC $< 2L$.

It follows that, by adopting Einstein synchronization in order for the local speed of light to be c also in the return trip, non-conservation of simultaneity introduces a space-time discontinuity between the two frames S'_- and S'_+ that eliminates the section BK $= \delta t' c = (2v/c)L$ and shortens the return light path L to $L(1-2v/c)$. The term $\delta t'$ is related to Kassner's ad hoc 'time gap' [12], criticized for being 'unphysical' by Gift [16], among many other authors [18]. Of course, if the section BK is not eliminated and the contour is closed, result (4) is obviously given by,

invariant speed (closed contour)

$$T_{round} = T_{CB} + T_{BK} + T_{KC} = \frac{CB}{c} + \frac{BK}{c} + \frac{KC}{c} \quad (5)$$

$$= \frac{L}{c} + \frac{2v}{c}\frac{L}{c} + \frac{L(1-2v/c)}{c} = \frac{2L}{c}, \quad (6)$$

in agreement with Einstein's second postulate but in conflict with what is measured by the clock.

The partial time intervals T_{out} and T_{ret} in (4) are proper time intervals measured by clock C. T_{out} is measured by C when at rest on the inertial frame S'_+, and T_{ret} when at rest on the inertial frame S'_-. Both T_{out} and T_{ret} are well-defined observable measurements that can be made independently: T_{out} is the proper time elapsed from the origin when the light pulse is emitted until the moment when clock C passes from the lower to upper belt section. While T_{ret} is the proper time elapsed from the moment when clock C passes from the lower to upper belt section until the moment when the light pulse reaches again clock C. Thus, an interpretation of T_{out} made from frame S'_+, where C is at rest during that time interval, is physical meaningful and appropriate for a theory based on either absolute or relative simultaneity. Similarly, an interpretation of T_{ret} made from frame S'_-, where C is at rest during that time interval, is equally physical meaningful and appropriate for a theory based on either absolute or relative simultaneity. Two reference frames are involved, separately, because the two measurements of the clock occur, separately, on two different frames. The theory should be able to interpret consistently the two measurements. We showed that, with absolute simultaneity, the interpretation of T_{out} made from frame S'_+ is consistent with the interpretation of T_{ret} made from frame S'_- (the ground path covered on each section is L, leading to the total closed path $2L$, as required, and the local ground speed is consistent with absolute simultaneity). However, with relative simultaneity, the interpretation of T_{out} made from frame S'_+ is not consistent with the interpretation of T_{ret} made from frame S'_- (if the local ground speed is assumed to be c, as required by the LT with relative simultaneity, the ground path covered on one of the sections is less than L, leading to an open covered total path, less than $2L$).

We see no valid physical reasons that could invalidate the interpretation of well-defined observables, such as T_{out} and T_{ret}, for the mere circumstance that clock C passes from frame S'_+ to frame S'_-. In any event, as shown in the next section, the same results (and problems) are obtained even when clock C keeps stationary on a single frame. Thus the mentioned problems are not attributable to the description involving two frames. We highlight that the inconsistencies we found with relative simultaneity can be related with the 'undesirable consequences' pointed out recently by Lee [23] for light propagation in (closed) cylindrical spacetime. As already shown by Selleri [3], by us [6], and now by Lee, these inconsistencies, or undesirable consequences, disappear when absolute synchronization is adopted. Thus, the work of Lee, for being published in a didactic journal, is an indication that the problems with the LT and relative simultaneity, well known to specialists, start to reach the wider audience.

2.2. Second example

In our second example shown in Figure 3, for simplicity we start with clock C initially at the position of pulley B on the belt lower section co-moving with the frame S'_+ (actually, we can have the clock at any different initial position along AB, but the calculations are more complicated). In this example, we are dealing with the most representative common case, because, with $v < < c$, the clock can keep moving with uniform motion while the photon completes one or more round trips.

In Figure 3, the counter-moving photon leaves the clock at $t'_+ = 0$, turns around the pulley B in a short negligible time and covers the ground path of the belt upper section. When the photon reaches pulley A, it turns around and in the return trip covers the ground path of the belt lower section until it reaches again the clock C after completing the round trip in the time interval T. Our assumptions here are that the one-way speed of light is c in frame S'_+ and the photon and clock C coincide at B when $t'_+ = 0$. Thus, relative to frame S'_+ and as shown in Figure 3(a), the world line of the photon going from B to A in the upper belt section is $L - ct'_+$ and that of the centre of the pulley A moving toward the photon is vt'_+, v being the speed of the centre of pulley A relative to frame S'_+. Then, from the equation, $L - ct'_+ = vt'_+$, for the outward trip time interval we find

$$T'_{+out} = \frac{L}{c+v}, \quad (7)$$

and the photon is now at the position of the pulley A, $x'_{+A} = vT'_{+out}$. In the return trip, as shown in Figure 3(b) the ground distance to be covered by the photon between A and the clock C is $L - vT'_{+out} = L/(1 + v/c)$. At ground speed c, we find that the photon covers this distance in the time interval,

$$T'_{+ret} = \frac{L}{c} - \frac{v}{c}T'_{+out} = \frac{L}{c+v}, \quad (8)$$

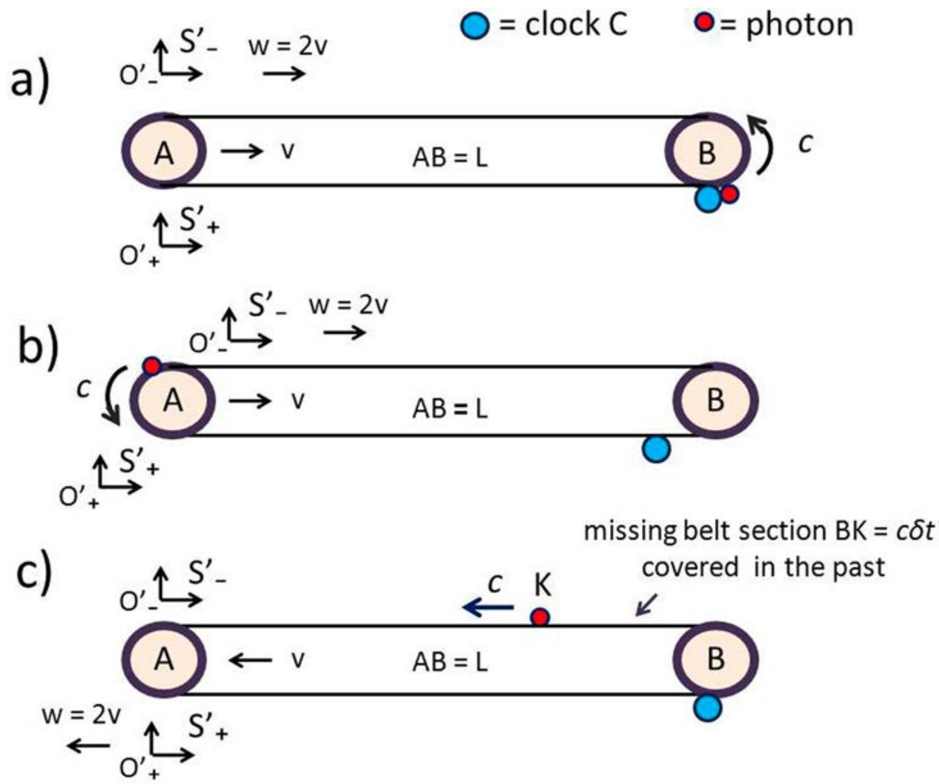

Figure 3. The clock C and the photon initially at the pulley B before the photon starts travelling on the belt upper section toward the pulley A. Relative to frame S'_+ where C is stationary the photon has speed c, the arm AB of the pulleys A and B has speed v and frame S'_- co-moving with the belt upper section has speed w. (b) The photon at pulley A before returning toward clock C when the arm AB has moved by vT'_{+out} relative to S'_+. (c) The initial position of the photon as seen from frame S'_-. At $t'_- = 0$ the photon is at point K, having covered during the past time interval $\delta t'$ the distance $BK = c\delta t' = (w/c)L$.

and the expected round-trip time, in agreement with observation [1], is,

$$T = T'_{+out} + T'_{+ret} = \frac{2L}{c+v}. \quad (9)$$

The derivation of (9) has been made entirely within the inertial frame S'_+ where the measuring apparatus (clock C) is at rest and, thus, it is independent of the type of synchronization (absolute or Einstein) adopted in frame S'_-. In order to verify the consistency of the theory, we need to check the ground path covered in every section of the closed contour. We show now that, if the speed of light is c in frame S'_+ and with the derived round-trip result T in agreement with observation, the kinematics of the photon and the synchronization to be adopted in S'_- are determined. The clock at the origin O'_- of S'_- and that at the origin O'_+ of S'_+ are set at $t'_- = t'_+ = 0$ when they approximately coincide. This operation can be performed, as done in the First Example, by accelerating a clock from S'_+ to S'_- in the negligible time interval η or simply by setting the clock at the origin O'_- of S'_- at $t'_- = 0$ when it passes by the origin O'_+ of S'_+.

With relative simultaneity, in frame S'_- the photon reaches pulley A at the time T'_{-out} that can be obtained from the time transform of the LT from S'_+ to S'_- in (3) and is given by $T'_{-out} = \gamma_w(T'_{+out} - wx'_{+out}/c^2)$. Since the photon is at pulley A at the time T'_{+out} and $x'_{+out} = x'_{+A} = vT'_{+out}$, then, $T'_{-out} = \gamma_w T'_{+out}(1 - wv/c^2) \simeq T'_{+out}$. With absolute simultaneity we have to use the time transform $t'_- = t'_+/\gamma_w \simeq t'_+$ of the LTA, which gives $T'_{-out} \simeq T'_{+out}$. Hence, the result $T'_{-out} = T'_{+out}$, valid for the outward trip to first order in v/c, is in agreement with the proper time measurements of the clock at the origin O'_-, which are related to the time t'_+ of S'_+ by the usual relation $\tau = t'_- = t'_+/\gamma_w$. Therefore, for both absolute and relative simultaneity, in the outward trip the time variations T'_{-out} and T'_{+out} of the clocks of O'_- and O'_+ may be expressed as

$$T'_{-out} = T'_{+out} = \frac{L}{c+v}, \quad t'_- = t'_+, \quad (10)$$

Events occurring in the past ($t'_- < 0$) for S'_- take place before origin O'_- meets origin O'_+ at $t'_- = t'_+ = 0$. Therefore, according to (10), if the clock of O'_- marks $t'_- < 0$, the clock of O'_+ must mark $t'_+ = t'_- < 0$ for S'_+.

Ground speed and kinematic constraint.

In the inertial frame S'_- the kinematic constraint on the ground speed of a photon is expressed as

$$c'_- = \frac{L'_-}{T'_-}, \tag{11}$$

where L'_- is the length of the ground path covered by the photon and T'_- the time interval taken for covering it.

Independently of the synchronization adopted in frame S'_-, according to frame S'_+, the return ground path is,

$$L_{+ret} = cT'_{+ret} = L/(1+v/c) = L(1-v/c), \tag{12}$$

valid to the first order in v/c, while space-time continuity requires the outward ground path to be

$$L_{-out} = 2L - L_{+ret} = L(1+v/c). \tag{13}$$

Since, for frame S'_+, the length $L'_- = L_{-out}$ is determined by (13) and, on account of (10), $T'_- = T'_{-out} = T'_{+out}$, physics determinism requires the outward ground speed in frame S'_- to be

$$c'_- = \frac{L'_-}{T'_-} = \frac{L_{-out}}{T'_{-out}} = \frac{L_{-out}}{T'_{+out}} = \frac{L(1+v/c)}{L/(c+v)} \simeq c + 2v, \tag{14}$$

which is superluminal.

Let us check the validity of result (14) when absolute, or relative simultaneity, is assumed between S'_- and S'_+.

Absolute simultaneity.

In this case result (14) agrees (to first order in v/c) with the Galilei composition of velocities requiring that in the outward trip the 'ground' speed c'_- of the photon in frame S'_- is $c'_- = c + w = c + 2v$. Simultaneity is conserved and the ground path L_{-out}, covered in the time interval T'_{-out} at the ground speed c'_-, is given by $L_{-out} = c'_- T'_{-out} = (c+2v)L/(c+v) = L(1+v/c)$ in agreement with (13). In the return trip, the ground speed is c and the corresponding ground path covered by the photon is given by (12). Then, for the closed contour of length $2L$, the total ground path covered is

$$L_{round} = L_{-out} + L_{+ret} = 2L\,, \tag{15}$$

as expected. The average speed over the path $2L$ is $c + v$.

Relative simultaneity.

With relative simultaneity and Einstein synchronization the speed of the photon in frame S'_- is required to be $c'_- = c$. Therefore, for the outward path in frame S'_- we find

$$(L'_-)_{rs} = (L_{-out})_{rs} = cT'_{-out} = \frac{L}{1+v/c} = L\left(1 - \frac{v}{c}\right), \tag{16}$$

and the total ground path covered in the round trip is

$$(L_{round})_{rs} = (L_{-out})_{rs} + L_{+ret} = 2L\left(1 - \frac{v}{c}\right) < 2L. \tag{17}$$

The difference between the actual length $2L$ (belt length) of the closed contour and the covered total ground path is

$$2L - (L_{round})_{sr} = 2(v/c)L = (w/c)L = c\delta t', \tag{18}$$

where $\delta t' = wL/c^2$ is the time gap corresponding to the time discontinuity between Einstein synchronized clocks of the two frames S'_- and S'_+ at the distance L from the origin.

Let us find out why, for standard SR, the path $(L_{round})_{sr}$ needs to be shorter than the closed contour length $2L$. If the section $c\delta t'$ is not missing and the outward ground path is $L'_- = L(1-v/c) + c\delta t' = L(1+v/c)$ as in (13), the total ground path covered is $2L$, but then, if $T'_{-out} = T'_{+out}$ as derived in (14), the ground speed in the outward trip is $c'_- = c + 2v$, which is superluminal. Hence, the price to pay for keeping $c'_- = c$ invariant is an outward ground path shorter than L'_- in (14). Standard SR attains light speed invariance by placing the photon at B in the past at the negative time $-\delta t'$ by means of the mechanism of relative simultaneity of the LT (3), which (at $t'_+ = 0$ and $x'_+ = L$) gives $t'_- = \gamma_w(0 - wL/c^2) \simeq -wL/c^2 = -\delta t'$. This way, as shown in Figure 3(c), at time $t'_- = 0$ in the present the photon has already moved by $c\delta t' = wL/c$ at point K and the outward path $(L'_-)_{rs} = \mathrm{KA} = L(1-v/c) = L/(1+v/c)$ is now shorter than L, and can be covered by the photon in the time interval $T'_{-out} = (L'_-)_{rs}/c = L/(c+v)$ in agreement with (10).

Absolute versus relative simultaneity.

According to relative simultaneity, for frame S'_- the missing path BK is covered by the photon in the past ($t'_- < 0$) and in the future ($t'_+ > 0$) for frame S'_+. If the missing path BK is added to (17), the total path covered is now $2L$ (as required) and the photon traverses the path $L'_- = L(1+v/c)$ at speed c in the time interval $\Delta t'_- = T'_{-out} + \delta t'$. However, from the relation $t'_- = t'_+$ in (10), we have $\Delta t'_- = \Delta t'_+ = T'_{+out} + \delta t'$. Thus, if $\delta t'$ is added to T'_{-out}, it must be added to T'_{+out} as well.

In fact, simultaneity requires that, when in frame S'_- the photon is at pulley B at $t'_- = -\delta t' < 0$, all (synchronized) clocks of S'_- (including the one of O'_-) display simultaneously the same time. Moreover, in agreement with (10), the clock of O'_+ must display the time $t'_+ = t'_- = -\delta t' < 0$. However, since simultaneity in frame S'_+

requires all of its (synchronized) clocks to display the same time $t'_+ = -\delta t'$, we must infer that, in its motion along the belt lower section, at $t'_+ = -\delta t' < 0$ in the past, the photon is not at point B, but at the distance $c\delta t'$ from B on its way toward it. Thus, if we wish to consider events in the past and take $\delta t'$ into account by adding it to T'_{-out}, the relation $\Delta t'_- = \Delta t'_+$ coherently requires that $\delta t'$ be added to T'_{+out} in (9) and the resulting round-trip time is now $T_{round} = T + \delta t' = 2L/c$ (as in (5) in the First Example), in disagreement with observation. It follows that, if standard SR requires the photon to travel at ground speed c, in order to agree with observation, the total ground path covered must be open and less than $2L$ as in (17).

That the total path covered is less than $2L$ if the local speed is c, can be deduced considering the average speed. The result (9),

$$T = \frac{2L}{c+v} = \frac{2L(1-v/c)}{c}, \tag{19}$$

in agreement with observation and valid to first order in v/c, indicates that, if the path $2L$ ($2L = 2\pi r$, for the circular Sagnac effect) is covered by the photon at the average speed $c+v$ in the time interval T, at the lower average speed c the photon must cover the shorter path $2L(1-v/c)$ in the same time interval T.

In conclusion, in the scenario of an objective physics reality, the following assumptions: simultaneity in S'_+, validity of the relation $t'_- = t'_+$ in (10) for the clocks at the origins O'_- and O'_+, and simultaneity in S'_-, are in conflict with and exclude relative simultaneity.

Final remarks.

The missing section $BK = c\delta t' = (w/c)L$ surges as a consequence of the time discontinuity of relative simultaneity and is not present in (17) because it has been covered by the photon in the past, while expression (10) refers to time variations taking place in the future. It follows that the interpretation of the Second Example involving the missing path section is essentially the same as the one of the First Example. In both Examples, the constraint imposed by the assumption of light speed invariance requires the covered ground path to be an open path, shorter than the closed contour.

According to the derivation made in frame S'_+, the photon covers the total ground path $2L$ in the round-trip time T of (9) measured by the clock C. However, due to the requirement of light speed invariance, with relative simultaneity the effective ground path $(L_{round})_{rs}$ in (17), covered at speed c, is shorter than expected. Although the interpretation based on relative simultaneity is mathematically feasible, the absence of the section $BK = c\delta t'$ in (17) is unacceptable and certainly quite controversial from a physical point of view. Deterministic physics, space-time continuity and objective physical reality indicate that the kinematics of the motion of the photon is determined by the derivation made in frame S'_+. The result is that, as coherently described by means of absolute simultaneity, the total ground path covered by the photon is $L_{round} = 2L$ in (15) and, if the local speed of light is c in frame S'_+, in the outward trip the local ground speed in frame S'_- must be $c'_- = c + 2v$, i.e. superluminal.

If we are allowed to stretch the physical interpretation, in the First Example the controversial aspect could refer to the fact that, as the clock C turns around the pulley and moves from the lower to the upper frame, the velocity change has the effect of making the photon perform a sudden jump that places it closer to the clock. In the Second Example, we find that, if a clock of frame S'_+, placed at B, is moved to the frame S'_- it finds itself retroceding to the past because of the time discontinuity $\delta t'_-$. Thus, referring to the corresponding behaviour of the photon moving from the belt lower section to the upper section, we find that the photon suddenly retrocedes back in time by $\delta t'_-$ in order to be able to cover in the past at ground speed c the missing section BK.

More realistic may sound the alternative that Einstein procedure does not actually correspond to a real synchronization. Citing Lundberg [*8*]:

'I have said several times that Einstein specified a procedure for coordinating clocks, but I have not said that he specified a procedure for synchronizing clocks. 'Coordinating' is a safe and unproblematic word in this context. To coordinate clocks is merely to connect the clock settings in some way or other, so that they are not independent of each other.' Thus Einstein procedure could simply be an elegant method for coordinating clocks in such a way that the speed c keeps invariant in different inertial frames. In any event, in the case of the Sagnac effect, it appears that the simple objective interpretation based on absolute simultaneity is more coherent and less controversial than the interpretation based on relative simultaneity.

3. The circular Sagnac effect in the framework of the theory of relativity

Standard Einstein synchronization has been used by Lee [*23*] to describe light propagation in the framework of (closed) cylindrical spacetime. In his article, published in a didactic journal, Lee points out that 'undesirable consequences' emerge in describing light propagation in moving frames with standard synchrony, an aspect highlighted by other physicists, e.g. Klauber [*17*] in the same didactic journal, Gift [*16*], and the authors of [*6*] in relation to light propagation on closed moving contour (Sagnac effect). Furthermore, Lee shows that the mentioned 'undesirable consequences' disappear if, in cylindrical space time, absolute synchrony is adopted in lieu of standard synchrony.

As done by Lee in cylindrical spacetime, we assume in this section that, at least in principle (or hypothetically), both the LT and LTA, expressed in cylindrical coordinates, can be used to describe light propagation along the circumference of the rotating platform of the circular Sagnac effect. Then, with this assumption, we wish to verify if the undesirable consequences emerging in the circular Sagnac effect are the same as, or have the same common origin as, the inconsistencies we found in Section 2 for the linear effect. Our results show that, by adopting the LT and relative simultaneity, the inconsistencies that emerge in describing the circular effect, are the same that emerge in describing the linear effect. We find that the theoretical result foreseen by the LT agrees with observation if, and only if, the local light speed is c along an 'open' section of the closed contour. In fact, by adopting the LT and relative simultaneity in describing the circular effect, we are met with the same missing section (due to the time discontinuity of relative simultaneity) we met in describing light propagation in the linear effect in Section 2. Thus our result seems to imply that the common origin of the inconsistencies emerging in the linear and circular Sagnac effect is attributable to Einstein synchronization with relative simultaneity, because no inconsistencies emerge by adopting the synchronization based on absolute simultaneity ($t' = t/\gamma$ or, equivalently, $t' = t$).

In the laboratory frame S, the standard flat Lorentz–Minkowski metric in cylindrical coordinates is, $d\sigma^2 = c^2\mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\phi^2 - \mathrm{d}z^2$. With $\mathrm{d}r = \mathrm{d}z = 0$, $\mathrm{d}t' = \mathrm{d}t$, and $\phi = \acute{\phi} + \omega t$, in the case of the rotating disk of the Sagnac effect and for a photon, or a signal, moving at the tangential speed u, we obtain the Langevin–Landau–Lifshitz metric [24],

$$\begin{aligned}\mathrm{d}\sigma^2 &= c^2\mathrm{d}t^2 - r^2\mathrm{d}\phi^2 = c^2\mathrm{d}t^2 - u^2\mathrm{d}t^2\\ &= c^2\mathrm{d}t^2 - (u' + \omega r)^2\mathrm{d}t^2\\ &= (c^2 - \omega^2 r^2)\mathrm{d}t^2 - r^2\mathrm{d}\acute{\phi}^2 - 2r^2\omega\mathrm{d}\acute{\phi}\mathrm{d}t,\end{aligned} \quad (20)$$

where in (20) $u' = u - \omega r$ and $u'dt = rd\acute{\phi}$. In terms of the coordinates of frame S and when, with $u = c$, $rd\phi = \pm cdt$ for light propagation at the speed c in frame S, the metric (20) may be set to zero for representing the null geodesic condition $d\sigma^2 = 0$.

Accelerating frame in General Relativity.

According to Malykin [25], the approaches to the circular Sagnac effect that make use of General Relativity provide the same results as SR. General Relativity is useful only when gravitational fields are present. In the paper by Benedetto et al. [26], the authors make interesting considerations on fundamental physics that touch directly the controversial interpretation of the Sagnac effect. By means of the Einstein Equivalence Principle, they derive the effect of the acceleration $\omega^2 r$ in terms of an equivalent gravitational field, described through the Riemann curvature tensor R_{ijkl}. The resulting spatial metric indicates that the effect of space–time curvature modifies the length of the circumference $2\pi r$ to $2\pi r(1 - \omega^2 r^2/c^2)^{-1/2}$, involving correction terms of second order. If the local speed of light is c in the accelerating frame of the clock also, we may conclude that, in changing from $t' = 2\pi r/c$ to $t' = 2\pi r/[c(1 - \omega^2 r^2/c^2)^{1/2}]$, the clock round trip proper time is modified by the effect of the space-time curvature at most by second order terms only. Then, the treatment of the Sagnac effect in the framework of General Relativity shows that the corrections to the time t' due to the acceleration of the non-inertial rest frame of the clock are of the same order as the special relativistic effects, linked to the relativistic γ factor and typically of the order of v^2/c^2. Therefore, eventual variations of first order in v/c are attributable only to the clock synchronization procedure adopted and, in any event, unrelated to the acceleration of the clock. Consequently, the correct 'natural' synchronization is the one that can provide a coherent interpretation of the Sagnac effect, for both its circular and linear versions, in flat space-time.

3.1. Synchronizing clocks on the circumference of a rotating platform

Let us consider a platform of radius r rotating with angular velocity ω relative to the stationary laboratory reference frame. Starting from the position of a clock fixed on the rotating circumference of length $2\pi r$, on the rotating platform we may use the curvilinear circular coordinate $s' = s'(t')$ to indicate the distance of a point (or a photon) moving on the circumference. We assume that clocks along s' have been synchronized by means of a clock synchronization procedure to be specified below. The synchronization procedure must be applied along the whole circumference in order to guarantee simultaneity for the clock readings along the circular light path. If two clocks separated by the distance ds' are synchronized and a photon takes the time dt' to cover ds', the local speed of the photon can then be expressed as $u' = \mathrm{d}s'/\mathrm{d}t'$. Considering the circular symmetry of the system and uniform rotational motion, all synchronized clocks along s' share the same acceleration and, to the first order in v/c, will maintain synchrony as time evolves. Therefore, to the first order in v/c, if the motion of the photon is uniform, the measured local speed u' will be the same along every section ds' of the circumference. It follows that, if T' is the measured round trip proper time, we have, $T' = \int dt' = \int_0^{2\pi r} ds'/u' = 2\pi r/u'$, and the average speed u' for the round trip over the closed contour $2\pi r$ coincides with the local speed. This is nothing other than what Sagnac claimed for his experiment where, for a counter-propagating light signal, the measured round trip proper time is $T' = 2\pi r/(c+\omega r) = 2\pi r/c'$, indicating that the local light speed c' is not constant.

The discussion could end here, as we know of no valid argument that can rebut Sagnac's claim. However, for completeness, in the next sections we show how, in the context of kinematics, the conclusion that c' cannot be constant can be reached after applying absolute and Einstein synchronization to the circular Sagnac effect. The many attempts aimed to rebut Sagnac's rational claim are quite understandable from some perspectives, although it is unreasonable to try to justify and preserve the paradigm of the constancy of c at any cost, for example, by claiming that synchronization is conventional. Considering that Sagnac's effect can be coherently interpreted with absolute simultaneity, then it is difficult to support the conventional arguments that Einstein synchronization is also confirmed [12–15,27] when relative simultaneity is met with the discontinuity of the time gap $\delta t'$. Moreover, some of these attempts seek to place the Sagnac effect outside its context within simple natural kinematics. This is the case for the intricate scalar potential of the gravitational field, as introduced to describe the Coriolis acceleration [27]. Appealing to 'Occam's razor', the simplest approach is to allow inconstancy of the speed of light, in this case, is $c' = c + \omega r$, in line with absolute synchronization and the kinematics of light propagation. In addition, we highlight that any coherent physical interpretation of the Sagnac effect has to be valid for both the circular and linear versions. Thus ad hoc arguments related to the rotational motion of the platform or dynamical effects, such as the Coriolis acceleration, have to be discarded, as they do not apply to the linear Sagnac effect where the clock is essentially always in uniform linear motion.

Rotating platform: absolute and Einstein synchronization in flat space-time.

Absolute synchronization.

For motion on the $x-y$ plane, let the inertial frame $S'(t', x', y')$ be instantaneously co-moving with a point on the rim of the rotating disk and $S(t, x, y)$ be the laboratory inertial frame where the centre of the disk is stationary. Let us assume that, at $t' = t = 0$, the origin O$'$ of S' is at $x = x' = 0$ and $y = r$, instantaneously coinciding with the origin O of S in the direction of motion. The relation $\phi = \phi' + \omega t$ implies $r d\phi' = r\, d\phi - \omega r\, dt$, or the curvilinear transformation $ds' = ds - \omega r\, dt$ between frame S_c and S'_c, equivalent to the transformation $dx' = dx - v\, dt$ between S and S' for infinitesimal variations. Absolute synchronization and simultaneity can be achieved with no problem by external synchronization of clocks of frame S'_c with clocks of frame S_c. The proper time τ of the clock (or measuring apparatus) can be taken to be, $t' = \tau = t/\gamma = t(1 - v^2/c^2)^{-1/2} \simeq t$, and $dt = dt'$ to first order in v/c. Let us consider a signal propagating at the local speed u' along s' in the rotating frame. Then, as also remarked by several authors, the relation between the local speed u in S_c and the local speed u' in S'_c on the rotating disk, is given by the Galilean composition of velocities $u' = u - \omega r$, because $ds'/dt' = ds/dt - \omega r$.

Considering that $ds' = r d\phi' = u' dt'$, after multiplying by dt the velocity composition, $u = u' + \omega r$, we find

$$u\, dt = u'\, dt' + \omega r\, dt = r\, d\phi' + \omega r\, dt. \tag{21}$$

If $u = c$, after integrating (21) over ϕ' from zero to 2π we obtain, for counter-propagation, $T'_+ = 2\pi r/u' = T_+ = 2\pi r/(c + \omega r)$, and, without the need of the metric (20), we have recovered result (2). Getting back to the metric (20), if the speed of the signal is $u = c$ in frame S, the null geodesic condition gives $d\sigma^2 = 0$ in (20). Then, expression (21) with $u = c$ represents the obvious Newtonian solution to the metric (20) that can be written as

$$c^2 dt^2 - (r\, d\phi' + \omega r\, dt)^2 = 0. \tag{22}$$

Therefore, the time intervals $T'_\pm$ of (2), solutions to the Langevin–Landau–Lifshitz metric (20), are linked to the Galilean addition of velocities $u' = \pm c + \omega r$, corresponding to the absolute synchronization and simultaneity of the time transformation $t' = t$, and not to the relativistic addition of velocities of the LT. In fact, the approach just described, which has been widely used in the literature even by supporters of standard special relativity [13–15,19,20,26], relies on a Newtonian based absolute simultaneity, which is not consistent with the relativistic invariant $d\sigma^2$. Indeed, with $t' = t$ and for an infinitesimal time variation $dt' = dt$, expression (20) is not invariant in form. If $u' = c$ represents the local invariant light speed along the arc $ds' = r\, d\phi' = c\, dt$ in the rotating frame, (20) becomes $d\sigma^2 = c^2 dt^2 - (c^2 dt^2 + 2\omega rc\, dt^2) \simeq 2\omega rc\, dt^2 \neq 0$, and not the expected invariant $d\sigma^2 = c^2 dt'^2 - (r\, d\phi')^2 = 0$. Therefore, with $t' = t$ and the speed of light c in the lab frame S, the Langevin–Landau–Lifshitz metric (20) in the rotating frame is formally consistent with a non-invariant and non-isotropic local speed $u' = c' \neq c$ only.

Einstein synchronization.

The application of Einstein synchronization along the curvilinear circular arc s' can either be physically meaningful or not. Considering that the application of absolute synchronization along s' is physically meaningful, if Einstein synchronization is not, then it can be argued that, between absolute and Einstein, the absolute is the natural preferable synchronization for describing natural phenomena. In any event, since Einstein synchronization can be used in cylindrical spacetime (as done by Lee [23]), the same procedure, expressed in cylindrical coordinates, can be applied in principle also on the circumference of a rotating platform. Thus let us apply Einstein synchronization along s' in order to see what kind of results emerge in the interpretation of the Sagnac effect.

With a single clock it is impossible to measure the instantaneous local speed of a photon or a light pulse. The local speed of a photon propagating along the x' axis, or the curvilinear s' axis on the disk circumference, is the speed measured by spatially separated synchronized clocks. If Einstein synchronization is applied along the spatial elements dx and dx' in an infinitesimal time interval, the corresponding relation between $\mathrm{d}t'$ and $\mathrm{d}t$ is given by, $\mathrm{d}t' = \gamma(\mathrm{d}t - v\,\mathrm{d}x/c^2)$ for Cartesian coordinates. When two clocks, spatially separated by $\mathrm{d}x'$, mark simultaneously the same time t' in S', we have $dt' = 0$ and an observer in S perceives the two events as non-simultaneous and separated in time by the interval $\mathrm{d}t = \gamma v\,\mathrm{d}x'/c^2$. It is this term, related to nonconservation of simultaneity, that leads to the velocity composition, $u' = (u + v)/(1 + uv/c^2)$, which provides the invariant relation $c' = c$ in S', if $u = c$ in S. If Einstein synchronization is applied along ds and $\mathrm{d}s'$, the corresponding time relation is $dt' = \gamma(\mathrm{d}t - \omega r\,\mathrm{d}s/c^2)$ for the curvilinear coordinate. Because of the space-dependent term $dt' = -\gamma\omega r ds/c^2$, related to nonconservation of simultaneity, we obtain the velocity composition, $u' = (u + \omega r)/(1 + u\omega r/c^2)$, which provides the invariant relation $c' = c$ in S'_c when $u = c$ in S_c. Therefore, it is unreasonable to claim, as done by many authors, that, with the Galilean relativity used in the metric (20) (providing $c' \neq c$), the local speed of light is nevertheless c in the rotating platform. From a mathematical formal point of view, Einstein synchronization along s' can be extended and applied throughout the whole circumference of the rotating disk, leading to the transformations, $s' = \gamma(s - \omega rt)$, $t' = \gamma(t - \omega rs/c^2)$. Then, as formally required and expected, the flat Lorentz–Minkowski metric in cylindrical coordinates, invariant in form, after some simple algebra, transforms as

$$\mathrm{d}\sigma^2 = c^2\mathrm{d}t^2 - \mathrm{d}s^2 = c^2\mathrm{d}t'^2 - \mathrm{d}s'^2 = c^2\mathrm{d}t'^2 - r^2\mathrm{d}\phi^2, \tag{23}$$

regardless of the speed of the signal. If along s the speed $u = c$, we find $u' = c$ and $cdt' = r\,\mathrm{d}\acute{\phi} = \mathrm{d}s'$ along s', so that on both the geodesics of expression (23) we have, $d\sigma^2 = 0$. Thus, in this case, with Einstein synchronization and with the invariant local speed of light $c' = c$ on the rim of the disk, after integrating over $\acute{\phi}$ we obtain, $T'_{\pm} = 2\pi r/c$, and $\Delta T' = 0$, a result that is obviously in contrast with experiment, as Sagnac claimed. In order to achieve agreement with experiment, for example for a counter-propagating signal with local speed c, as for the case of the linear Sagnac effect, the closed contour $2\pi r$ must be reduced to the open path $2\pi r/(1 + \omega r/c)$, leading to the observed value $T'_{+} = 2\pi r/(c + \omega r)$.

Still, the resulting time transformation $t' = \gamma(t - \omega rs/c^2)$ for synchronized clocks along s' leads to the contradiction that, as observed from the lab frame S_c, at $t = 0$, the time displayed by the moving clock at the origin $s = 0$ is $t'(0) = 0$ while the same clock, thought of as being at $s \simeq 2\pi r$, must display the time $t'(2\pi r) \simeq -2\pi r\omega r/c^2$. This well-known fact is referred to by stating that Einstein synchronization fails when applied to a closed contour [6,8,9,16–18,20] and this unphysical prediction of a discontinuity in time on a rotating disc arising from Einstein synchronization has been appropriately described by Klauber [17] as 'bizarre', as it amounts to a clock being out of synchronization with itself! Therefore, Einstein synchronization, or more precisely the equivalent assumption of the invariance of c, leads to a result in disagreement with observation.

In conclusion, by adopting Einstein synchronization, for the circular Sagnac effect we find the same difficulties and inconsistencies that emerge in the interpretation of the linear Sagnac effect considered in the previous section. Considering that two different synchronization procedures can be deemed to be physically equivalent if both interpret physical reality equally well and coherently, an immediate consequence is that Einstein synchronization is not equivalent to absolute synchronization, as proved independently in [5]. It is not that Einstein synchronization cannot be applied to a rotating platform or to an inertial frame. Einstein synchronization can be formally applied to both the circular and linear Sagnac effects. However, in both cases it fails to provide a coherent physical interpretation because, if the local light speed is c along the length $2L$ (or $2\pi r$) of the whole contour, the expected result is $T_{\pm} = 2L/c$, in contrast with observation. The theoretical result agrees with observation if the local light speed is c along an open section only of the whole contour, as shown for both the linear and circular effect. Thus, the bottom line is that a coherent interpretation of the Sagnac effect favours the 'natural' absolute synchronization and simultaneity, as shown also by Lee [23] for special relativity in cylindrical spacetime.

4. The preferred frame of transformations based on absolute simultaneity

We sketch here possible scenarios where transformations based on absolute simultaneity can be applied, even though more research is needed in this area.

According to Maxwell, his equations are valid in the preferred frame at rest with the medium, ether or vacuum, where electromagnetic waves may propagate. The value c of the speed of light is related to the value of the dielectric and magnetic permeability constants in vacuo, ε_0 and μ_0 respectively. If the light speed is c in a given inertial reference frame in vacuo (empty space), supporters of SR argue that it must be c also in any other frame in motion relative to it, because the physical conditions of the vacuum are unchanged. However, this argument only applies to the ontological, or epistemological, non-physical 'vacuum' that, as such, is not characterized by any physical measurable attributes or physical constants. Thus the argument does not apply to Maxwell's medium where light may propagate at the speed c because the vacuum possesses the adequate physical properties, such as ε_0 and μ_0, that *a priori* might vary for an observer in relative motion, as happens for the common case of light propagating in water with refractive index n at the speed c/n. A vacuum with physical attributes is also conceived in other scenarios, such as the one of Quantum Electrodynamics and Gravitation. Thus, in the context of relativistic theories adopting the LTA, we conveniently denote Maxwell's vacuum by the term 'physical space,' a space with physical attributes.

As per the conventionalist thesis [2], the LTA can be considered physically equivalent to the LT and, therefore, the preferred frame is not 'identifiable' and can be chosen arbitrarily. However, since the Sagnac effect can be described coherently by means of absolute simultaneity, but not with relative simultaneity, absolute and Einstein synchronization cannot be physically equivalent, as has been proved independently in the work of [5]. Then, the preferred reference frame of absolute simultaneity, associated with the LTA, or the compatible time transform $t' = t$, has to be the unique 'identifiable' preferred rest frame where space is isotropic and the one-way speed of light is c. The natural properties of this 'identifiable' preferred frame make it possible to single it out by means of the approach described in [5] that invalidates the conventionalist thesis, so that the preferred frame of the LTA cannot be chosen arbitrarily. In some relativistic preferred frame models the preferred frame is the one where the cosmic background radiation is isotropic [2], but there are other possibilities considered below.

Other models, where transformations with absolute simultaneity can be used, rely on a modern version of the Stokes–Planck theory [28] that was introduced to explain optical phenomena including the phenomenon of stellar aberration. In the Stokes–Planck theory, the classical ether, where light propagates, is dragged by massive bodies (planets and stars) in their motion in such a way that it can be practically locally at rest with the body. In a modern version of the theory, also the Stokes–Planck ether can be replaced by the concept of 'physical space'. We may assume that the gravitational field of a massive body creates a space curvature, such that the corresponding physical space, embedded by the gravitational field, is 'dragged' and bound to be co-moving with the body. In this scenario, the basic hypothesis consists of assuming that the physical space is the actual medium where the speed of light propagates at, or nearly to, the local speed c. Although gravitation may affect radially the speed c, the rotation of the body about its axis should not affect locally the isotropy of space if the body has spherical symmetry. In this model, the physical space can be tentatively described by means of an ideal centred inertial frame (fixed to the centre of mass of the body) where, within a finite range, space is isotropic and the speed of light is locally c. Therefore, within the centred inertial frame of a huge massive body, we may expect as being unlikely the existence of an effect analogous to the historical 'ether wind'. In this case, this centred inertial frame could assume the properties of a preferred frame where the one-way speed of light is c, at least locally and within the range of its gravitational field. Of course, if a light ray leaves the massive body to move in intergalactic space, its local speed there will be determined and affected by the average gravitational fields of nearby celestial bodies and galaxies and, in this intergalactic 'nearly empty' physical space, the cosmic background radiation might be isotropic. Then, if the speed of a light ray is c locally in the preferred frame of the intergalactic physical space, its speed will no longer be c relative to the centred inertial frame of a massive body in relative motion, but it will be c locally as soon as the light ray enters the physical space of the massive body. In a way, the concept of a constant light speed c is maintained, although with the limitation that it is valid locally within the 'nearly empty' intergalactic space or the 'dragged' physical space of massive bodies.

In the case of the Earth and in the context of the application of the Global Positioning System (GPS), the frame that represents the physical space where the local speed of light c is isotropic, is denoted as the Earth Centred Inertial (ECI) frame. It is centred on the Earth's axis and it moves with the Earth in its orbit around the Sun but does not share its rotation. Also the Sun has its own centred inertial frame (SCI), as elaborated by Field [9] in the context of optical experiments. As considered by Gift [16], Ashby [29], and other authors [3,30,31] the existence of the ECI is supported by the fact that it clarifies the problem of clock synchronization on the Earth. Indeed for achieving the clock synchronization with Einstein synchronization in the GPS and maintaining accuracy, the GPS must apply a Sagnac velocity correction to the propagation of its electromagnetic signals. This can be understood by considering that, if the speed of light is c locally in the ECI frame, it turns out to be $c \pm v$ on the rotating Earth surface (at the distance R from its centre) because of the tangential velocity $v = \omega R$ [16,30,31]. Thus the GPS algorithm seems to be supportive of the ECI frame and absolute synchronization for maintaining global accuracy among synchronized clocks. The result is a world-wide network of precisely synchronized clocks that are within 4 ns of 'perfect synchronization' with global simultaneity within the GPS [16].

5. Conclusion

The claim by Sagnac [1], Selleri [3] and many other authors [6,8,9,14,16–18,27,31] is that the result of the Sagnac experiment, when considered by an observer co-moving with the measuring apparatus, is not consistent with a constant *local* light speed c along the closed contour, in contrast with Einstein's second postulate on the invariance of the speed of light. These claims have been considered and addressed in our paper in a rational way by adopting both absolute and Einstein synchronization and verifying rigorously whether they, and the related fundamental concepts of absolute and relative simultaneity, permit coherent interpretation of the Sagnac effect or not. Most readers are aware that there is a strong and understandable tendency, adopted by physicists, in favour of maintaining the current paradigm based on relative simultaneity and the related invariance of the speed of light. We find that this tendency is kept in several of the interpretations of the Sagnac effect in the context of standard Special Relativity, where, for example, the authors use a non-invariant metric for the rotating platform, and some might not even be aware that they are actually adopting absolute instead of Einstein's relative simultaneity. In other approaches, the authors that adhere to the conventionalist thesis are forced into formulating ad hoc unphysical assumptions, such as that of the time gap [12], in the attempt to reconcile relative and absolute simultaneity. Nevertheless, after Sagnac, for several decades there has been a recognition, visible in recent literature [6,8,9,12,16–18,23] that conservation of simultaneity and the related transformations offer a simpler and physically meaningful way to interpret optical experiments. We consider that this recognition is confirmed by our analysis of the Sagnac effect in its circular and linear versions where we assume the non-equivalence of Einstein and absolute synchronization, proved in Ref. [5]. The conclusion is that a coherent interpretation of the Sagnac effect favours absolute over Einstein synchronization, indicating that transformations based on absolute simultaneity (such as the LTA) are likely candidates for describing the whole body of natural phenomena. We believe that models and theories of modern physics based on relativistic transformations, should be explored using transformations based on absolute simultaneity, either rather than or beside the LT. A comparison between the two will help restore rationality and avoid the development of a modern physics based on unreasonable ad hoc assumptions.

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