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ADAPTIVE LIMIT ANALYSIS OF HISTORICAL MASONRY STRUCTURES MODELED AS NURBS SOLIDS

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Abstract. *In this work, we propose an adaptive upper bound limit analysis based on the representation of geometry through NURBS-solids. A NURBS-solid is a closed volume identified by boundary NURBS surfaces (Non-Uniform Rational Bezier Spline). Differently from using NURBS surfaces representing masonry shell-elements, NURBS-solids allow an accurate representation of masonry three-dimensional macro-blocks, such as vaults or walls with variable thickness. The initial model is subdivided into very few macro-elements, each one is still a NURBS solid and is considered as a rigid block. Since dissipation occurs only along interfaces, NURBS boundary surfaces represent possible fracture zones. An upper bound limit analysis is applied. The minimum kinematic multiplier is found by modifying the initial subdivision of solids until the real collapse mechanism is reproduced. This automatic research is performed through a Genetic Algorithm. A simple numerical example is finally reported.*

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1 INTRODUCTION

The field of historical masonry structures includes a wide range of different masonry construction typologies, such as vaulted structures, churches, towers, castles, monumental buildings, and aggregates.

The interest in the study of these structures through new advanced computational strategies became higher after recent seismic events, which underlined the strong vulnerabilities of historical masonry constructions to horizontal loads [1–3]. In the analysis of masonry structures, the peculiar characteristics of historical masonry structures have to be taken into account: the nonlinearities of masonry material (i.e. negligible tensile stress compared with good compressive behavior), typical presence of curved geometries, uncertainties on interlockings, coexistence of different and irregular masonry textures and presence of damage. Above all methods adopted [4], one of the most suited numerical tools is limit analysis. The limit analysis of masonry structures has been introduced for the first time by Heyman [5], who suggested to treat masonry as a no-tension material. In this theory, the structure is simplified as an assembly of rigid bodies with null tensile resistance and both crushing and sliding failures avoided. This allowed to analyze of the statics of curved masonry structures through simple procedures based on the static theorem of limit analysis [6,7], such as the construction of the thrust surface for the analysis of masonry arches. Among the methods proposed in last years, it is worth mentioning the so-called Thrust Network Method [8], which consists of a lower bound limit analysis of masonry vaults through the construction of the thrust surface, which is the three-dimensional extension of the thrust line.

As regards methods based on the kinematic theorem of limit analysis, procedures based on the finite element method have been proposed [9]. Here, rigid elements with nonlinear interfaces are often adopted. Nonlinearities are represented by assigning a three-dimensional failure domain which allows a refined representation of mechanical parameters of masonry. Moreover, recently some homogenization procedures have been presented for a reliable determination of the three-dimensional failure surface, opening the access to a wide field of applications [10–12].

However, analyses based on the kinematic theorem of limit analysis can lead to inaccurate results if the geometrical representation and the mechanical models are not able to reproduce the real collapse mechanism. In order to overcome such limitations, a new adaptive upper bound limit analysis has been proposed by the authors [13]. In this method, the geometry of a masonry vault is modeled through NURBS surfaces (Non-Uniform Rational Bezier Spline, [14]). The use of NURBS is particularly suited for curved masonry structures, because it allows an accurate representation of complex geometries [15]. The masonry vault is discretized through very few shell elements, each one is still a NURBS surface. An upper bound limit analysis is applied. Elements are idealized as curved rigid blocks and jump of velocities are allowed only along interfaces according to an associated flow rule. A properly defined homogenized failure domain is required. Therefore, interfaces assume the meaning of possible fracture lines. In order to find the real collapse mechanism, i.e. the real position of fracture lines, a mesh adaptation procedure is applied. By maintaining a low total number of elements, the mesh is adaptively modified until the minimum kinematic multiplier is reached. This optimization procedure is conducted through a Genetic Algorithm (GA) [16], even if other meta-heuristic approaches can be followed [17]. Different typologies of masonry structures have been studied through this GA-NURBS limit analysis [18–26].

Despite the great versatility of the method, it is still limited to constructions which can be described through plate or shell elements. However, there are many masonry structures which do not behave as assemblies of surfaces, such as arches and vaults with variable thickness, monumental triumphal arches, and complex buildings. Therefore, a new adaptive upper bound limit analysis based on the use of NURBS solids is presented. A NURBS solid is a closed region of space delimited by NURBS boundary surfaces.

The use of solids, which can be characterized by curved boundaries, allows the representation of curved objects without geometrical limitations. In this new method, a masonry structure is modeled in Rhinoceros by using NURBS solid-entities and then imported into MATLAB, in which an adaptive upper bound limit analysis is applied. A simple example on a masonry arch with variable thickness is presented to prove the effectiveness of the method.

2 NURBS SOLIDS: GEOMETRIC DESCRIPTION

In the geometrical description of three-dimensional objects, NURBS functions (Non-Uniform Rational Bezier Spline, [14]) are commonly adopted in the modern modeling software, such as Rhinoceros or SolidWorks. NURBS basis functions are built on B-splines basis functions, which are piecewise polynomial functions defined by a non-uniform knots vector (i.e. non-equidistant points in a parametric domain) $\Xi = \left\{\xi_1, \xi_2, ...\xi_{n+p+1}\right\}$, where p and n denote respectively the polynomial order and the total number of basis functions. Given a set of weights w_i and the i-th B-spline basis function $N_{i,p}$, then the NURBS basis function $R_{i,p}$ can be written as follows:

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi)}{\sum_{j=1}^{n} N_{j,p}(\xi)\,\mathrm{w}_j} \tag{1}$$

Starting by NURBS basis function, a NURBS surface $\mathbf{S}(u,v)$ of degree p in the u-direction and q in the v-direction is a parametric surface in the three-dimensional Euclidean space defined as follows:

$$\mathbf{S}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} R_{i,\mathrm{j}}(u,v)\mathbf{B}_{i,j} \tag{2}$$

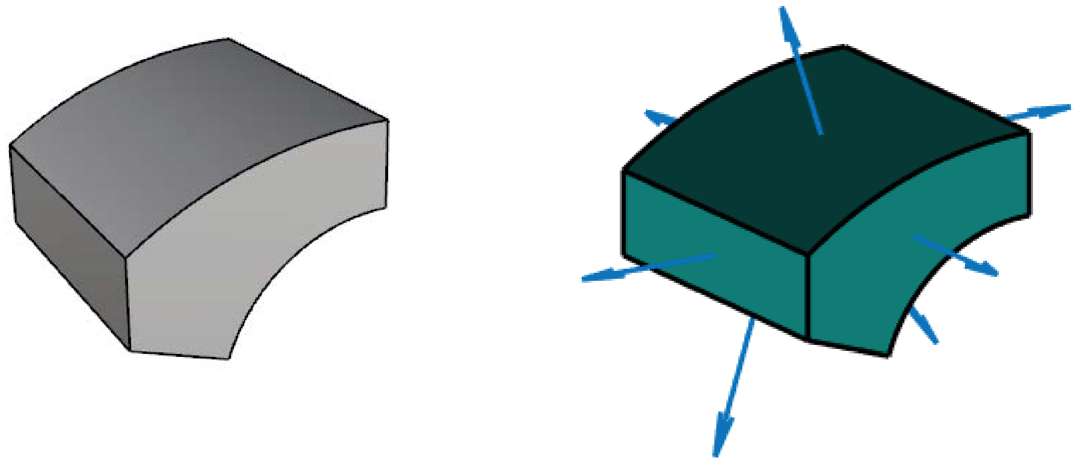
where $\mathbf{B}_{i,j}$ are the control points.

Consider now a limited region of space $D \subset \mathbb{R}^3$ whose frontier ∂D is a piecewise regular and oriented surface. A NURBS solid is defined as a closed region of space D in which the boundary ∂D is an assembly of NURBS surfaces and no information is provided for the space internal to the boundary. In other words, a NURBS solid is exhaustively described by its frontier ∂D as follows:

$$D = \partial D = \bigcup_{k=1}^{M} \mathbf{S}_k\left(u,v\right) \tag{3}$$

where M is the total number of boundary NURBS surfaces, and each boundary surface is identified by the classical NURBS description (see Equation 2).

Geometrical models composed of NURBS surfaces can be easily imported in MATLAB from Rhinoceros through the IGES standard file [27] and the available functions within the IGES toolbox [28]. However, this is not the case of NURBS solids, which are not supported by the IGES standard file. Therefore, a new IGES format, called "IGESsolid" and a new function "iges2matlab" [29] has been written specifically to import solids in the MATLAB environment. An example of NURBS solid in Rhinoceros and in MATLAB, representing a portion of a masonry arch with variable thickness, is reported in Figure 1.



(a) (b)

Figure 1. NURBS solid in (a) Rhinoceros and (b) MATLAB**.**

A NURBS solid can be subdivided into one or more solids directly within MATLAB. The subdivision of solids is a fundamental step for the adaptive limit analysis procedure which will be described in the next Section. This operation is conducted through a new algorithm written by the authors specifically for the subdivision of solids in MATLAB. Without going in detail about this algorithm, the subdivision is performed by defining an additional NURBS surface, which represents the "splitting object". For each boundary surface, a surface-surface intersection problem (SSI) is solved. Once the common curve between the boundary surface and the splitting surface is determined, the boundary comes split into two (or more) boundaries, in which each one is a partition of the original boundary. After a procedure of re-assembling of the new sub-boundaries, two new NURBS solids are obtained (see Figure 2).
Therefore, the core of this subdivision procedure is the SSI algorithm. The SSI problem has been widely investigated in the scientific literature [30], the procedure here implemented has been developed by improving the formulation presented in [31].

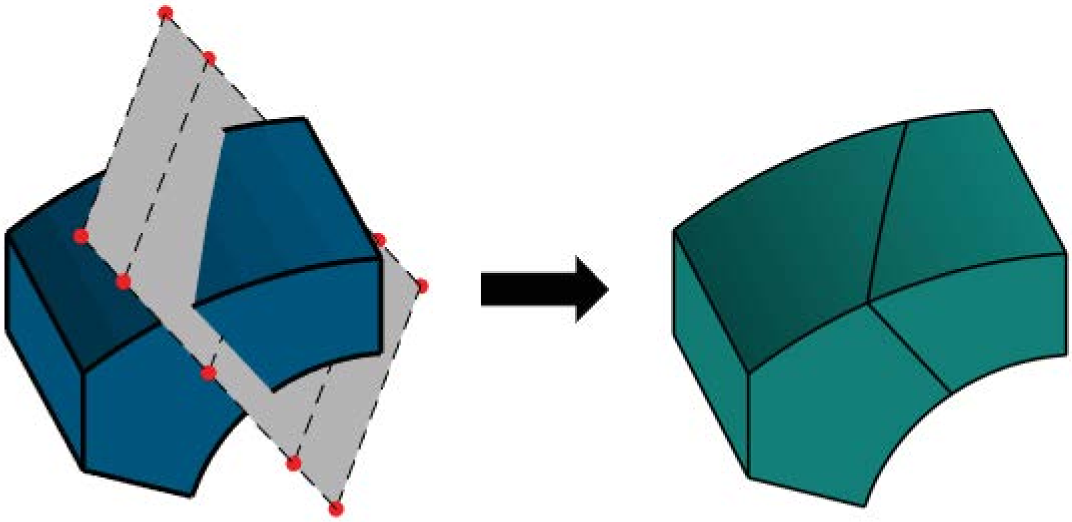


Figure 2. NURBS solid subdivided through a planar NURBS surface.

Once the subdivision of solids is completed, the volume V and the centroid $\mathbf{G}$ of each solid can be evaluated. As it is well known, these properties are determined as follows:

$$V=\iiint_D dxdydz \tag{4}$$

$$\mathbf{G}=\frac{\iiint_D \nabla\cdot\mathbf{F}(x,y,z)\,dxdydz}{V}\text{, such that } \nabla\cdot\mathbf{F}(x,y,z)=x\mathbf{i}+y\mathbf{j}+z\mathbf{k} \tag{5}$$

These volume integrals can be easily converted in surface integrals by applying the divergence theorem (also known as the Gauss's theorem), for which:

$$\iiint_D \nabla\cdot\mathbf{F}(x,y,z)\,dxdydz=\iint_{\partial D}\mathbf{F}(x,y,z)\cdot\mathbf{n}_e dS \tag{6}$$

where $\mathbf{n}_e$ is the normal external unit vector for ∂D. For NURBS surfaces, the normal unit vector is so defined:

$$\mathbf{n}_e=\frac{\mathbf{S}_u(u,v)\times\mathbf{S}_v(u,v)}{\left\|\mathbf{S}_u(u,v)\times\mathbf{S}_v(u,v)\right\|} \tag{7}$$

where $\mathbf{S}_u(u,v)$ and $\mathbf{S}_v(u,v)$ denote respectively the two partial derivative vectors. Provided that in a NURBS solid the frontier ∂D is a composition of NURBS surfaces (Equation 1), Equation 6 can be re-written as follows:

$$\iiint_D \nabla\cdot\mathbf{F}(x,y,z)dxdydz=\sum_{k=1}^{M}\iint_{Q_k}\mathbf{F}\left(x(u,v),y(u,v),z(u,v)\right)\cdot\left(\mathbf{S}_{k,u}(u,v)\times\mathbf{S}_{k,v}(u,v)\right)dudv \tag{8}$$

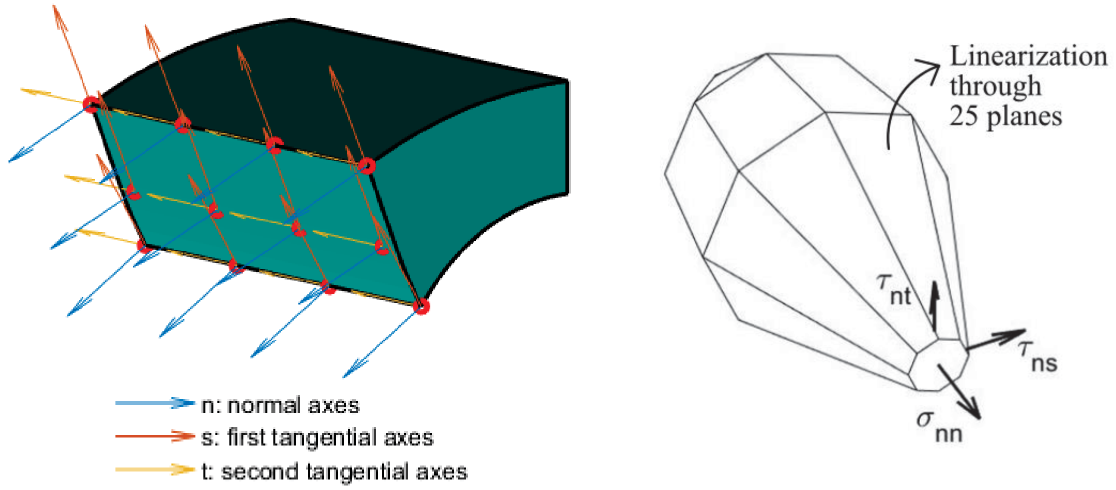
in which Q_k is the standard parametric domain associated to the *k*-th surface. This integral can be solved in MATLAB through the Gauss integration. Equation 8 is applied to solve all volume integrals on NURBS solids. Therefore, it is adopted in the evaluation of volumes, weights, centroids, and in the application of volume forces.

3 ADAPTIVE KINEMATIC LIMIT ANALYSIS

A masonry structure is here represented as an assembly of few rigid blocks, in which each block is a NURBS solid. A procedure of kinematic limit analysis is defined. The kinematic of each block is defined through the six degrees of freedom $\left\{u_x^i,u_y^i,u_z^i,\Phi_x^i,\Phi_y^i,\Phi_z^i\right\}$ of its centroid in the three-dimensional space $Oxyz$. Interfaces between elements, which are the NURBS surfaces of the common boundaries, are the only zones where jumps of velocities are supposed to occur. Therefore, dissipation is allowed only at interface according to a rigid-plastic behavior. A three-dimensional failure surface, which consists of a Mohr-Coulomb failure surface with tension cut-off and linear cap in compression [32], is assigned to the masonry material. In this way, crushing and sliding failures are automatically included in the computation. It is clear that, by using high values of ultimate compression strength and friction angle, the typical Heyman condition [5] can be reproduced. The internal plastic dissipation at the generic interface is evaluated according to the associated flow rule:

$$\Delta\tilde{\mathbf{u}}=\left(\dot{\boldsymbol{\lambda}}^T\frac{\partial f}{\partial\boldsymbol{\sigma}}\right)^T \tag{9}$$

where $\Delta\tilde{\mathbf{u}}$ is the jump of velocity in the local reference system, $\dot{\boldsymbol{\lambda}}$ is the vector containing the non-negative plastic multiplier, f is the three-dimensional failure domain, and $\boldsymbol{\sigma}$ is the stress vector. The associated flow rule is applied by discretizing the interface into a series of points. Since the use of NURBS allows the presence of curved interfaces, the discretization is conducted by defining a triangularization of the NURBS surface (the procedure described in [33] is used). In Figure 3(a) and example of discretization with local reference systems is showed, whereas Figure 3(b) reproduces a linearization of the adopted three-dimensional failure surface.



(a) (b)

Figure 3. (a) Discretization of the curved interface into points and local reference systems, (b) linearized three-dimensional failure surface.

The total internal dissipation is thus determined as follows:

$$P_{\text{int}} = \sum_i \int_{S_i} \left(\boldsymbol{\sigma} \cdot \Delta \tilde{\mathbf{u}} \right) dS = \sum_i \int_{S_i} \dot{\boldsymbol{\lambda}}^T \cdot \left(\boldsymbol{\sigma}^T \frac{\partial f}{\partial \boldsymbol{\sigma}} \right)^T dS \tag{10}$$

A configuration of loads, distinguished in dead loads (permanent) $\mathbf{F}_0$ and live loads (depending on a multiplier λ) $\lambda \cdot \boldsymbol{\Gamma}_0$, is defined. If the vector of velocities is indicated as $\mathbf{U}$, the power dissipated by the external loads is defined as:

$$P_{ext} = \left(\mathbf{F}_0 + \lambda \cdot \boldsymbol{\Gamma}_0 \right)^T \mathbf{U} \tag{11}$$

The solution of the kinematic problem is obtained by applying the Principle of Virtual Powers. By normalizing the power dissipated by live loads, i.e.:

$$\boldsymbol{\Gamma}_0^{\ T} \mathbf{U} = 1 \tag{12}$$

the problem can be written with the following linear programming formulation:

$$\min \left\{ \lambda = P_{\text{int}} - \mathbf{F}_0^T \mathbf{U} \right\} \text{ such that } \begin{cases} \mathbf{A}\mathbf{x} = \mathbf{b} \\ \dot{\boldsymbol{\lambda}} \geq 0 \end{cases} \tag{13}$$

where $\mathbf{x} = \left[\mathbf{U}, \dot{\boldsymbol{\lambda}} \right]$ is the vector of unknowns, $\mathbf{A}$ and $\mathbf{b}$ are respectively the overall equality constraints matrix (which include geometric constraints, compatibility constraints and normality condition) and the corresponding right hand side vector.

The presented upper bound formulation provides the configuration of velocities of each block at failure, i.e. a kinematic mechanism, and an associated load multiplier, which is an upper bound of the collapse multiplier. In order to optimize the kinematic multiplier, the real collapse mechanism has to be found.

The mechanism depends on how solids have been initially subdivided. Therefore, changing the position and/or the shape of the NURBS surfaces adopted as splitting objects, different mechanisms can be found. A genetic algorithm (GA) [16] is applied to move these NURBS surfaces and find the subdivision associated to the minimum kinematic multiplier. Given the lower number of parameters involved in the meta-heuristic optimization, the optimized mechanism is usually found after very few iterations.

4 NUMERICAL EXAMPLE

The first result obtained through the novel NURBS solids-based formulation is here presented. A masonry arch characterized by variable thickness is here analyzed under a simple pointed load. Geometry and load condition are depicted in Figure 4(a). This type of arch has been originally presented by Lamé and Clapeyron [34] and successively studied in [35]. These arches are constructed with a layer of radial voussoirs disposed at the intrados and additional horizontal layers of masonry above. Therefore, the shape of the arch is different between intrados and extrados, resulting in a variable thickness. In presence of simple vertical load, Lamé and Clapeyron observed that this kind of arches fails by developing fracture lines that start radially at the intrados (i.e. following the radial joints given by voussoirs), and then propagate upward becoming almost vertical. A representative scheme of this crack pattern is shown in Figure 4(b). This observation has been confirmed in [35].

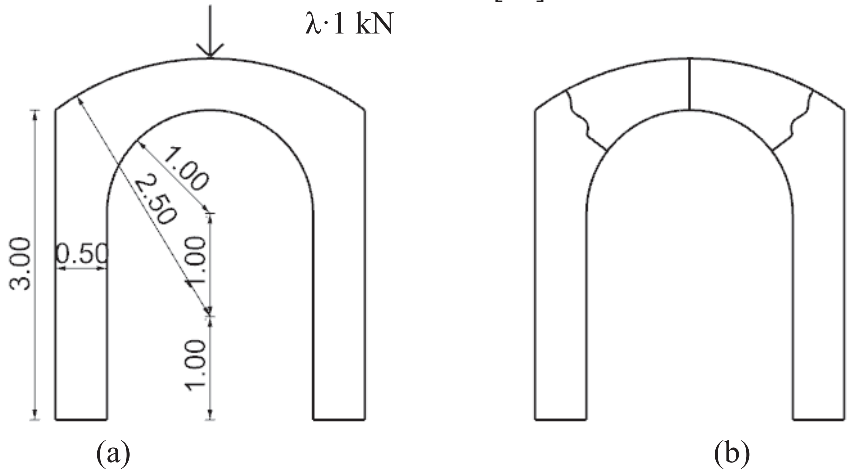


Figure 4. Arch with radial voussoirs at the intrados and a horizontal layer of masonry above: (a) geometry and load condition analyzed, and (b) typical crack pattern under vertical loads [34,35].

As a preliminary evaluation, the masonry arch has been analyzed as 2D problem (width has been assumed unitary) through a heterogeneous kinematic limit analysis. Two different subdivisions into rigid blocks have been adopted. In the first one, the arch is subdivided through radial joints starting from the intrados, whereas horizontal joints have been adopted for piers, see Figure 5(a). In the second one, two different mesh have been adopted for the intrados and the extrados, in order to better take into account the two different layers of voussoirs. Whereas radial joints have been maintained at the intrados, triangular blocks have been assigned to the masonry material located above the radial voussoirs, see Figure 5(b). A value of 18 kN/m3 has been adopted as specific weight. A null tensile strength has been assigned, cohesion and friction angle are respectively equal to 0.02 MPa and 30°, and finally the compression strength has been assumed equal to 2.6 MPa. Results obtained for both the mesh are shown in Figure 5(c,d). It can be noted that a lower load multiplier has been found for the second mesh, in which the crack pattern observed in [34,35] can be better reproduced.

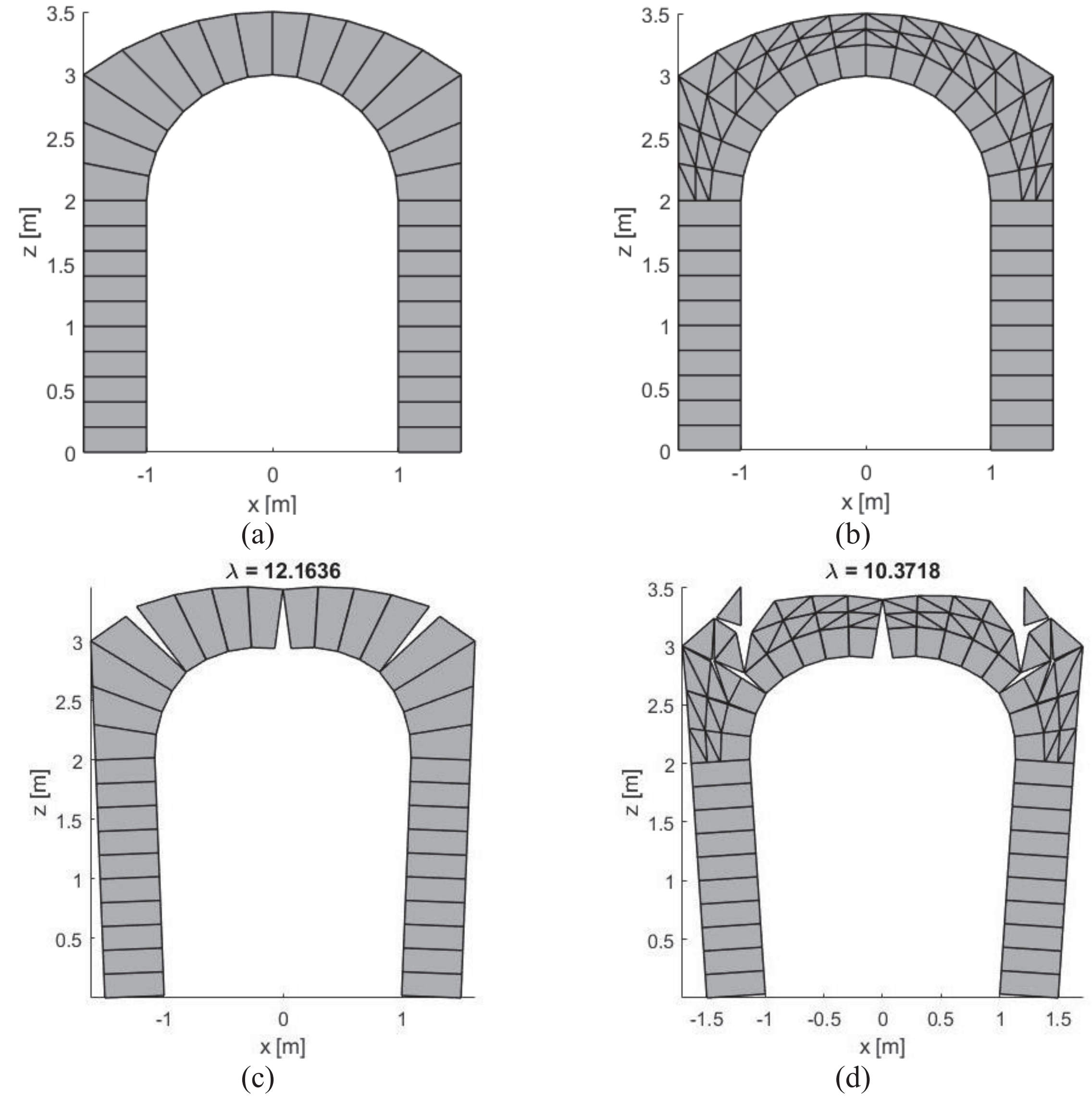
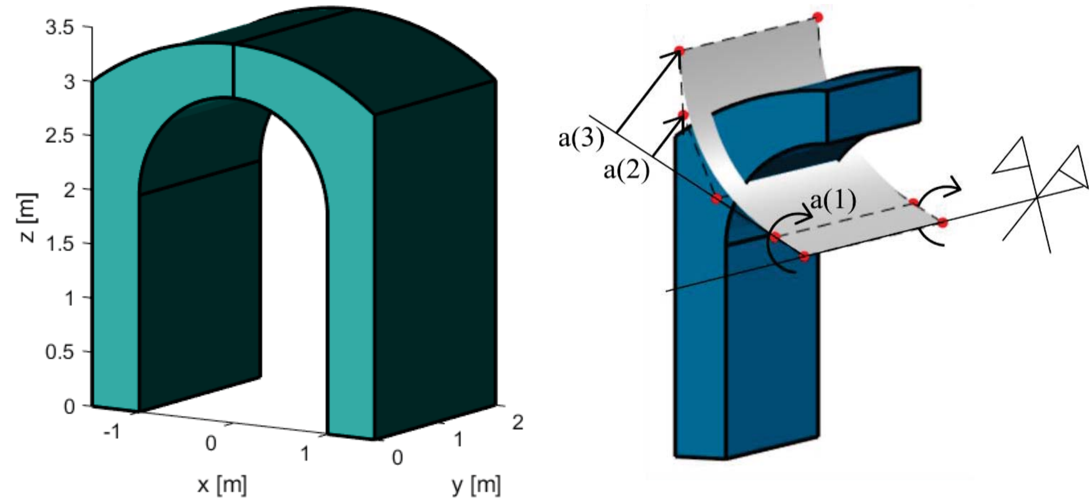


Figure 5. Limit analysis of the masonry arch with heterogeneous approach: (a,b) two different initial mesh, and (c,d) corresponding collapse mechanism and load multiplier.

The study through adaptive limit analysis through NURBS solids is now presented. A representation through two NURBS solids is reported in Figure 6(a). A width equal to 2 m has been assigned. In order to correctly compare this example to the previous ones, the live load condition here used consists of a linear load equals to 1 kN/m distributed along the extrados line belonging to the symmetry plane of the arch. According to the symmetry of the problem, the subdivision of the masonry arch is governed by two planar NURBS surfaces that move symmetrically. The shape of these surfaces is governed by three parameters, summarized by the vector $\mathbf{a}$ and depicted in Figure 6(b). It can be observed that, if the second or the third parameter are not null, the surface is not more planar and the resulting fracture surface on the solid is curved. The same material parameters have been adopted for this model.



(a) (b)

Figure 6. (a) NURBS model of the arch, and (b) NURBS surfaces adopted for the subdivision.

Results are shown in Figure 7. Figure 7(a, b) depicts the collapse mechanism found and the corresponding live load multiplier, whereas in Figure 7(c) the convergence diagram deriving from the application of the GA is reported. As it was expected, the arch collapses for the formation of curved cracks that are almost vertical near the extrados. The load multiplier resulted lower than values obtained through the heterogeneous approaches (9.37 compared with 10.37 and 12.16), denoting a higher precision in the definition of the collapse mechanism for the adaptive approach.

This method is computationally cheap because it allows to study complex geometries by using a very reduced number of elements, still maintaining the exact geometry of the case study.

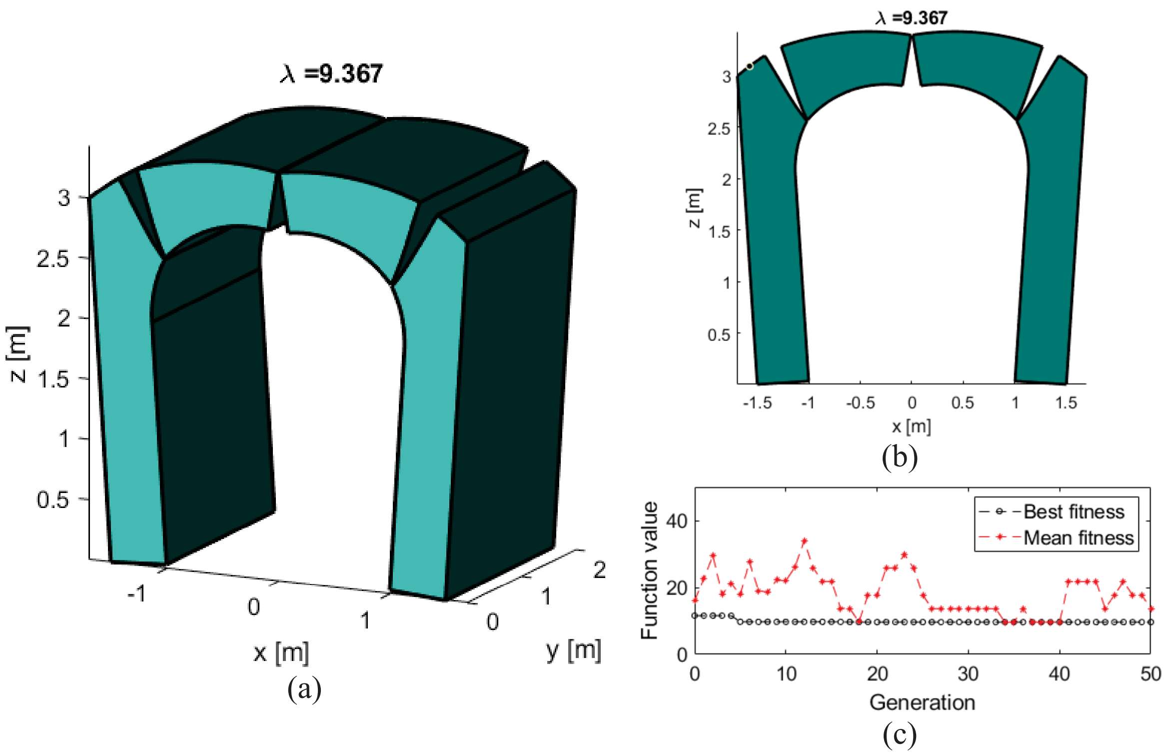


Figure 7. Results obtained through adaptive limit analysis: load multiplier and collapse mechanism in (a) axonometric and (b) frontal view, and (c) convergence diagram.

5 CONCLUSIONS

A new adaptive upper bound limit analysis based on the use of NURBS solids has been presented. A given masonry structure is modeled through NURBS solids and then imported into MATLAB. The model is then subdivided through NURBS surface and analyzed through an adaptive upper bound limit analysis is applied. By using a Genetic Algorithm, the initial subdivision is then modified to reproduce the real collapse mechanism and identify the collapse multiplier.

The presented method has been applied on a masonry arch with variable thickness and results have been compared with classical heterogeneous limit analysis procedures. A good agreement has been observed between results obtained. The use of NURBS solids allowed the exact representation of curved geometries of structures that cannot be represented with standard shell elements. The collapse mechanism is identified with good precision thanks to the automatic mesh adaptation. Moreover, the subdivision of solids through NURBS surfaces allows the identification of curved fracture zones within the macroblocks in easy way.

Future research will be focused on the use of the presented method for the analysis of monumental masonry constructions subjected to horizontal actions.

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