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Exact and inexact decompositions of trade price indices

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Abstract

A reasonable concept for the true trade price index in situations where low-price countries capture market shares from high-price countries is the average price paid by importers for the same quality of good or service from all exporting countries. However, decompositions of trade price indices are usually *inexact* in the sense that the average price used as the underlying aggregator formula is *not* exactly reproduced. In this paper, we compare analytically exact and inexact decompositions of trade price indices, paying particular attention to the bias in aggregate inflation occurring from applying the first-order Taylor series approximation and not the quadratic approximation lemma to a geometric average price. Our calculations, using the Norwegian clothing industry as an illustration, reveal that the bias in aggregate inflation over the sample period of 1997–2016 is quite substantial and as much as 0.6 percentage point in some years. We therefore conclude that the quadratic approximation lemma should be used in practice to exactly reproduce the underlying aggregator formula.

Keywords Trade price indices · Exact and inexact decompositions · First- and second-order Taylor series approximations · Quadratic approximation lemma · Bias in aggregate inflation

JEL Classifications C43 · E31 · F14

1 Introduction

Index number theory generally recommends the use of superlative price index formulae, including the Fisher, Walsh and Törnqvist price indices, when aggregating prices of internationally traded goods and services; see, for example, ILO et al. (2009,

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p. 34).[1] These indices yield good approximations of the true inflationary effects of international trade given the central assumptions that the importing countries are free to choose among all goods and services and that changes in country composition of imports follow from *changes* in relative prices. In practice, however, import patterns have changed over the past few decades as a result of a gradual liberalisation of international trade along with large initial price *level* differences among exporting countries. The observed increase in the share of imports from low-price countries, China in particular, has accordingly provided additional deflationary effects of international trade. Because the economic mechanism behind the deflationary effects is attributable to trade liberalisation and price level differences and not to changes in relative prices, aggregating prices by means of superlative price index formulae will potentially not capture the true inflationary effects of imports increasingly originating from low-price countries.

The additional deflationary effects of international trade are closely related to what the Boskin Commission calls the outlet substitution bias, which occurs in classical price index formulae due to failure to adequately account for situations where discount outlets capture market shares from high-cost retailers; see Boskin et al. (1996, p. 5).[2] As argued by Silver (2010), Hausman and Leibtag (2009), White (2000), Diewert (1998) and Reinsdorf (1993), among others, if differences in the goods or services provided by the discount and the high-cost retailers are negligible (homogenous products), then a reasonable concept for the "true" price index is the average price paid by consumers over all outlets. The System of National Accounts also advocates that the price relatives used for index number calculation when there is price variation for the same quality of good or service should be defined as the ratio of the weighted average price in two consecutive periods, the weights being the relative quantities sold at each price; see European Commission et al. (2009, p. 303, paragraph 15.68).

Similarly, a reasonable concept for the true trade price index in situations where low-price countries capture market shares from high-price countries is the average price paid by importers for the same quality of good or service from all exporting countries, see, for example, ILO et al. (2009, p. 75). Of course, the underlying premise of truly homogenous products may not be the case in practice, but for the literature that has developed a framework for analysing how changing import patterns impact trade price indices, it *is* the case by assumption. For instance, Nickell (2005), ECB (2006), Pain (2006), Kamin et al. (2006), Wheeler (2008), MacCoille (2008) and Thomas and Marquez (2009) seek to include the deflationary effects of the observed shifts of imports towards low-price countries by employing either a geometric or an arithmetic average price. However, because a first-order Taylor series approximation is used, the

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[1] See Diewert (1976) for the economic theory and the definitions underlying the superlative price indices. There is also a consensus among economists that the most appropriate aggregator formulae to use in empirical applications, at least in principle, are the superlative price indices; see, for example, ILO et al. (2009, p. 28).

[2] The Boskin Commission estimated outlet substitution bias to contribute 0.1 percentage point per year to the overall upward bias of 1.1% in the US consumer price index (CPI). A later study by Gordon (2006) finds that the outlet substitution bias remains at 0.1 percentage point per year and that the overall upward bias in the CPI is reduced to 0.8%.

decompositions of trade price indices in Nickell (2005), among others, are *inexact* in the sense that the underlying aggregator formula is *not* exactly reproduced.

In this paper, we take the assumption of truly homogenous products as a starting point and motivate the use of the geometric average price by building on the theoretical model of consumer behaviour in Hausman and Leibtag (2009). We contribute to the existing literature by illustrating the importance of conducting an exact decomposition of a geometric average price. Specifically, we compare analytically exact and inexact decompositions of trade price indices, paying particular attention to the bias in aggregate inflation occurring from using the first-order Taylor series approximation and not the quadratic approximation lemma by Diewert (1976) to a geometric average price. We show that the bias in aggregate inflation vanishes only in the special cases when inflation rates are equal across exporting countries and/or when no switching of imports occurs from high-price to low-price countries or vice versa. As an empirical illustration, we estimate the bias in aggregate inflation using annual data from the Norwegian clothing industry, which has experienced massive trade liberalisation and increasing imports from China and other low-price countries since the Uruguay Round Agreement starting in the mid 1980s. Our calculations reveal that the bias in aggregate inflation over the sample period of $1997 - 2016$ is quite substantial and as much as 0.6 percentage point in some years. We therefore argue that the quadratic approximation lemma should be applied in practice for decomposing a geometric average price.

The rest of the paper is organised as follows: Sect. 2 outlines the theoretical background behind the use of the geometric average price as the true trade price index. Section 3 compares analytically the exact and inexact decompositions to the geometric average price. Section 4 presents the empirical illustration. Section 5 provides a conclusion.

2 Theoretical background

Using a two-stage utility consistent consumer choice model to account for outlet substitution bias in the CPI for identical food items, Hausman and Leibtag (2009) show that the true price index is an expenditure weighted average of the high price of the supermarkets and the low price of the supercenters. Likewise, the two-stage choice model by Hausman and Leibtag (2009) can be applied in the context of a cost-minimising establishment that chooses to import goods of the same quality from either a high-price country or a low-price country.

Hence, we apply a version of the two-stage choice model in which the establishment first at the lower stage considers its importing behaviour conditional on type of destination choice, high-price or low-price country, and then at the upper stage decides which type of country to import, say clothing, from. At the lower stage, the establishment has a conditional expenditure function

$$y = e(\mathbf{p}_0, p^j; \bar{u}), \tag{1}$$

where $\mathbf{p}_0$ is a vector of prices of all nonclothing goods, assumed the same for the destination choice, $p^j = \{p_1^j, \ldots, p_n^j\}$ are the prices of the n clothing goods from the two types of destination choice denoted by the superscript $j = 1$ (high-price country) and $j = 2$ (low-price country), and $\bar{u}$ is the production level. The conditional quantity of imports for each type of clothing good i, depending on the type of destination j chosen, is

$$x_i^j = \frac{\partial e(\mathbf{p}_0, p^j; \bar{u})}{\partial p_i^j} = -\frac{\partial v(\mathbf{p}_0, p^j, y)/\partial p_i^j}{\partial v(\mathbf{p}_0, p^j, y)/\partial y}, \quad i = 1, \ldots, n, \tag{2}$$

where the indirect function $v(p, y)$ is derived from the duality relationship with the expenditure function. Using duality further, the minimum expenditure required to achieve $\bar{u}$, is given by

$$E(p^j; \bar{u}) = e(p^j; \bar{u}) = y^j(p^j; \bar{u}) = y^j = \sum_{i=1}^{n} p_i^j x_i^j. \tag{3}$$

An average price, $\bar{p}^j$, can now be calculated by dividing y^j by a quantity index, $\bar{x}^j$, such that $y^j = \bar{p}^j \bar{x}^j$. At the upper stage, the establishment's destination choice can be considered by means of a binominal logit model in which the probability of choosing the high-price country is

$$pr(j = 1) = \frac{1}{1 + \exp[\beta_0 + \beta_1(\bar{p}^1 - \bar{p}^2)]}. \tag{4}$$

Assuming that the overall units of a good are the same, we can simplify such that the overall quantity of good i becomes

$$\hat{x}_i(\mathbf{p}_0, p^1, p^2, y) = pr(j = 1)x_i^1(\mathbf{p}_0, p^1, y) + pr(j = 2)x_i^2(\mathbf{p}_0, p^2, y), \tag{5}$$

where x_i^1 and x_i^2 are the conditional quantities of imports from (2). Similarly to the lower stage, the unconditional price for good i can be calculated by

$$\hat{p}_i = \frac{E_i(\mathbf{p}_0, p^1, p^2, y)}{\hat{x}_i(\mathbf{p}_0, p^1, p^2, y)}, \tag{6}$$

where E_i denotes the overall expenditure on good i and $\hat{x}_i$ is the overall quantity of good i from (5). Clearly, (6) demonstrates that the true price index in a situation where both a high-price country and a low-price country are available to the establishment is an expenditure weighted average of the two prices of the high-price country and the low-price country.

The two-stage choice model by Hausman and Leibtag (2009) thus shows that a weighted average of prices is a reasonable concept for a true price index when consumers are shifting from one shopping outlet to another or when importers are shifting from a high-price to a low-price country. In the following, we apply a geometric average price to facilitate an explicit comparison of the inexact decomposition applied in the literature and the corresponding exact decomposition.

3 Analytical comparison

As pointed out by Diewert (2002), it is well known that a second-order Taylor series approximation to a quadratic function, evaluated at two points, will exactly reproduce the quadratic function. It is not so well known, however, that the arithmetic average of two first-order Taylor series approximations evaluated at two points will also exactly reproduce a quadratic function, a result called the quadratic approximation lemma by Diewert (1976). We utilise these properties in our context, as a means of comparing the exact and inexact decompositions, by first writing the geometric average price used by Nickell (2005), among others, as a quadratic function of the form

$$F(S_t, p_t) = \sum_{n=1}^{N} S_{nt} p_{nt}, \tag{7}$$

where $(S_{1t}, \ldots, S_{Nt}) \equiv S_t$ is a set of N value shares of imports of a commodity group of interest in period t, $0 \leq S_{nt} \leq 1$ and $\sum_{n=1}^{N} S_{nt} = 1, \forall t$, and $(p_{1t}, \ldots, p_{Nt}) \equiv p_t$ is a set of N (logarithmic) price levels of a particular good or service in period t.[3]

The second-order Taylor series approximation to $F(S_t, p_t)$ evaluated around period $t-1$ is

$$\begin{aligned}\Delta F(S_t, p_t) = &\sum_{n=1}^{N} F_{Sn}(S_{t-1}, p_{t-1})\Delta S_{nt} + \sum_{n=1}^{N} F_{p_n}(S_{t-1}, p_{t-1})\Delta p_{nt}\\ &+ \sum_{n=1}^{N} F_{S_n p_n}(S_{t-1}, p_{t-1})\Delta S_{nt}\Delta p_{nt},\end{aligned} \tag{8}$$

where Δ denotes the difference operator, $F_{S_n}(S_{t-1}, p_{t-1})$ and $F_{p_n}(S_{t-1}, p_{t-1})$ are the first-order partial derivatives of $F(S_t, p_t)$ with respect to S_n and p_n, respectively, evaluated at period $t-1$, and $F_{S_n p_n}(S_{t-1}, p_{t-1})$ are the second-order partial derivatives of $F(S_t, p_t)$ with respect to S_n and p_n, evaluated at period $t-1$.[4]

Similarly, the second-order Taylor series approximation to $F(S_t, p_t)$ evaluated around period t is

$$\begin{aligned}\Delta F(S_t, p_t) = &\sum_{n=1}^{N} F_{S_n}(S_t, p_t)\Delta S_{nt} + \sum_{n=1}^{N} F_{p_n}(S_t, p_t)\Delta p_{nt}\\ &- \sum_{n=1}^{N} F_{S_n p_n}(S_t, p_t)\Delta S_{nt}\Delta p_{nt}.\end{aligned} \tag{9}$$

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[3] Our analytical framework below builds on Diewert (2002). Whereas Diewert (2002) considers a quadratic function $F(z_1, \ldots, z_N)$ consisting of one set of N variables defined as $(z_1, \ldots, z_N) \equiv z$, we consider two sets of N variables in (7). In the following, lower case letters indicate natural logarithms of a variable.

[4] The two expressions for the other second-order partial derivatives, $F_{S_n S_n}(S_{t-1}, p_{t-1})$ and $F_{p_n p_n}(S_{t-1}, p_{t-1})$, are both equal to zero for all n.

Now we can apply the quadratic approximation lemma by taking the arithmetic average of the first-order Taylor series approximations inherent in (8) and (9) to obtain

$$\Delta F(S_t, p_t) = \sum_{n=1}^{N} (1/2)[F_{S_n}(S_{t-1}, p_{t-1}) + F_{S_n}(S_t, p_t)]\Delta S_{nt} + \sum_{n=1}^{N} (1/2)[F_{p_n}(S_{t-1}, p_{t-1}) + F_{p_n}(S_t, p_t)]\Delta p_{nt}. \tag{10}$$

Since (8–10) are equivalent and yield *exact* decompositions of (7), it follows that $\sum_{n=1}^{N} F_{S_n p_n}(S_{t-1}, p_{t-1})\Delta S_{nt}\Delta p_{nt}$ from (8) and $\sum_{n=1}^{N} F_{S_n p_n}(S_t, p_t)\Delta S_{nt}\Delta p_{nt}$ from (9) define the bias in aggregate inflation, but with opposite signs. We can simplify the expressions for the bias in absolute value, B_t, as

$$B_t = \left| \sum_{n=1}^{N} \Delta S_{nt}\Delta p_{nt} \right|, \tag{11}$$

because $F_{S_n p_n}(S_{t-1}, p_{t-1}) = F_{S_n p_n}(S_t, p_t) = 1$. Hence, the bias from using *inexact* decompositions of (7) is equal to a weighted sum of underlying country-specific inflation rates with the changes in the respective value shares of imports as weights.[5] As such, $B_t = 0$ only in the special cases when the inflation rates are equal across exporting countries and/or no switching of imports occurs from high-price to low-price countries or vice versa.

To compare the exact and inexact decompositions in more detail, we assume one low-price and one high-price country, apply (10) to (7) and write the *exact* decomposition of aggregate inflation, Δp_t, as

$$\Delta p_t = \overline{S_{1t}}\Delta p_{1t} + (1 - \overline{S_{1t}})\Delta p_{2t} + \Delta S_{1t}(\overline{p_{1t}} - \overline{p_{2t}}), \tag{12}$$

where Δp_{1t} and Δp_{2t} are the inflation rates in the low-price and the high-price country, respectively, in period t, $\overline{p_{1t}}$ and $\overline{p_{2t}}$ are the average price for periods t and $t-1$ in the low-price and the high-price country, respectively, and $\overline{S_{1t}}$ is the low-price country's average value share of imports for periods t and $t-1$.[6] The first two terms on the right hand side of (12) correspond to aggregate inflation when the Törnqvist price index is used as the underlying aggregator formula. The last term, $\Delta S_{1t}(\overline{p_{1t}} - \overline{p_{2t}})$, constitutes the deflationary effects of the shifts of imports from the high-price to the low-price country due to the lowering of trade barriers. The greater the change in the import share and the greater the difference in relative price levels, the greater the deflationary effects in Δp_t. Although the cross-country distribution of the deflationary effects is

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[5] Note that $\Delta S_{nt} = S_{nt} - S_{nt-1}$ and that $\Delta p_{nt} = p_{nt} - p_{nt-1}$, which is, due to the use of natural logarithms, approximately equal to the inflation rate given by $(P_{nt} - P_{nt-1})/P_{nt-1}$.

[6] To derive (12), we have utilised the facts that $\overline{S_{2t}} = 1 - \overline{S_{1t}}$ and $\Delta S_{2t} = -\Delta S_{1t}$.

sensitive to the choice of numeraire country, the size of the aggregate deflationary effects is not affected when more than two countries are involved in the calculations.[7]

Note that the deflationary effects are zero only in the special cases when the import share is constant ($\Delta S_{1t} = 0$) and/or when the composition of trade changes between countries with identical price levels ($\overline{p_{1t}} - \overline{p_{2t}} = 0$). It is therefore likely that the Törnqvist price index, or any other classical index number formula, for that matter, fails to account for the deflationary effects in (12). Suppose that the low-price country has relatively high inflation for a particular tradable good and that barriers to trade are reduced. As a result, imports from the low-price country increase at the expense of imports from the high-price country. Using the Törnqvist price index as an import price index will thus capture only the higher inflation and not the lower price level due to the shift in imports. The Törnqvist price index therefore does not represent the *true* inflationary effects of imports in this case. It is noteworthy, however, that the potential inappropriateness of the Törnqvist price index rests on the premise of quantity switches of truly homogenous products from high to low-price countries. As index number theory tells us, superlative price indices are superior for heterogeneous products; see, for example, Silver (2010) and the references cited therein.

When the first-order Taylor series approximations from (8) and (9) rather than the quadratic approximation lemma are applied to (7), the comparable *inexact* decompositions of Δp_t evaluated at period $t-1$ and t become

$$\Delta p_t \approx S_{1t-1}\Delta p_{1t} + (1 - S_{1t-1})\Delta p_{2t} + \Delta S_{1t}(p_{1t-1} - p_{2t-1}) \qquad (13)$$

and

$$\Delta p_t \approx S_{1t}\Delta p_{1t} + (1 - S_{1t})\Delta p_{2t} + \Delta S_{1t}(p_{1t} - p_{2t}), \qquad (14)$$

respectively. The first two terms on the right hand side of (13) and (14) now correspond to aggregate inflation when the geometric Laspeyres and the geometric Paasche price indices are used as the underlying aggregator formula. The deflationary effects in (13) and (14) are also somewhat different from those in (12), as relative price levels in period $t-1$ and t are not the same as the relative arithmetic mean of price levels in period t. It follows from (11) in the case of one low-price and one high-price country that the bias in aggregate inflation when first-order Taylor series approximations rather than the quadratic approximation lemma are used for decomposition of (7) is

$$B_t = \mid \Delta S_{1t}(\Delta p_{1t} - \Delta p_{2t}) \mid . \qquad (15)$$

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[7] Using a high-price country as the numeraire country will increase the size of the deflationary effects from a low-price country with a rising import share, whereas using a low-price country as the numeraire country will increase the size of the deflationary effects from a high-price country with a falling import share. That said, it can be shown that the evolution of the deflationary effects in (12) can be decomposed into the relative price levels in the base period and the relative inflation rates in period t between the low-price and the high-price country, see Benedictow and Boug (2017). Hence, higher inflation over time in the low-price country with a rising import share will dampen the deflationary effects from the base period over time and vice versa.

Because (14) is used by Nickell (2005), among others, it is implicitly assumed in existing analyses of the impact of imports from emerging countries on inflation in developed countries that $B_t = 0$ or negligible.[8] Having established the analytical framework for comparing the exact and inexact decompositions of trade price indices based on (7), we now turn to the empirical illustration to shed light on the potential significance of the bias in aggregate inflation in practice.

4 Empirical illustration

As noted in the introduction, we use annual data from the Norwegian clothing industry over the sample period of 1997–2016. Our empirical illustration is motivated by the fact that the Norwegian clothing industry has undergone massive trade liberalisation since the Uruguay Round Agreement starting in the mid 1980s, which has increased imports of clothing from China and other low-price countries at the expense of imports from high-price countries, the euro area in particular.[9] The significant shift in trade pattern over the last three decades or so has contributed to reduced purchasing prices for Norwegian importers of clothing and therefore also to consumer prices for clothing.

The underlying data are price indices (measured in local currencies) for the main exporters of clothing to Norway: the euro area (*ea*), Denmark (*dk*), Sweden (*se*), UK (*uk*), Turkey (*tr*), China (*cn*), Hong Kong (*hk*), Vietnam (*vn*), Bangladesh (*ba*) and India (*in*).[10] Together these countries accounted for about 85% of Norwegian imports of clothing through the sample period.[11] Data on clothing prices for China are only available from 1997, defining the starting point of the sample period. The clothing price indices are converted into a common currency, the Norwegian krone (NOK), by means of bilateral exchange rates, and the import weights are defined as the value shares of clothing imports from the countries listed above. Price indices and exchange rates are acquired from Macrobond and import values are acquired from Statistics Norway's foreign trade statistics. Finally, relative price levels between countries in period t are calculated by means of the formula

$$\frac{P_{nt}}{P_{eat}} = \frac{P_{n2011}}{P_{ea2011}} \frac{I_{nt}}{I_{eat}}, \tag{16}$$

where $\frac{P_{n2011}}{P_{ea2011}}$ are relative clothing price levels adjusted for purchasing power parity in 2011 with the euro area (*ea*) as the numeraire country, based on the OECD statistics and the international comparison programme by the World Bank,[12] and $I_{nt} = \frac{P_{nt}}{P_{n2011}}$ and $I_{eat} = \frac{P_{eat}}{P_{ea2011}}$ are clothing price indices in period t with 2011 as the base year, $\forall t$.

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[8] See equation (1) in Nickell (2005).

[9] See Høegh-Omdal and Wilhelmsen (2002) for a summary of the trade policy liberalisation of the Norwegian clothing industry.

[10] We simplify matters by treating the euro area as one country. Note that export prices for Vietnam, Bangladesh and India are proxied by consumer prices due to lack of price data on clothing for these countries.

[11] The remaining exports of clothing to Norway come from countries with relatively small import shares.

[12] See OECD (2011) and WB (2015).

Table 1 Summary of statistics

Country(n)[1]	Prices		Weights[4]		
	P_{n2011}/P^{2}_{ea2011}	$\overline{\Delta p_n}^{3}$	1997	2016	$\overline{\Delta S_n}^{5}$
Sweden (*se*)	1.25	1.8	4	2	−0.1
Denmark (*dk*)	1.24	2.8	10	1	−0.5
Euro area (*ea*)	1.00	1.7	41	16	−1.3
UK (*uk*)	0.79	1.6	8	1	−0.4
Turkey (*tr*)	0.64	1.0	3	7	0.2
China (*cn*)	0.58	2.9	24	51	1.4
Hong Kong (*hk*)	0.47	1.6	5	0	−0.3
Bangladesh (*ba*)	0.30	4.7	1	9	0.4
Vietnam (*vn*)	0.26	4.4	1	8	0.3
India (*in*)	0.22	4.4	2	4	0.1

The Norwegian clothing industry (see Appendix for data definitions and sources)

[1] Together these countries covered close to 85% of Norwegian imports of clothing throughout the sample period of 1997–2016, $n \equiv (se, dk, ea, uk, tr, cn, hk, ba, vn, in)$

[2] Purchasing power parity adjusted relative price levels for clothing in 2011 with the euro area (*ea*) as the numeraire country; see OECD (2011) and WB (2015)

[3] Average annual export price inflation of clothing, measured in the Norwegian currency (NOK), per cent

[4] Value shares of imports of clothing, per cent, do not sum to unity due to rounding errors

[5] Average annual change in value shares of imports of clothing, percentage points

Table 1 summarises the price and weight statistics used in the calculations over the sample period. Using the euro area as the numeraire country implies that $\frac{P_{ea2011}}{P_{ea2011}}$ equals unity. The relative clothing price levels in 2011 are thus easy to interpret. For instance, the price level in India was around 20% of that in the euro area in 2011. The corresponding figure for Sweden was around 125%. The UK, Turkey, China, Hong Kong, Bangladesh, Vietnam and India accordingly stand out as low-price countries and Sweden and Denmark as high-price countries. It is further evident that average annual export price inflation has varied considerably across the countries. Relatively high inflation in most of the low-price countries throughout the sample period implies significant catch-up effects in export price levels. After China abandoned the USD peg in 2005, leading to a substantial appreciation of the yuan against the USD, Chinese export prices increased rapidly. Import shares have also changed markedly across countries. Most importantly, the share of imports from China has increased by 27 percentage points, from a level of around 25% in 1997, mainly at the expense of the share of imports from the euro area. The Chinese import share accelerated from 2001 when China joined the WTO, but peaked around 2012 at 55%. The shares of imports from most of the other low-price countries have also increased significantly through the sample period, mainly at the expense of the shares of imports from the high-price countries.[13] To sum up, the significant differences in the inflation rates and

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[13] Imports of clothing from the UK have fallen considerably, consistent with the export price level approaching the export price level of the euro area towards the end of the sample period. That clothing imports from

the changing import shares across the exporting countries illustrate how a first-order Taylor series approximation to (7) imposes a likely bias in aggregate inflation.

Figure 1 shows the exact and inexact decompositions of (7) based on (12) and (14), respectively, together with the Törnqvist price index based on (12) and the bias in aggregate inflation based on (15). Particularly high aggregate deflation is evident in 2002 and is mainly attributable to high deflation rates in low-price countries due to NOK appreciation of more than 10% that year. The aggregate inflation of close to 20% in 2015 is similarly explained mainly by high inflation rates in low-price countries due to NOK depreciation of close to 30% in the wake of the huge drop in the oil price in 2014. The discrepancy in several individual years between aggregate inflation calculated by (12) and the Törnqvist price index is rather significant. For instance, the discrepancy is as much as 5 percentage points in 2009 as the Törnqvist price index does not take into account the deflationary effects generated by the switch in imports towards low-price countries. The deflationary effects, which are dominated by China, push down aggregate inflation by an annual average of 2.1 percentage points over the sample period. As a result, the *total* effects based on (12) and the inflationary effects alone based on the Törnqvist price index contribute annual averages of 0.5 and 2.6 percentage points, respectively, to aggregate inflation from 1997 to 2016.

Our calculations also reveal that the bias in aggregate inflation over the sample period is quite substantial and as much as 0.6 percentage point in some years when a first-order Taylor series approximation rather than the quadratic approximation lemma is applied to (7). The magnitude of the annual bias in aggregate inflation may have important implications for the estimation of pricing-to-market models of clothing import prices and for the conduct of monetary policy by the inflation targeting central bank.

5 Conclusions

In this paper, we have compared analytically the exact and inexact decompositions of trade price indices based on a geometric average price and derived an expression for the bias in aggregate inflation arising from applying the first-order Taylor series approximation and not the quadratic approximation lemma. We have shown that the bias in aggregate inflation is zero only in the special cases when inflation rates are equal across exporting countries and/or when no switching of imports occurs from high-price to low-price countries or vice versa. Hence, the bias may be significant in practice as import patterns have changed dramatically over time following massive trade liberalisation in many countries. Our empirical illustration, using annual data from the Norwegian clothing industry over the sample period of 1997 − 2016, revealed that the bias in aggregate inflation is quite substantial and as much as 0.6 percentage point in some years. We therefore conclude that the quadratic approximation lemma should be applied in practice for decomposing trade price indices based on a geometric average price.

Footnote 13 continued
Hong Kong have diminished, despite it being a relatively low-price country, may be explained by reasons other than price, for instance changing preferences among Norwegian consumers of clothing.

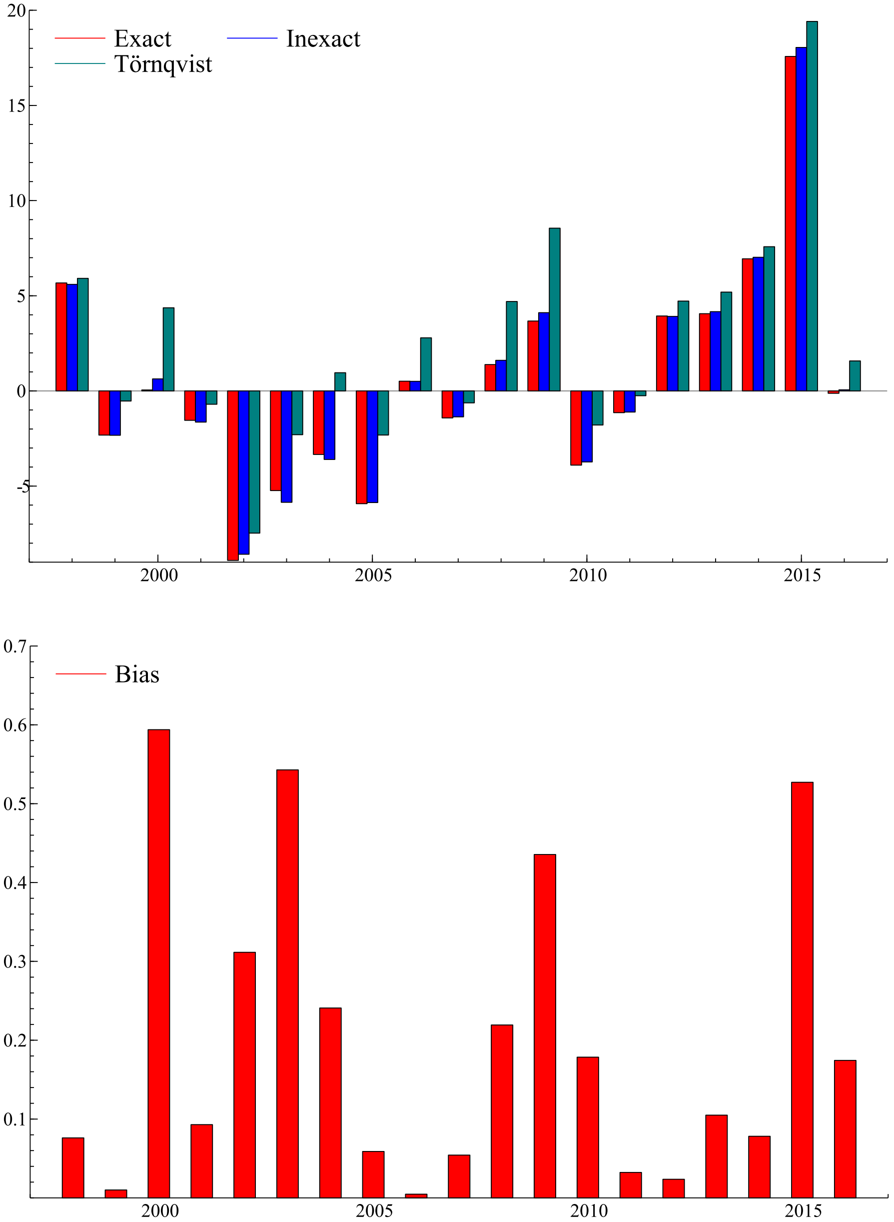

Fig. 1 Exact and inexact decompositions of Δp_t, Törnqvist price index and bias. Data from the Norwegian clothing industry. The exact decomposition and the Törnqvist price index are based on (12), the inexact decomposition is based on (14) and the bias in aggregate inflation is based on (15). Upper panel in per cent and lower panel in percentage points

Although not important for the purpose of this paper, we should emphasise that the usefulness of a ratio of a geometric average price (like a unit value index) and the potential inappropriateness of a price index formula (like a superlative price index) rests on the premise of quantity switches of truly homogenous products from high to low-price countries. A ratio of a geometric average price may thus be accepted as the true trade price index for homogenous products whereas a superlative price index, as well established in the index number literature, is superior for heterogeneous products. We should also remark that the failure of the identity test of the axiomatic approach to index numbers[14], which *de facto* arises with a geometric average price, is appropriate since the deflationary effects of international trade are driven by trade liberalisation and price level differences among countries rather than by changes in relative prices.

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Declarations

Conflicts of interest The authors declare that they have no conflicts of interest.

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Appendix

I_{dkt}: Domestic supply price index of apparel and accessories except knitwear from $t = 1997, \ldots, 2000$, producer price index of textiles and leather products from $t = 2000, \ldots, 2005$ and producer price index of wearing apparel for foreign markets from $t = 2005, \ldots, 2016$, measured in local currency (DKK), 2011=1. Source: Macrobond.

I_{set}: Export price index of textiles and wearing apparel from $t = 1997, \ldots, 2016$, measured in local currency (SEK), 2011=1. Source: Macrobond.

I_{ukt}: Producer price index of wearing apparel from $t = 1997, 1998$ and export price index of clothing and footwear from $t = 1998, \ldots, 2016$, measured in local currency (GBP), 2011=1. Source: Macrobond.

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[14] The identity (or constant prices) test states that a price index should equal unity, no matter what the quantities are, if the price of every good is identical in two consecutive periods; see, for example, ILO et al. (2004, p. 293).

I_{eat}: Producer price index of textiles, leather and wearing apparel from $t = 1997, \ldots, 2016$, measured in local currency (EUR), 2011=1. Source: Macrobond.

I_{trt}: Producer price index of textiles and wearing apparel from $t = 1997, \ldots, 2016$, measured in local currency (TRY), 2011=1. Source: Macrobond.

I_{cnt}: Producer price index of clothing from $t = 1997, \ldots, 2016$, measured in local currency (CNY), 2011=1. Source: Macrobond.

I_{hkt}: Consumer price index (total) from $t = 1997, \ldots, 2005$ and producer price index of wearing apparel from $t = 2005, \ldots, 2016$, measured in local currency (HKD), 2011=1. Source: Macrobond.

I_{vnt}: Consumer price index (total) from $t = 1997, \ldots, 2016$, measured in local currency (VND), 2011=1. Source: Macrobond.

I_{bat}: Consumer price index (total) from $t = 1997, \ldots, 2016$, measured in local currency (BDT), 2011=1. Source: Macrobond.

I_{int}: Consumer price index (total) from $t = 1997, \ldots, 2016$, measured in local currency (INR), 2011=1. Source: Macrobond.

S_{nt}: Value share of imports from country n in Norwegian clothing imports in period t, $n \equiv (se, dk, ea, uk, tr, cn, hk, ba, vn, in)$. Source: Foreign trade statistics, Statistics Norway.

$Bilateral\ exchange\ rates$: $\frac{USD}{DKK}, \frac{USD}{SEK}, \frac{USD}{GBP}, \frac{USD}{EUR}, \frac{USD}{TRY}, \frac{USD}{CNY}, \frac{USD}{HKD}, \frac{USD}{VND}, \frac{USD}{BDT}$ and $\frac{USD}{INR}$ are used to convert the prices of clothing measured in local currencies into USD. $\frac{NOK}{USD}$ is then used to convert the country specific prices in USD into NOK. Source: Macrobond.

$\frac{P_{n2011}}{P_{ea2011}}$: Relative clothing price levels among country n and the numeraire country ea, the euro area, adjusted for purchasing power parity in 2011, $n \equiv (se, dk, ea, uk, tr, cn, hk, ba, vn, in)$. Source: OECD (2011) and WB (2015).

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