




A Lesson Study with Mathematics Teachers: Learning about Communication in the Classroom¹

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ABSTRACT

Background: Orchestrating classroom communication is one of the challenges for teachers in exploratory lessons. **Objectives:** We aim to understand how the participation of secondary school teachers in a lesson study based on an exploratory approach promotes the development of their knowledge about classroom communication. **Design:** The research is qualitative and interpretative. **Setting and Participants:** The participants are three secondary school teachers who were willing to participate. **Data collection and analysis:** Data were collected through participant observation (with research journal and audio/videorecording), document collection, and a focus group interview. **Results:** During the students' autonomous work, although with different actions, *closed progress details* and *open progress initiatives*, the teachers listened to small-group discussions and used them as a starting point to support the students, asking them to explain their answers without pointing out the strategy to follow, according to what they discussed in the planning sessions. During the whole-class discussion, the teachers promoted the comparison of different representations and *invited* the students to *explain* and justify their answers, as they had planned, but also considered answers that they did not anticipate. After observing the research lesson, the teachers reflected about the students' explanations, valuing their clarity, and referring to them as an opportunity for the teacher to understand how the students were thinking. **Conclusions:** The results suggest that lesson study is a professional development process in which secondary school teachers can deepen their knowledge about classroom communication.

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Um Estudo de Aula com Professoras de Matemática: Aprendizagens Sobre a Comunicação em Sala de Aula

RESUMO

Contexto: A gestão da comunicação em sala de aula é um dos desafios dos professores em aulas exploratórias. **Objetivos:** Este artigo tem como objetivo compreender como a participação de professoras do ensino secundário num estudo de aula com base na abordagem exploratória promove o desenvolvimento do seu conhecimento sobre a comunicação em sala de aula. **Design:** Investigação qualitativa e interpretativa. **Ambiente e participantes:** As participantes são três professoras que lecionam o ensino secundário e que se disponibilizaram para participar. **Coleta e análise de dados:** Os dados foram coletados por observação participante (com diário de bordo e gravações áudio/vídeo), recolha documental e entrevista em grupo focal. **Resultados:** Durante o trabalho autónomo, embora com ações diferentes, *perguntar apontando a direção* e *perguntar sem apontar a direção*, as professoras ouviram as discussões entre os alunos e partiram delas para os apoiar, pedindo-lhes explicações, sem lhes indicar a estratégia a seguir, tal como discutiram nas sessões de planeamento. Durante a discussão coletiva, as professoras promoveram o confronto de diferentes representações e *convidaram* os alunos a *explicar* e justificar as suas respostas, como planearam, mas consideraram também respostas não antecipadas. Depois de observarem a aula, as professoras refletiram sobre as explicações dos alunos, valorizando a sua clareza e referindo que são uma oportunidade para o professor perceber a forma como eles pensaram. **Conclusões:** Os resultados sugerem que o estudo de aula é um processo de desenvolvimento profissional em que professores do ensino secundário têm oportunidade de aprofundarem o seu conhecimento sobre a comunicação em aula.

Palavras-chave: Abordagem exploratória; comunicação; conhecimento didático; estudo de aula; ensino secundário.

INTRODUCTION

Classroom communication has a major impact on students' learning opportunities (Franke et al., 2007). In an exploratory teaching-learning lesson, the moments of discussion with students, based on their work on the task, are crucial to their learning (Ponte, 2005). This teaching strategy "has shown potential for guiding students towards a better learning of Mathematics" (Quaresma & Ponte, 2017, p. 45), however its accomplishment is a challenge for teachers (Guerreiro et al., 2016; Quaresma & Ponte, 2016, 2017). More specifically, it is necessary to deepen the type of interventions that the teacher

can make during a mathematics lesson to support students' work in small groups (Hofmann & Mercer, 2016; Webb et al., 2009) and in whole-class discussions (Ponte & Quaresma, 2016; Stein, Engle, Smith, & Hughes, 2008).

In lesson study, a collaborative professional development process, teachers can plan and teach exploratory lessons to rethink their practices, their role and the role of the students in the classroom (Quaresma & Ponte, 2016), and to reflect on students' learning through observation of their work in the classroom. The participation of teachers in lesson study is an opportunity for them to develop their knowledge, namely regarding tasks (Barber, 2018; Verhoef et al., 2015) and classroom communication (Barber, 2018; Ni Shuilleabhain & Seery, 2017; Quaresma & Ponte, 2016). However, most of the research conducted with in-service teachers has involved elementary school teachers working with students with less mathematical knowledge than secondary school students. Secondary school teachers generally propose more complex tasks, the solution of which often involves a greater number of steps than those proposed for elementary school students. The way these teachers manage classroom communication based on the diversity of their students' answers is a question that has been little investigated and calls for further analysis (Kooloos et al., 2020; van Leeuwen & Janssen, 2019).

Several authors have identified and analysed teachers' actions in leading whole-class discussions (e.g., Ponte, Mata-Pereira, & Quaresma, 2013), while others have analysed teachers' practices during students' autonomous work (e.g., Hofmann & Mercer, 2016). To understand how the participation of teachers in a lesson study promotes the development of their knowledge about communication in the classroom, this paper analyses a lesson study with three teachers, and the interactions between two of them and their students (16-17 years of age) while working on a task and in the subsequent whole-class discussion, in a research lesson.

TASKS AND COMMUNICATION

The selection and design of tasks and the type of communication established in the classroom influence what students will learn and how they will learn it (Franke et al., 2007; Ponte & Quaresma, 2016). They are part of the knowledge of teaching practice, one of the four dimensions of the teachers' didactic knowledge model proposed by Ponte (2012). In his model, the author also takes the teacher's knowledge of mathematics for teaching and his/her knowledge and management of the curriculum into consideration. The students'

knowledge and their learning are another feature, where the author includes knowledge of the strategies that students can follow, of the learning processes and the students' most common difficulties.

The student's role during the lesson and the teacher's role and his/her proposed tasks are related to the underlying teaching approach. From the perspective of exploratory teaching-learning (Ponte, 2005), the teacher should select challenging tasks (Quaresma & Ponte, 2016; Stein et al., 2008; Stein & Smith, 2009), for which the students do not have an immediate solution and can start from their prior knowledge to begin their work. Students may solve these tasks using different mathematical representations and following various strategies "that may be compared and evaluated, resulting in interesting mathematical discussions in the classroom" (Quaresma & Ponte, 2016, p. 299). Hence, the teacher tends to assume the role of a coordinator of the learning processes, encouraging students to explain their ideas and to question the ideas of others, thereby having an important role in the management of discussions during students' autonomous work and whole-class discussions (Kooloos et al., 2020).

Several studies claim that students have more opportunities to learn if the teacher supports them during autonomous work (Kooloos et al., 2020; van Leeuwen & Janssen, 2019; Webb et al., 2009), without evaluating their answers (Hofmann & Mercer, 2016). However, when the teacher proposes tasks that can be solved using different representations, a variety of answers may arise, making it difficult for him/her to support the students. Therefore, when planning the lesson, besides anticipating possible student difficulties, the teacher should think about questions or comments that may be advanced to support the students in solving the task, both to help those with more difficulties and to encourage those who manage to complete the original task to make extensions (Vale et al., 2019).

Upon identification of students' correct and incorrect solutions that meet the objectives of the lesson, the teacher should then sequence them for the whole-class discussion (Stein et al., 2008) and conduct the discussion with the students, seeking to promote their mathematical learning (Kooloos et al., 2020; Ponte, 2005). The purpose of the discussion is to establish connections between the students' answers (Stein et al., 2008), focusing on exploring incomplete or inaccurate answers and introducing the representations that did not arise in their autonomous work.

To analyse classroom communication with secondary school students, Kooloos et al. (2020) propose an analytical framework for teacher and student

actions in lessons involving tasks that can be solved through different strategies. The authors present twelve teacher actions organised into four categories which, in turn, are subdivided into subcategories (Table 1): (i) divergent actions, in which the teacher gives students the opportunity to participate and conducts the discussion based on their ideas; (ii) convergent actions, in which the teacher takes control of the ideas discussed; (iii) encouraging actions for students to proceed with their explanations and (iv) regulating actions, related to communication management. The authors also propose eight student actions, whenever they interact with the teacher, on the strategies they are using to solve the proposed task (Table 2).

Table 1

Teacher Actions Table. (Kooloos et al., 2020, Table 1)

Divergent actions	Convergent actions	Encouraging actions	Regulating actions
Request explanation	Demonstration	Confirmation	Rules of classroom discourse
Request clarification	Reformulate	Encouragement	
Open progress initiatives	Closed progress details		
External general	Set aside		
External directed			

Table 2

Student Actions Table. (Kooloos et al., 2020, Table 2)

(Steps of) Solution Method	Evade answer
Explanation	Remark about solution method
Partial answer	External
Teacher-led response	Question

LESSON STUDY

Originating in Japan, lesson study (*jugyoukenkyuu*) is a formative process of a collaborative nature disseminated in the West by the book *The Teaching Gap*, by Stigler and Hiebert (1999). In a lesson study, a group of

teachers carefully plan a lesson, a research lesson, to help their students overcome a learning problem they have identified. In the planning sessions, the teachers try to anticipate the strategies and representations the students may use, the difficulties they may have, and the mistakes they may make, which also allows them to think about how they can intervene to support the students while working on the task (Vale et al., 2019). The research lesson is taught by one of the teachers, while the others observe and focus on the students' work. After the lesson, the teachers have another meeting to share and reflect on their observations of the students' work and try to draw conclusions about the students' learning.

Several studies refer to lesson study as an opportunity for teachers to develop their knowledge on the design and planning of tasks (e.g., Barber, 2018; Verhoef et al., 2015) and on classroom communication (e.g., Barber, 2018; Ni Shuilleabhain & Seery, 2017). Barber (2018) mentions that before participating in the lesson study, one teacher used to explain the answer to the students when they did not give her the answers she expected. However, throughout the lesson study, the teacher acknowledged the importance of questioning the students and focused on questions she could ask to help them explain and justify their answers. Ni Shuilleabhain and Seery (2017) state that after participating in four cycles of lesson studies in the same academic year, the teachers rethought their practices and began to include group work, consisting of small groups, in other lessons, promoting the autonomous work of the students and inter-group discussion. Regarding communication management, these teachers encouraged the participation of the students and the presentation of their work to their peers beyond the context of the research lesson, thus changing their teaching practices and adopting the role of learning facilitators, encouraging the participation of the students and changing the tasks accordingly.

METHODOLOGY

This research follows a qualitative and interpretative approach (Bogdan & Biklen, 1994), since its aim is to understand the extent to which three secondary school teachers developed their knowledge of communication in the classroom, based on their work in a lesson study held at their school. The teachers, Sofia, Branca, and Luz (pseudonyms), with over 25 years of teaching practice, agreed to participate in the lesson study. The first author of the article, who facilitated the lesson study and assumed the role of participant, is a teacher at the same school and had previously worked with these teachers, including in

other training processes. This research was approved by the Ethical Committee of the Instituto de Educação da Universidade de Lisboa. An informed consent for the data collection was requested from participants

The lesson study was held between November 2019 and February 2020 and consisted of 19 sessions with variable duration (between 20 and 125 minutes). During this particular academic year, the three teachers were working with 11th-grade students (16-17 years of age, also presented herein under pseudonyms) who had almost all been in the same class since the 10th grade and were already used to working in groups, usually with two to five members that they themselves formed.

At the beginning of the lesson study, the teachers decided to give attention to communication in the classroom, namely its management, and encourage students to communicate their ideas, as this was a widely discussed issue in their curriculum group. They also decided that the topic for the research lesson would be to “study the sign of rational functions, given by expressions of the form, $\frac{P(x)}{Q(x)}$, where P and Q are polynomials”, based on their experience and the difficulties students tend to have in the “Real functions of a real variable” domain, a cross-cutting theme in the curriculum since the 7th grade.

In line with the exploratory approach, the teachers planned the lesson considering an introduction to the task, students’ autonomous work, whole-class discussion and a synthesis. The research lesson lasted 90 minutes and was taught by two teachers, Branca and Sofia, in two classes (RL1.1 and RL1.2). After each lesson, they reflected on the students’ work, the strategies had used and how they had explained their answers. Following on from this work, the teachers decided to plan a new research lesson that was taught by the three teachers in their classes.

The data collection included documentary material, videorecordings of the research lessons and audio recordings of the planning sessions and the semi-structured interview, in a focus group, at the end of the lesson study. For this paper, excerpts of the conversations between the teachers in the planning sessions of the first research lesson, in the post-lesson reflection (Reflection 1 and Reflection 2) and in the interview were selected, where features of communication in the classroom were discussed. An illustrative moment of the teacher and students’ interventions and actions in each of the research lessons (RL1.1 and RL1.2) was also selected, during autonomous work and the whole-class discussion. The data analysis was performed with the support of the NVivo software, considering the analytical framework for communication in

the classroom proposed by Kooloos et al. (2020) and the framework proposed by Ponte (2012) to analyse the development of the teachers' knowledge.

RESULTS

Planning sessions

Task. This paper refers to the first part of a task (Figure 1) designed by the teachers to introduce the topic on “the study of rational functions given by expressions of the form $\frac{P(x)}{Q(x)}$, where P and Q are polynomials”, and was planned for the first 45 minutes of the research lesson. The second part had four more conditions of the type $\frac{P(x)}{Q(x)}$, with varying degrees of P and/or Q .

Figure 1

Formulation of the first part of the task.

Solve, in \mathbb{R} , the condition $\frac{x+3}{x-2} > 0$

While they were designing the task, Branca suggested asking the students to present the graphical representation alongside the algebraic representation to enhance the comparison of the two, as had been done previously in another proposed task: “Could we give it the same form, or not?” (Session 5). This suggestion led the teachers to discuss the wording of the task:

Paula: But do you think we should give this indication (...) or should we let them do it... and will we use different solutions in the discussion phase?

Branca: I think their first question will be: “Are we supposed to do this by calculation, or should we use a calculator?”

Sofia: As they have already done another [task] more guided ... they can do as they please...

Branca: They are going to ask... And we should say... “choose yourselves” (...) Ok. Let us see what they do (...)

Sofia: And in the comparison too, right? And the comparison is interesting... if some [students] do it analytically and others graphically, trying to understand why they do not coincide... or why they do... (Session 5)

After a period of discussion, the teachers decided not to guide the students in their work, contrary to what they had previously done. Letting the students decide, they were not limiting the representations they could use and the strategies they could follow, thus enriching the whole-class discussion.

Autonomous work. Although Sofia regularly monitors her students' work while they approach the tasks she proposes, she posed a question on the role of the teacher during autonomous work in an exploratory lesson: "In autonomous work, are we supposed to monitor the work and give them clues, or not? (...) give them some guidance ..." (Sofia, Session 5).

Paula then suggested returning to a text they had already discussed (Guerreiro et al., 2016), in which the authors refer to the teacher's role in an exploratory lesson, supporting students while they work on the task. The teachers then discussed the type of intervention they could make during autonomous work to support the students, without lowering the task's level of difficulty:

Branca: Are we going to give those clues... you know, open, right?

Sofia: OK... give some guidance without "killing" the task, right? (Session 5)

The teachers then thought about comments and questions they could include in the lesson plan based on their anticipation of students' answers and difficulties. For example, to support the students who use algebraic representation and only consider that the numerator and denominator must both be positive, Sofia suggested the following: "Here we can say (...) 'what if you replaced the x with -4 , making $\frac{-1}{-6}$. Is that a solution or not?'" (Session 5)

With this type of intervention, the students would be able to realise that the case in which the numerator and denominator are both negative should also be considered. Another possibility would be to suggest a comparison between the algebraic and graphical representations: "Going in that direction, they [the students] should get it: 'Hold on. No, we could not do it that way. Some of the solutions are missing here'" (Sofia, Session 5).

Thus, the discussion on the role of the teacher while accompanying the students during the task, based on an intervention by Sofia, contributed to deepening the teachers' knowledge on conducting exploratory tasks. In particular, Sofia suggested questions that could be posed to the students to support them during their autonomous work, without lowering the task's level of difficulty and encouraging them to explain their solutions and to compare the various representations.

Whole-class discussion. Following on from this work, the teachers discussed the selection of answers for the whole-class discussion.

But listen, are we also going to present the incorrect solutions? That idea of drawing attention to the error? (Sofia, session 5).

Sofia's intervention was the starting point for the teachers to discuss whether they should select answers with errors for the whole-class discussion phase. The teachers considered it important that students were able to correct their answers while working on the task, but that during the whole-class discussion, they should talk about the mistakes they had made, even if they had corrected them:

Branca: [The students should] Explain how they thought. (...) How they overcame the error...

Sofia: Exactly. And how they overcame this difficulty. It is not just about presenting the final product but the whole process. "We started out by thinking this, but ..." (...) It's not: "it's wrong, don't show your peers". (...)

Branca: Shown us so we can understand what is wrong. (Session 5)

These teachers' discussions allowed them to consider the selection and sequencing of answers for the whole-class discussion from a different angle. They decided to include incomplete or incorrect solutions in the discussion, asking the students to explain how they had overcome their difficulties, which could also help other students not to make the same mistakes or to complete their solutions.

Regarding the selection of students to explain their answers, Branca asked her colleagues what they should say to the students during the introduction of the task so that they would be prepared to go to the board to present their solutions:

We are supposed to choose one of each of the situations here, but we will ask them [the students] to present it, right? We should probably say this in the presentation. So that they also know what our aim is, don't you think? (...) then it is not the teacher who will correct and they are just going to confirm ... no! (Branca, Session 5)

Branca's observation and her evident need to inform the students that they would be presenting their solutions appear to suggest lessons in which the students first solve the tasks proposed by the teacher, which she then proceeds to correct. Branca also asked if a member from each group should go to the board, to which Sofia replied that "We can only say that afterwards, in the end, some students... you are no longer saying one from each group (...) Some students will be called to the board to present their solutions. (...) And not all the groups have to go", to which Luz added, "We must have different solutions there".

By means of this discussion, Branca was able to rethink the process following students' autonomous work, when the teacher selects some of them to explain and justify their solutions to the class, with "different solutions", as mentioned by Luz, thus deepening their knowledge on conducting exploratory tasks, namely on conducting whole-class discussions.

Research lessons

Two episodes are presented for each of the classes, RL1.1 and RL1.2, conducted by Branca and Sofia, respectively, in which the teacher and students' actions are analysed. In the first episode of each of these classes (Episodes 1 and 3), a conversation between the teacher and a group of students during autonomous work is presented. Then, one of the students in this group presents his work, and this moment of whole-class discussion is analysed (Episodes 2 and 4).

Episode 1 – Autonomous work in RL1.1

A group of students were trying to solve the task by drawing analogies with the solution of equations of the type $\frac{A(x)}{B(x)} = 0$, where A and B are polynomials, on which they had worked in the two previous lessons. While

monitoring the students' work, Branca observed what the group was doing and tried to help them:

1 Gustavo: Here, do we have to do $x + 3 = 0$ and... like in the equations? (*Question*)

2 Teacher: What was the aim [in the solution of fractional equations]? (*Closed progress details*)

3 Gustavo: It was $A(x) = 0$ and $B(x) \neq 0$. (*Teacher-led response*)

4 Teacher: What was the purpose of all that? When is a fraction equal to zero? (*Closed progress details*)

5 Rodrigo: When the A is zero. (*Teacher-led response*)

6 Teacher: When the $A(x) = 0$. (*Reformulate*) But what do I actually have here? A polynomial divided by another has to be greater than zero. Should you just do the same thing? What does it mean when the quotient is greater than zero? What is "greater than zero"? (*Closed progress details*)

7 Gustavo: They can both be positive. (*Partial answer*)

8 Teacher: Then... when does the quotient of two numbers lead to a positive number? (*Closed progress details*)

9 Gustavo: Or both negatives... (*Teacher-led response*)

10 Teacher: So, what do you need to write? [she moves away and leaves the students to work] (*Open progress initiatives*) (audio recording)

In this episode, Branca overheard the conversation of a group of students who were working on an idea that was not correct. One of the students asked her a question, trying to get her to validate the answer. Instead of doing so, Branca focused on the students' work, trying to make them realise that the task they were working on involved other procedures: "But what do we have here? (...) Is it just a question of doing the same thing?" (Line 6). In response to the teacher's question "What does it mean when the quotient is greater than zero?" (Line 6), one of the students gave an incomplete answer (line 7), which had been anticipated during the planning of the lesson. As planned, instead of giving the students an answer, Branca asked them another question, trying to

make them realise that they should also consider the case when the numerator and denominator are both negative and then let them work on the task.

Although the teacher's questions do not appear to leave much room for students' interventions (*Closed progress details*), she managed to guide them and make them progress in the solution of the task without telling them how to solve it and without specifying the representations they could use.

Episode 2 – Whole-class discussion in RL1.1

Branca decided to initiate the discussion with a trial-and-error solution, which had not been anticipated during the planning stage. She then invited a group who began by writing only that the numerator and denominator would both have to be positive. Although the students had solved the problem correctly, the teacher asked them to present and explain only their initial solution and invited another group to complete the solution:

Teacher: Gustavo is adding [the student writes the solution on the board] the conditions that this group had initially forgotten, but they managed to get there in the end. (*External directed*)

Gustavo: So, in this part, we followed the same reasoning as them, but... as everyone knows [pointing to the graphical representation and the trial-and-error solution on the board], when we have got a fraction, it is greater than zero when two positive or two negative numbers are divided. We did this first part [pointing to $x + 3 > 0$ and $x - 2 > 0$] and then we did lower than zero and it came to this [pointing to $x < -3$ and $x < 2$] and then it was finding the solution set (...) (*Explanation*) (videorecording)

In addition to sharing his solution, the student explained it to his peers, without the teacher feeling the need to intervene. Basing his explanation on the conversation with the teacher during autonomous work (Episode 1) and the explanation of his peers' solutions, the student explained that he considered the quotient to be "two positive numbers or two negative numbers", completing the solution of the group that preceded his explanation.

Branca ended the whole-class discussion by asking a student who had used tabular representation to present and explain his answer. Then, as discussed during the planning of the lesson, the teacher synthesised the work done, referring to the various representations and strategies that could be used

by the students to study the sign of rational functions given by expressions of the form $\frac{P(x)}{Q(x)}$, where P and Q are polynomials.

For the whole-class discussion, Branca selected the students who had used different representations and followed different strategies, instead of asking all the groups to present their answer, as they have discussed in the planning sessions. In addition, she selected a group that presented an incomplete solution and a group with a solution that had not been anticipated. Thus, the planning of the research lesson was an opportunity for Branca to deepen her knowledge about conducting whole-class discussions.

Episode 3 – Autonomous work in RL1.2

A group of students used tabular representation to solve the task. After solving the task correctly, one of the students said that she did not know why the zeros of the numerator and denominator of the fraction $\frac{x+3}{x-2}$ were calculated to study its sign. Sofia, the teacher, decided to intervene:

1 Teacher: What you want to know is why we consider the zero of the numerator and the zero of the denominator. (...) When you did the table... $x + 3$... at some point, Pedro said: “Here it is zero, in -3 it is zero. The $x - 2$, here in 2 , we can put the zero straight in the table.” And then you completed [the table] with the signs. Why did we choose zero then? We have continuous functions here and (...) when is there a change of sign? (*Closed progress details*)

2 Guilherme: When they pass in zero. (*Teacher led response*)

3 Teacher: That is why we are determining zeros. (...) And why is the sign of interest to us? (*Closed progress details*)

4 Guilherme: To know when it [the quotient] is greater than zero. (*Teacher-led response*)

5 Teacher: To know the sign of the quotient. (*Reformulate*) (videorecording)

To support the students, Sofia used their answer and the statements made by one of them (Line 1) as the starting point. Although the teacher took over a considerable part of the intervention (*Closed progress details*), the questions she asked helped the students understand why they should determine

the zeros of the numerator and denominator of the fraction and then study the sign of a quotient.

During this conversation with the students, Sofia realised that in addition to tabular representation, they were also working with a graphical representation, and so, she challenged them to explain what they were doing (Line 6):

6 Teacher: (...) And what other approach could we take? Guilherme, for instance, has his calculator here. But he has two straight lines here [$y = x + 3$ e $y = x - 2$]. What other approach could we take in addition to this table? (*Open progress initiatives*)

7 Guilherme: We could put $\frac{x+3}{x-2}$ in the calculator. ((*Steps of solution method*))

8 Teacher: (...) Try it then. Graphically, for example, (...) how can we resolve this issue? (*Open progress initiatives*)

9 Guilherme: It would be a hyperbola. (*Partial answer*)

10 Teacher: (...) You have already worked with them [rational functions] at a graphical level. You have determined asymptotes [in the graph] and everything. So, look to see if graphically we effectively have the same solution. And do not erase the lines, Guilherme (...), so that we can see what happens with each one of them. (*Open progress initiatives*)

[The students try to obtain a graphical representation on the calculator.]

11 Pedro: Teacher, couldn't we also do that thing of dividing this [$x + 3$] by this [$x - 2$] ...? That would then give the function, but in a different way... $1 + \frac{\text{something}}{x}$... (*External*)

12 Teacher: Try it. (*Open progress initiatives*) (videorecording)

One of the students, Guilherme, graphically represented two straight lines, but after the teacher's question, he soon realised that the problem could be solved with the graphical representation of $f(x) = \frac{x+3}{x+2}$ (line 7). Without pointing him in the direction to be followed, Sofia challenged him to do a graphical representation (Line 8). She also told him not to erase the lines he had

drawn (Line 10), as this might help them clarify their doubt (Line 1). Another student from the same group, Pedro, took the initiative and asked if he could write the fraction $\frac{x+3}{x+2}$ in another way (Line 11), recalling content that had already been worked on. Based on this question, Sofia challenged the students to try, leaving them to work on the different representations.

Thus, the teacher's actions were in line with what had been discussed in the planning sessions. In her intervention, Sofia supported her students, helping them to clarify their doubts regarding tabular representation and encouraging them to explain their solutions. In addition, she used the students' work as a starting point to challenge them to answer the question using a different representation, without erasing the straight lines that one of the students had drawn (Line 10) to enhance the comparison between the graphical representation and the tabular representation that they had already used. This episode suggests that planning and conducting the research lesson was an opportunity for Sofia to deepen her knowledge of how to conduct exploratory tasks.

Episode 4 – Whole-class discussion in RL1.2

Similar to what happened in RL1.1, Sofia opened the whole-class discussion when almost all the students had solved the task. Before the discussion to which this episode refers, she invited a student to present and explain the solution of the task with an algebraic representation and, contrary to what had occurred in RL1.1, she also asked the student to explain how she had obtained the solution set, taking the opportunity to remind the class of when they should use the union or intersection of sets. As foreseen in the lesson planning sessions, the teacher invited another student to present her solution using a graphical representation. She then invited another student, Guilherme, to present and explain his group's solution. The student wrote the solution on the board before explaining it (Figure 2).

Figure 2

Table drawn by Guilherme

x	$-\infty$	-3	2	$+\infty$	
$x+3$	-	0	+	+	+
$x-2$	-	-	-	0	+
$f(x)$	+	0	-	0	+

1 Teacher: Let us listen to Guilherme now, then. (*External directed*)

2 Guilherme: We have got this condition, and we need to find out when this condition is greater than zero. So, to do this, we calculate the zero of $x + 3$ and the zero of $x - 2$. (...), we made a sign table [Figure 2], to see when the function was positive and when it was negative. And the zeros allowed us to understand when there was a change of sign in the function. (...) then we put the zero in -3 , and the zero in 2 (...) Then the zero in the numerator [pointing to the numerator of the fraction] with a negative number that would give a zero [pointing to the respective column of the table]. Then, a plus with a minus gave a minus [pointing to the respective column of the table] and a number divided by 0 does not exist [pointing to the fraction's denominator and to the column of the table corresponding to his discourse]. So, it is undefined. (...) So, we knew that the solution set was from minus infinity to -3 union with 2 to plus infinity. (*Explanation*)

3 Teacher: Exactly, because we must look for the signs... (*Closed progress details*)

4 Guilherme: The signs of the function. (*Teacher-led response*)

5 Teacher: Yes. The plus or minus signs? (*Closed progress details*)

6 Guilherme: Plus. Because the function required numbers greater than zero. (*Explanation*)

7 Teacher: Well done. You just need to explain (...) why it is this distribution of minus-plus signs here [pointing to the line in the table corresponding to the sign of $x + 3$] (...) (*Request explanation*) (videorecording)

The student explained his solution using tabular representation (Line 2). In his explanation, he mentioned that he and his group had determined the zeros of the numerator and denominator to identify the quotient signs, recalling what they had discussed with the teacher during autonomous work (Episode 3). Sofia did not explore the other representations made by the group as other students had already been presented them, but asked the student for explanations about the signs he had written on the second and third lines of the table (line 7).

To close the discussion, Sofia wanted the students to understand the advantages of using the tabular representation, as it helps solve other conditions in which the numerator and/or the denominator are not degree 1 polynomials:

12 Teacher: And there we are, we have three procedures [pointing to the algebraic, graphical, and tabular representations]. The procedure we are going to give priority to in the solution is this one [table]. And why this one? [When what we had to do was] consider the sign of the numerator and denominator, both positive or both negative, it was well solved (...). Imagine a situation like this [writing $\frac{A \times B}{C \times D}$ on the board]. Imagine now: I wanted this to be positive. How many possibilities (...) would there be for the fraction to be positive? What would have to happen? (*Open progress initiatives*)

13 Daniel: A and B would have to be positive. (*Partial answer*)

14 Teacher: For example, the A positive... (*External general*)

15 David: There are several ways of multiplying this. The A and B can both be negative and the C and D both positive too, because minus times minus gives a plus on top. (*Explanation*)

16 Teacher: (...) So many possibilities! We would be here all day. If we use the table, it is easy. [Drawing a table on the board, identical to that of Figure 3] We draw a table in which

we put factor A, factor B, factor C and factor D, we analyse each of their signs, and we get there easily. It is far more practical than considering all those possibilities. OK? (*Demonstration*) (videorecording)

Sofia requested the collaboration of the students (Line 12) who, based on the work they had done and the whole-class discussion, managed to understand that it would be necessary to consider several possibilities for the signs of A, B, C and D (Line 15). Finally, the teacher told them that tabular representation could facilitate the solution of tasks such as the one they had solved, without limiting the representations the students could use or the strategies they could follow, as discussed in the planning sessions.

Reflection on communication in the research lesson

In the reflection that followed the class taught by Branca, the teachers reflected on the students' work and felt the need to emphasise the use of tabular representation:

If we really stress the importance of the table, they will not get lost (...) For this $\left[\frac{x^2-4}{x-3}\right]$, those who broke it down $[(x-2)(x+2)]$ (...) got lost. (...) They get lost on the intersections and unions. (...) If we have more factors, then it gets really complicated, that is why we should focus on the table to solve it. (Sofia, Reflection 1).

Hence, to conduct the synthesis referred to in Episode 4, Sofia fell back on the reflection that followed RL1.1, but also on the work done in the planning sessions, referring to the different representations that could be used by the students.

After Sofia's class, the teachers reflected on the whole-class discussions of the lessons they had taught and observed and valued the clarity of the students' explanations. In the following dialogue, they are discussing Guilherme's explanations (Episode 4):

Sofia: But I think Guilherme is good at explaining! I think they all really understood. When he went to the board (...)

Luz: But he said something that was important: why they had calculated the zeros. He did not say it at first, but then it just

came out... I think that is what made the real click! (Reflection 2)

Sofia, who had supported Guilherme's group (Episode 3), said that the discussions among the students during autonomous work had probably helped the student to explain the various aspects of their solution in the whole-class discussion:

Sofia: And he remembered! Let me explain as this problem also emerged in the group...

Branca: He explained to the others.

Luz: He even said, "it was to see..." he even made gestures! "Where the function was... above or below". (Reflection 2)

The teachers referred to the fact that the student had explained the several stages of the solution in detail, in a manner that had appeared clear to them, and that they particularly liked the explanation he had given for $x = 2$: "As he emphasised this... 'Ah! Here it's not defined [$x = 2$] because any number divided by zero...' Then he carried on with 'positive with negative'... he explained that in great detail..." (Sofia, Reflection 2). Recalling the whole-class discussion in her lesson, Branca said that this aspect had not been clarified for her class: "Rodrigo didn't explain this, and I didn't draw attention to it either. (...) [but] as nobody said anything, we assumed that they had all understood and perhaps they had not..." (Reflection 2).

In these discussions, the teachers were able to reflect on the students' explanations for the representations they had used, the strategies they had followed and the way in which they had given these explanations. They also reflected on the importance of checking whether students had understood important aspects of the solutions that had been presented and discussed. Thus, observing the lessons and reflecting on them were opportunities for the teachers to deepen their knowledge about communication in the classroom, namely the whole-class discussion and synthesis moments.

Reflecting on the planning sessions, Luz mentioned in the interview that, in addition to thinking about ways to support the students, their work had enabled her to think about "even the way we ask [the students] a question". Sofia added:

What is most interesting is how they get there themselves, but allowing them to rethink... the idea of not killing the task (...) sometimes they get blocked and we need to do a bit of

unblocking (...) it is important to have a small group discussion, where the teacher can question the students on what they have done, and give them the opportunity to explain (...) And this is an interesting phase because this is where it's possible to break down the questions so that the teacher can understand the underlying reasoning (interview).

In the planning sessions, Sofia had expressed uncertainty about the teacher's role during autonomous work, but in the lesson she had taught, she had supported her students by using their work and their explanations (Episode 3). In the interview, she spoke about the importance of the teacher monitoring the students' work and supporting them, considering this an opportunity for the teacher to understand how they think. Thus, the lesson study was an opportunity for Sofia to deepen her knowledge about communication in the classroom, namely the autonomous work moments.

Although they valued the explanations given by the students in the whole-class discussions in both of the lessons, in the interview, the teachers' concerns related to why they tend to assume the role of conducting communication after the students' autonomous work became evident. The first reason is associated with their need for classroom discourse to be as clear as possible and mathematically correct:

Well (...) because of mathematical correction of saying things, I do not know. They say things using their own Portuguese, right? Perhaps that is why (...) Because we are always obliged to correct their language, aren't we? Mathematical correction. (Luz, interview)

Sofia referred to another reason related to students' difficulty in providing explanations that are understood by their peers. She, therefore, considered it important to make their explanations clearer to the rest of the class:

And sometimes it is your way of ensuring everybody gets it... (...) Quite frequently (...) their peers do not understand very well... the way the student is using the language is not very precise. And the teacher thinks: 'OK, for everyone to follow, this has to be said in a more...more rigorous, objective...more precise or rigorous way, right?'" (Sofia, interview)

Sofia's statements appear to contradict what she herself had mentioned in the post-lesson reflection on the clarity of the students' explanations, both in

the class she had taught, and in the one she had observed, and may be a constraint to leading of whole-class discussions in her lessons. Conversely, Branca said that the students can explain their solutions to their peers, but that as they do not always use mathematical language correctly, this may give the teacher a different perception:

Maybe we just think that. Because when they are discussing, they understand each other ... and maybe when we hear them, we get that feeling: “No, that has not been stated correctly. Let me say this in a different way”. (Branca, interview)

Along the same lines as Branca’s intervention, Luz recalled an episode that she had shared in one of the planning sessions, in which the explanations of a student had been clearer to some of his peers than the teacher’s had been:

I was working on the board, giving lots of explanations, but had the feeling that (...) it was not going in. One [student] of them said: “Teacher, can I come up to the board?” And I said: “Of course”. And off he went... in that Portuguese of theirs... and the others: “Ah teacher! Is that it? Ah! Now I get it... I understand”. (Luz, interview)

Luz said that the student’s explanations had helped his peers to understand the solution of the task, in line with what Branca had already mentioned, and she added: “Sometimes they ask us to clarify a doubt. And we try to explain... and they say, ‘I don’t understand...’ And the student sitting next to them has already understood and says it in another way and ... ‘Ah!’” (Branca, interview).

Both Luz and Branca acknowledge the importance of listening to the students and letting them explain their thinking process to their peers. They also refer to situations where students understand the explanations of a peer more easily than the teacher’s explanations. This reflection was thus an opportunity for the teachers to reflect on the importance of listening to students’ explanations, even if they do not use mathematical language correctly.

CONCLUSION

In the planning sessions, the teachers designed a task for which the students could use different representations, and discussed the role of the teacher in supporting the students during the periods of autonomous work. During the students’ autonomous work (Episode 1), in the lesson taught by

Branca, the teacher supported them while they were working on ideas that were not correct, as highlighted by Hofmann and Mercer (2016) in their study. However, unlike the references made by these authors, the teacher did not ignore the students' incorrect answers or give them the solution to the task. Instead, Branca managed to make them move forward in the solution without giving them a strategy to follow and without lowering the task's level of difficulty, although their actions were mainly *Closed progress details*, followed by *Teacher-led responses*, appearing to have little potential to encourage students to explain their ideas. During the autonomous work periods in the class conducted by Sofia (Episode 3), listening to the students before intervening allowed her to use their answers as a basis to support them, as mentioned by Hofmann and Mercer (2016). Observing the students' work also led her to challenge them to make other representations, in line with what had been discussed in the planning sessions, thus enriching the whole-class discussion. As in Episode 1, the teacher placed the responsibility on the students to continue solving the task, as also referred to by Van Leeuwen and Janssen (2019), closing her intervention with divergent actions (*Open progress initiatives*).

Although Sofia had expressed some uncertainty in the planning sessions about her role during autonomous work, in the lesson she had taught, her actions had been mainly divergent, unlike that which had occurred in the lesson she had observed (RL1.1). In addition, reflecting on the moments of students' autonomous work, she referred to them as an opportunity for students to explain their ideas and for the teacher to understand their way of thinking. Thus, her participation in the lesson study gave her a new perspective on classroom communication when the students are working on a task.

In preparation for the whole-class discussion, in the planning sessions, the teachers discussed how they could select and sequence students' answers to encourage them to explain their solutions and compare the different representations. In the lesson taught by Branca, the whole-class discussion began with a trial-and-error solution that had not been foreseen in the planning sessions. Unlike what she had said in these sessions, she only asked the students who had used different representations to present their solutions, including a student who had relied on his peers' solution and the conversation with the teacher during autonomous work (Episode 1) to explain his solution to the class, completing the algebraic representation presented by another student. In RL1.2, the answers for which the students had used different representations were also presented and discussed. One of the students invited by Sofia to present his answer used tabular representation and explained it, referring to aspects that he and his group had discussed with the teacher during autonomous work (Episode

3). To end the whole-class discussion, Sofia requested the students' collaboration to make a synthesis, without limiting the representations they could use, based on the discussions in the planning sessions and on the reflection after the lesson taught by Branca. Following the lesson taught by Sofia, the teachers reflected again on the importance of making a synthesis at the end of the whole-class discussion, to ensure students' clear understanding of the targeted learning, taking the aims of the lesson into consideration and highlighting the importance of this moment in an exploratory lesson.

This study points to a deepening of the teachers' knowledge regarding their teaching practices, namely conducting whole-class discussions and giving students an active role in the construction of their knowledge. In particular, Branca, who had expressed some degree of uncertainty about the selection of students to present and explain their work, selected only those who had used different representations, in line with what had been discussed in the planning sessions, and even selected a trial-and-error solution which had not been foreseen.

Hence, the secondary school teachers' collaborative work in the lesson study brought them a new perspective on the communication established in the classroom, a central feature of the exploratory approach. On the one hand, the teachers gave several opportunities to the students to participate in the lessons and supported them during their autonomous work, based on their answers. On the other hand, they led the discussions based on the students' answers, some of which were incomplete, asking them for explanations, and ended the lesson with a synthesis, referring to the various representations that could be used. Moreover, the teachers reflected on the students' explanations, referring to situations where the students had understood their peers' explanations more easily than the teacher's explanation.

In the lessons referred to in this paper, the teachers worked with students who were already used to working in groups, who knew each other and who chose the peers with whom they wanted to work, which may influence teachers' interactions with students, as mentioned by Van Leeuwen and Janssen (2019). As for the students' explanations in the whole-class discussions, only occasionally did the teachers feel the need to ask other students to intervene, which may be related to the quality of the explanations given by the students, which included connections to the solutions already presented by their peers (as in Episode 3). Therefore, in future studies, it might be important to analyse teachers' interactions with students in lessons where they are not used to working in groups or do not know each other as well as the students in this

study. Furthermore, as stated by Ponte (2005), “learning to conduct discussions is not only a task for the teacher, but also a whole-class learning exercise to be carried out in every class” (p. 16). In fact, leading whole-class discussions is a challenge for teachers (Guerreiro et al., 2016; Kooloos et al., 2020). Thus, this is an issue to be deepened in future lesson studies.

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STATEMENT OF CONTRIBUTION OF AUTHORS

P.G. wrote the first version of the theoretical framework and of the research methodology. She also made the data collection and preliminary analysis of data. All authors, P.G., M.Q., and J.P.P., discussed all parts of the article, reviewed, and contributed for the final version of the work.

STATEMENT ABOUT AVAILABILITY OF DATA

The data collected for this study may be made available by the corresponding author, PG, in a request with adequate justification.

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