HUMANS-WITH-INTERNET OR INTERNET-WITH-HUMANS: A ROLE REVERSAL?[1]

SERES HUMANOS-COM-INTERNET OU INTERNET-COM-SERES HUMANOS: UMA TROCA DE PAPÉIS?

Daise Lago Pereira Souto
daise@unemat.br
Universidade do Estado de Mato Grosso

Marcelo de Carvalho Borba
marcelo.c.borba@unesp.br
Universidade Estadual Paulista

ABSTRACT

The aim of this article is to analyze possible influences of the internet on mathematics online distance education and to discuss the interrelationships between the human and non-human actors involved. To do so, we analyzed empirical data produced in a mathematics education course for teachers. The discussion is based on the approach of the Activity Theory and on the humans-with-media construct. As an analysis tool, we used the notion of mini-cyclones of expansive transformation. The results indicated that during the activity of "doing math online", the internet played two roles: artifact and community. This double role contributed to the transformation of the basis of the activity and leveraged distinct movements. In addition, we found that human and non-human actors interacted in such a way that both were able to play identical roles in an activity system.

Keywords: Mathematics teaching; Activity Theory*;* Humans-with-media; Distance Online Education; Expansive Transformations.

RESUMO

O objetivo deste artigo é analisar possíveis influências da internet na produção matemática a distância online e discutir as inter-relações entre atores humanos e não humanos envolvidos nessa produção. Para tanto, nos baseamos no exame de dados empíricos produzidos em um curso de Educação Matemática para professores e fundamentamos em uma vertente da teoria da atividade e no construto seres humanos-com-mídias. Como

[1] **NOTE -** This article was originally published in Portuguese in the Journal Latin American of Research in Educational Mathematics - Relime - DOI 10.12802/relime13.1924. *Revista Latinoamericana de Investigación en Matemática Educativa* (2016) *19* (2): 217 - 242.
Although the authors were involved in the translation with the help of a native English teacher, there are always small changes in emphasis and interpretation as a result of translation and the passing of time itself, which may change the perspective of the authors somewhat.

ferramenta de análise utilizamos a noção de miniciclones de transformações expansivas. Os resultados indicaram que durante o fazer matemática on-line, a internet desempenhou dois papéis: artefato e comunidade. Esse duplo desempenho contribuiu para a transformação dos motivos da atividade, e também alavancou movimentos distintos. Também verificamos que atores humanos e não humanos se inter-relacionam de tal forma que ambos podem desempenhar os mesmos papéis em um sistema de atividade.

Palavras-chave: Ensino de Matemática; Teoria da Atividade; Seres-humanos-com-mídias; Educação a Distância Online; Transformações Expansivas.

1. Introduction

Even though we experience the changes digital technologies imposes on our lives, we are still entrenched to a classroom culture that either does not allow internet access or resists it. This kind of culture reveals a conservative vision, i.e., the teacher is considered central in the process of knowledge production (Borba, 2012). As students, we usually resort to the teacher to answer our questions or to legitimize our conjectures.

Yet the internet advances unrelentingly and with exponential speed, gaining a foothold in education, particularly in the classroom. The internet unsettles some of our beliefs, because it generates changes in social rules relating to the roles each actor "could" or "should" play in the process of knowledge production.

The culture of our current society carries configurations and reconfigurations of several theoretical perspectives, which suggest that the human actor should not be seen as the primary or only entity responsible for knowledge production. Now we find an emphasis on the collective with the co-participation of non-humans in this process as well.

In the context of mathematics education, some studies developed in virtual classrooms discuss, among other issues, the role of non-human actors (media) in the production process of mathematics (e.g. Gracias, 2003; Borba & Villarreal, 2005; Santos, 2006; Zulatto, 2007; Malheiros, 2008; Rosa & Maltempi, 2010; Borba, Malheiros & Amaral, 2011; Villarreal & Borba, 2010). The results of these studies suggest that the media are also required in the production process of mathematical knowledge.

Our aim in this article is to look into these and other possible interrelationships among those actors, consequently and intentionally, we have placed a question mark in our title to emphasize the goal of this article: to prompt reflections on the contributions of digital technologies- particularly the role of the internet - in the process of mathematical knowledge production.

Scagnoli (2005) argues that by using the internet, teachers and students can consult books, quickly access data, acquire information, etc. In addition, it provides easy access to one’s peers, which allows for the development of intellectual work (collective thinking) in a collaborative way (Baldwin, 1998; Harasim, 1990; McDonald, 2002). In this regard, Scagnoli (2005, p. 6) states that "the internet becomes a virtual learning space, where knowledge is shared." Based on this concept, we discuss the role of the internet in an activity system made up of collective thinking, and thus illustrate the way the internet can expand to take on various roles, noting that this expansion, in turn, brings implications to the activity system itself.

The theoretical framework we decided to use to analyze our data is the Activity Theory (AT), particularly Engeström's ideas (1987), which propose a systemic organization to support discussions about understanding the collective nature of human activity. We also use the humans-with-media construct (S-H-C-M) (Borba &Villarreal, 2005) that emphasizes the role of the media in the production of knowledge, leaning on the authors of AT, ethnomathematics, and the philosophy of technology.

We adopt the concept of mini-cyclones of expansive transformation (Souto, 2013; Souto & Borba, 2013) that weaves together the ideas of AT and the S-H-C-M construct. This analysis tool helps in understanding the movement of actors in an activity system aimed at the production of online mathematical knowledge.

Therefore, we organized this article as follows: we begin by discussing the ideas of Engeström (1987), particularly those that have developed the Activity Theory. Then we highlight the ideas from Borba (1999) and Borba & Villarreal (2005). that discuss the humans-with-media construct. Finally, we discuss methodological and pedagogical aspects, followed by the data analysis and a synthesis of our ideas.

2. Activity Theory

The Activity Theory's central axis is the transformations that occur in the interrelationships between human beings and the environment in which the activities develop. It is based on the principles of the Soviet cultural-historical school of psychology.

According to Souto (2013), the genesis of this theory is hazy, mainly due to translation problems and its split development. However, Vygotsky, Leontiev, and Engeström are singled out as some of the leading theorists who contribute and/or contributed to its development.

In this article, we rely on the ideas of Engeström (1987, 1999, 2001), that propose a systemic organization to explain this theory (Fig. 1).

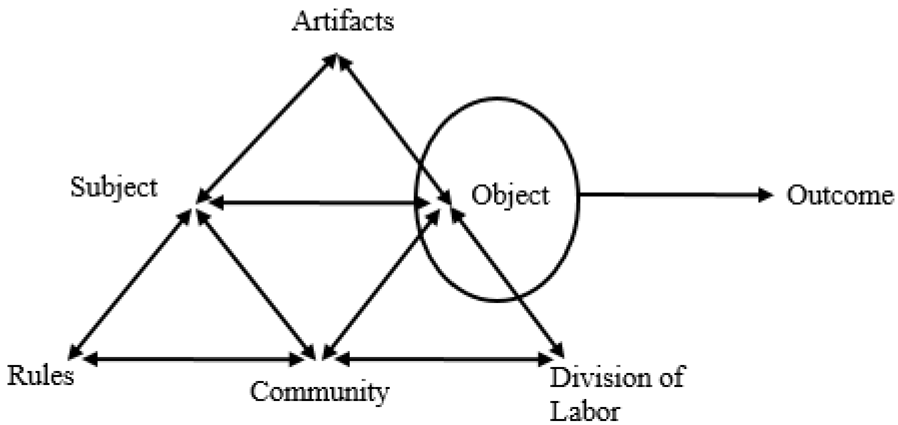

Figure 1. Representation of the human activity system

Using the triangular representation of Figure 1, Engeström (1987) seeks to integrate subjects (all of which have "agency" or power of action), objects ("raw material" or a "problem" to which the activity is directed), artifacts (tools and signs), rules (those defined by the system), community (all that mediates the relationship between the subject and the object of activity) and division of labor (negotiation of responsibilities based on the rules). He considers that these elements together form a unified whole. With this structure, which Engeström himself termed an "activity system," he highlights the collective nature of the activity and, at the same time, expands Vygotsky's concept of mediation of the cultural context.

In summary, Engeström explains that a system of activity should be understood as a continuous process of change and a movement arising from crises and disruptions, which are creatively interrelated and composed of elements, voices, multiple conceptions, transformations and innovations that are understood historically (Souto, 2013).

To help us understand his ideas on this process of change related to the movements of an activity system, Engeström (1999) proposes an analytical tool: the expansive learning cycle that usually begins with the socialization and training of apprentices (and occurs through questions, critiques, or the rejecting of current practices) to become members of the activity.

The expansive learning cycle often begins with the process of cultural reproduction. Then, individual changes slowly alter these cultural reproductions as they search for solutions to a given problem. As the cycle evolves, the design and implementation of a new representation for the activity may develop. In other words, the construction of a modified model or of a new idea can explain and offer a solution to a given problem. Then, the model or idea is evaluated in order to verify its potentialities and limitations. Once the best model, solution, or idea is found, it is implemented through a practical application.

Following these steps, an evaluative reflection is performed on the modified representation, and other practices may be consolidated. In other words, this new representation becomes stable and dominant.

3. The Construct Humans-with-media

The construct humans-with-media is based on the idea that knowledge is produced by human and non-human actors conducting "thinking collectives," (Thought-collective is a term used by Lévy to emphasize that knowledge is produced by collectives composed of human and non-human actors in which all actors play a central role). According to Borba & Villarreal (2005), there is no quality scale among the media that can classify them as better or worse. There are simply different kinds of media that have affected the production of different types of knowledge throughout history.

When interacting with some media, humans reorganize their thoughts according to both the multiple possibilities and the limitations that media offer. The presence or absence of media influences the kind of knowledge produced, and the use or the emergence of a particular media does not invalidate or eliminate another, although it often exists in a location other than where it had been earlier.

The above ideas are based on studies of Tikhomirov (1981) and Lévy (1993). Tikhomirov's studies are based on the idea of mediation of the Activity Theory, which is implicit in the concept of reorganization and is present in the process of human interaction with the environment. He proposes the establishment of a human-computer system. To encompass the technical and collective dimensions of the cognition, Lévy conceptualized the expression "cognitive ecology" and realized new possibilities for information technology. He suggested a system that goes beyond Tikhomirov's proposal, that composes a "thought-collective" of human things. Borba (1999) shifts these concepts to mathematics education, creating the construct humans-with-media. We understand that in this construct, the idea of mediation is extended to a mutual influence, where the media influence humans in the same way that technologies are understood as being impacted by humanity. The external-internal line dissolves and becomes "fuzzy." Knowledge is thought of as a human product, but knowledge with historically built technologies.

A central concept of this construct is the notion of an *intershaping relationship* (Borba, 1993, 1999) which, according to the feedback of a particular medium, influences the reasoning of humans who interact with it. In other words, media shape humans. However, humans themselves are also shaped as they use them. An example of this can be seen when students use a piece of software differently from the way the team that developed it had thought it might be used. On the other hand, a team that develops software seeks to expand a design by considering how students have modified or used it.

The notion of intershaping relationships proposed by Borba (1999) has a strong link with the Activity Theory, as human beings throughout history have invented tools and developed ways to adapt in order to ensure their survival. This process of creation and interaction with the environment is dialectic, since as the human beings transform the environment, the environment is transformed by humans. The difference emphasized in Borba's thinking lies in the emphasis on certain aspects of the interaction between humans on one side, and digital media on the other. For example, human actors receive feedback from a given medium that shapes (but does not determine) their actions, yet at the same time such actions affect and shape the possibilities that this medium itself offers. Therefore, it is possible to say that under the notion of the humans-with-media construct, the possibilities and constraints (conditions) that media offer, result in a process of knowledge production distinct from others in which different media are involved.

Ideas related to the humans-with-media construct were created and have been legitimized within the research group Technology, other Media and Mathematics Education Research Group - GPIMEM[2].

4. Methodological and Pedagogical Aspects

The goal of this paper, as stated earlier, is associated with the idea that searching for deep understanding goes beyond simple numerical results. Consequently, we opted for a qualitative research approach and have adopted participant observation as the main research procedure. Jaccoud and Mayer (2008) state that the advantage of this procedure is to allow for a deeper understanding of reality, and shows to be a method that reduces the distance between the discourse and the concrete practices of the social actors.

Yet, Lincoln & Guba (1985) highlight that the process of "participant observation" is not neutral, because the interaction between the observer and observed implies mutual influences that can configure "bias" in research. A way to minimize bias, then, is the use of additional procedures and different data sources. Therefore, we decided to conduct interviews and analyze the data produced in participants' chats, forums, and emails. These data were produced in the context of an online distance course offered by the university. This course lasted 32 hours, and 20 math teachers in this virtual learning environment Tidia-Ae[3] participated in discussions about mathematics education texts as well as about developing solutions to problems of analytic geometry, more specifically, about conic sections. In order to discuss conics sections, which is the focus of the analyses of this article, the teachers were divided into small groups to solve problems designed for the collective of teachers-with-GeoGebra[4]-internet-pencil-and-paper. Both authors of this article were online teachers of this course.

As a tool to analyzing the data, we proposed the notion mini-cyclones of expansive transformation (Souto, 2013; Souto & Borba, 2013). This perspective of analysis is based on the Activity Theory, particularly on the concept of the Expansive Learning Cycle proposed by Engeström (1999) along with the notion of intershaping relationships (Borba, 1993, 1999), which is central to the concept of humans-with-media. The idea of the mini-cyclone associates the movements of an activity system to a natural phenomenon: the cyclone.

---

[2] For a more in-depth reading on the work of GPIMEM we recommend Borba and Chiari (2013).

[3] http://tidia-ae.rc.unesp.br

[4] www.geogebra.org

A cyclone is a storm produced by large masses of air of increasing speed and rotation that transforms itself as it moves along. Our definition refers to this phenomenon because in addition to rotation (moving around oneself) and transforming (moving around to other systems of activity), it is not possible to determine or predict the direction a cyclone will take. In addition, we consider it appropriate to relate the idea of increasing speed to the process of knowledge production and its transformations.

A mini-cyclone can promote the understanding of the development of humans-with-media[5] systems that are developed in virtual learning environments, as they capture the internal contradictions and the occurrence of expansive transformations. The internal contradictions are not problems or conflicts, but rather structural tensions historically seen in activity systems. They can serve as sources of changes in the activity on one hand, or as sources of conflicts, clashes of opinions, or lack of acceptance on the other. Expansive transformations are understood as movements within a system of collective activity where humans-with-media critically seek a way to understand and/or rebuild knowledge about a specific problem or mathematical content (Souto, 2013) that had not been deduced in other situations or under other circumstances.

In general, a mini-cyclone begins to take shape when doubts and questions arise from a tension often caused by a need to solve a new situation or by the - even if unconscious - desire of breaking already established patterns of mathematics production.

The evolution of mini-cyclones takes place as thinking is reorganized. This reorganization is driven and influenced by the results of the media involved in the process. As the mini-cyclone progresses, work dynamics may change, and alternative paths and new tensions may arise. The object of the system may be better understood during the intershaping relationship process. In addition, conducting experiments and simulations of a given theory can intensify, indicating that expansive transformations may be in motion, that is until arguments arise to help provide a solution, which may indicate the mini-cyclone is close to an end. In other words, the process of developing a mini-cyclone results in qualitative changes in mathematics knowledge production of the activity's participants.

In order to find harmony among the methodological, epistemological, and pedagogical dimensions, we have structured the online course to give priority to collective, collaborative, and dialogic work, which has the potential of becoming a virtual learning environment to the degree that participants interact. In keeping with this thought, we sought a pedagogical model based on what Silva (2003) called the "interactive classroom," which encourages the student to create a theory or supposition and then search for alternative paths, i.e., whether to change, enlarge, and shape "doing math" (Souto, 2013).

We feel it is appropriate to consider that the relationship between the various actors has exceeded the limits of exchanges between humans, constituting a collective of teachers-students-internet-software. This kind of distance education model has been developed over

[5] Humans-with-media systems must be understood as systems of activity in which the collective of human beings-with-media would be not only the basic unit of production of knowledge, but also a part of the activity which becomes metamorphosed according to the movement (Souto & Borba, 2013).

the last 15 years, when the GPIMEM drafted and improved the concept of humans-with-media.

With respect to the study of conic sections with GeoGebra, we followed the experimental-with-technology approach proposed by Borba & Villarreal (2005). With this approach, the problem encourages students to the form conjectures and procedures of trial and error to build arguments that accept or refute initially formulated hypotheses.

This approach fits with the epistemological vision of the construct humans-with-media, because the media feedback generated during a trial allows those solving the problem to initiate debates, discussions, questions, ideas, and different possibilities. This perspective is rife with the notions of collective, collaborative, and dialogic work, because in this construct, humans and the media are seen as one knowledge-producing unit.

5. The course participants' voices

Our attempt to adjust the focus of math online production so the internet’s possible influences could emerge led us to analyze a data clipping produced during the course. To specify the movements that occurred during the study of conic sections, we divided the group into two small sections. For this analysis we selected the groups formed by Thais/Elza, and Virginia/Bianca (these are pseudonyms used to preserve their identities).

The participants received the first activity, whose goal was to construct a conic using some guidelines, followed by open questions that explored the drag tool of the software. As an example, we present the step-by-step construction of the parabola and its respective issues below.

> 1º) Create a line *d*, then create a point F out of line *d*.
> 2º) Create a point A on line *d* and by that point, create a perpendicular line *a*.
> 3º) Construct the perpendicular line segment bisector *m* between points F and A, and at the intersection of the bisector *m* with perpendicular line *a,* mark point P.
> 4º) Use the right click on point P, and in the window where it appears, click “enable trace.”
> 5º) Select the move tool, click on the point A, hold and drag. Note the trace point P and answer:
> a) What happens when you move the point?
> b) What can we affirm with respect to points A, F and P?
> c) On the view menu, enable the algebraic window. Once more, move the point and describe what occurs in this window.

The second part of this study presented problems usually found in textbooks. In our view, the process of solving these two problems illustrates the analysis of the influences of the internet on algebra online distance production. We will discuss this issue in this section.

Initially, the internet played what Activity Theory considers to be a “natural” role--i.e., the condition of an artifact (Fig. 1) influences the relationship of the subject to the object. This idea is also based on Engeström’s (2002) discussions about school learning, where he recognizes the role of the pencil and paper as artifacts (or media) and the role of students as they themselves explore developing their own study. Artifacts are mediational tools, and refer to machines, writing, speech, gestures, numbers, mnemonic resources, etc. In addition, the internet as a virtual learning environment (Scagnoli, 2005) shapes teachers' thinking (Borba & Villarreal, 2005). In other words, there is a relationship mediated by this media in which various reorganizations of thought express the same mathematical idea, yet in different ways.

With the development of mathematical work, new movements have occurred within the system, suggesting changes in the role of "artifact" such as the internet. This change of role can be seen in the mini-cyclone of expansive transformation that occurred. In the comment below, one can see Thais gives some indication that identifies the onset of a mini-cyclone. She explicitly states the desire to research something new or to build something previously unknown to her.

> Thais: Look Virginia ... I also must confess that I am disturbed with these constructs. I've never studied the parabola... I was tempted to look for theories, but I decided I wasn't going to, so I decided to get my own constructions. [Thais refers to the solution of problems usually found in textbooks that at that time her group was trying to solve based on geometric constructions that were performed earlier with GeoGebra] (Chat - 03/05/2011).

Thais is a professor and has worked with this topic for some time. It is important to highlight this experience, because, from her comment, one can verify that the way she approached the subject helped her to review, reconsider, and rebuild various notions from her own ideas. In this sense, we can say that there was, at that moment, relative destabilization of the activity system, because these movements were part of the process of the reorganization of thought (Tikhomirov, 1981). Considering that Thais was agreeing with Virginia, it is possible to infer that this was not an isolated opinion, but rather shared ideas in common.

Sometime during the study, Elza and Virginia discussed the following problem belonging to the second part of the study: *"given the equation $y = x^2$, trace the graph, determine the coordinates of the focus and the equation of the guideline."* To help the reader understand, we present here the feedback of GeoGebra for this problem, where the data were inserted into the input field. However, it is important to note that at that time, the teachers did not have this software image which, consequently, required a mental exercise to preview, and at the same time transform mathematical ideas in their mother tongue, as shown below in the following excerpt.

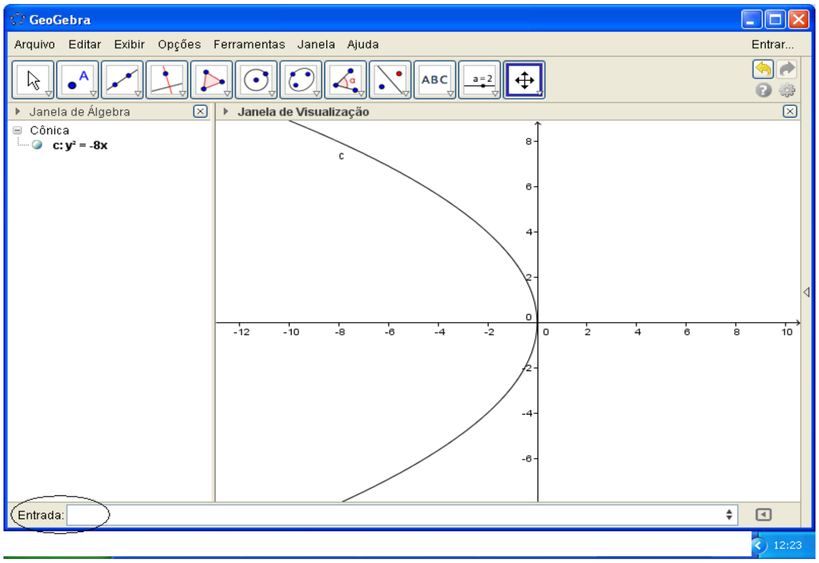

Figure 2. Solution provided by GeoGebra by using the input tool

The following dialogue, where the teachers refer to the question in Figure 2, exemplifies a mini-cyclone more clearly. As highlighted earlier, a mini-cyclone can start with doubts and questions arising from a tension caused by a need to address a situation never before encountered, or by the- perhaps unconscious - desire to break an already existing mathematical pattern of solving a given problem. Its development is based on the reorganization of thought caused by the responses of the media involved in the actions of the apprentices.

> Elza: Have you solved item j? [question represented in Fig. 2] I had problems ...
> Virginia: For me, it seems the focus should be on the x-axis, the left of y, i.e. it must be a negative x. With y = 0. What do you think, girls? Is this the right way? It seems to me that the focus should be equidistant from the two "arms" of the parabola, and for this to occur, you need to be on the x-axis. Girls, what do you think? I'm not able to do this alone, if you have any ideas, please write ... And in this case, the guideline must be parallel to the y-axis?
> Elza: Parallel? Why?
> Virginia: I don't know how to explain ... Thinking on the activity we did earlier, moving point A, when point P described the path of the parabola. Remember, thinking about that, it seems to me that we need to get this confirmation. [Virginia is referring to the geometric construction held earlier with GeoGebra. Point A is the intersection of the guideline of the parabola with the straight line perpendicular to the directrix, and point P is the intersection of the perpendicular bisector line between the focus of the parabola and point A, and the straight line perpendicular to the guideline. So, by moving the point on the line, the directive point P draws the parabola] (Chat - 03/05/2011).

As we saw in the synchronous debate, Elza requested help from her colleagues to solve the issue concerning the parabola. Her problem was in understanding the geometric representation of the equation's parameters. Might this be considered a tension in the activity of this group? The reaction of the teachers participating in the group indicates yes.

Virginia, in an attempt to contribute, shared an idea she had formed based on the study the group had performed in the previous step, where they had explored relationships between algebraic and geometric representations with the “drag” tool of GeoGebra. These conjectures had been intuitively built, because Virginia herself recognized there were no arguments to accept or refute them. The collaborative work began to be outlined, and in addition to Virginia, Thais also offered a solution with GeoGebra.

> Thais: I agree that the focus is a point like (x,0). So, I made a parameter to this value; [This action of Thais refers to the insertion of a selector in the GeoGebra].
> Virginia: Go ahead Thais, please, I'm anxious...
> Thais: But it's not possible, Virginia ... I will try to describe.
> Elza: What’s not possible??
> Thais: My tests ... Hold on, people. My idea is to fix the distance of a point (x,0) to the parabola of the other side and try to follow the procedures we did of the perpendicular and the perpendicular bisector. [This comment of Thais’ is related to the construction of the parabola performed by the Group].
> Virginia: An important point to create a parabola is the focus and for now, we are considering that it is on the *x*-axis (x,0). For the parabola to exist, another point must be about the guideline and both must be equidistant from any point of the path of the parabola. Yes, I stopped in this step. You can help me organize it, these are the data I have, but now we need organize it; from this, we can prove if the guideline is parallel to *y-axis* or not.
> Thais: It [the focus of the parabola] is on the *x* axis. I agree Virginia ... So, I'm trying to get this fixed point on the *x*-axis and fix the distance from it to the other side, in search of the guideline… But I'm getting lost in the procedures. Virginia ... It [guideline of the parable] is parallel to the *y*-axis.
> Virginia: That's my opinion ...
> Elza: Oh guys, I'm lost!! (Chat - 03/05/2011)

As the work progresses, Thais supports Virginia’s opinions, while Elza still cannot follow her colleagues’ reasoning, yet she remains interested in understanding the relationships and in finding a solution. Thais’ idea was to explore the software’s tool possibilities, fixing some elements of the parabola, using the drag tool in others, and performing simulations until she found the desired answer (Fig. 3).

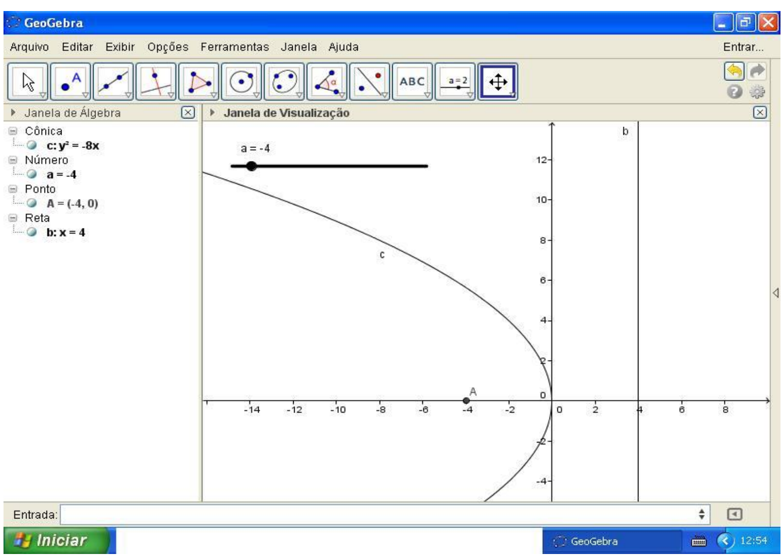

Figure 3. Partial construction of the solution prepared by teachers

Figure 3 illustrates the process of the dynamic solution constructed with GeoGebra was discussed in the preceding section.

Borba (2012) and Villarreal and Borba (2010) state that different intelligence technologies have been influencing the production of various kinds of knowledge throughout history. We understand that Thais' simulation made with GeoGebra would be hard to perform with other technology not having the same resources- for example speaking or writing.

These examples of movements which used reasoning with technology partially present an intershaping relationship process (Borba and Villarreal, 2005) because the actions of Thais and her colleagues were shaped by the feedback they received from GeoGebra, and reorganization of thought happened with each new simulation, which was subject to the capabilities the software offered. Later in a chat, Thais explained the procedures she used in the construction, and after synthesizing her ideas, she sent an e-mail highlighting once again the importance of the collective thinking that was reorganized based on the interactions with the software and her colleagues. The suggestions Thais proposed in the message she sent by email are shown below.

> Yesterday's class…
> I was thinking about what we had done last night, and I think it can be done an easier way.
> * Note: the parabola touches the *y*-axis at the origin... that guarantees us the focus *F* is a point on the *x*-axis and the guideline is parallel to the *y*-axis.
> Also, remember Bianca's conclusions about parallel lines. So, if the focus is F (*a*, 0), the guideline will be *y =-a*, in the Cartesian plane. Following the previous steps that we did in the activity, we can find what we wanted.
> Suggested resolution:
> Enter the parabola *y ^ 2 =-8 x*
> Type = -4
> Create point A by typing = (*a*, 0)
> Create the directive straight typing *x =-a*
> Go to algebra window and click the white ball number *a* that we created. (that way we will be able to adjust it)
> Create a point P on the line *x =- a* (just go with the tool point at any point of this line).
> Trace the perpendicular line to *x =-a* on P
> Trace the perpendicular bisector between A and P;
> Use the intersection of two objects and find the intersection of the last two lines that was built, generating a point B.
> To adjust the dial on the -2, move point P.
> Point B will pass exactly in the parabola we wanted ...
> Thais (mail-5/4/2011)

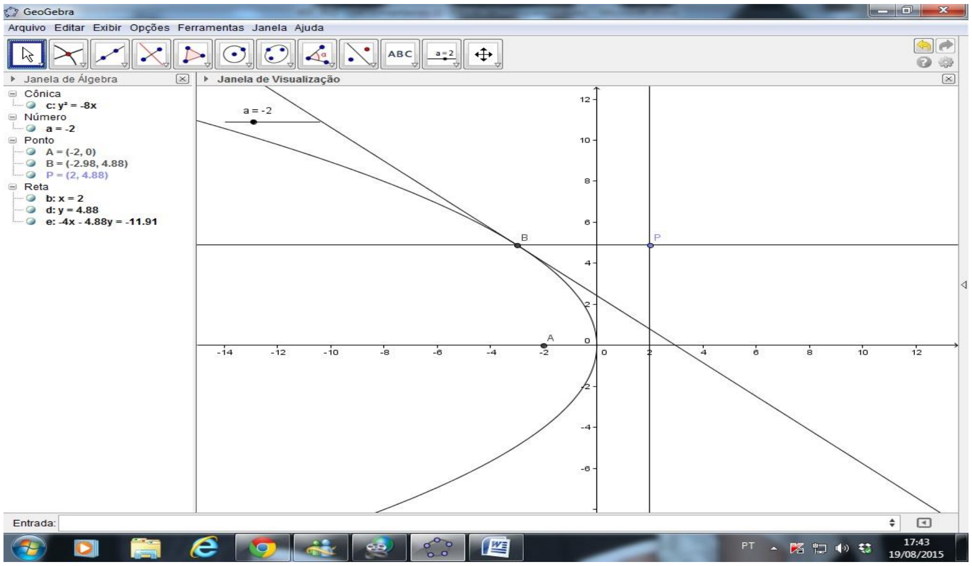

Figure 4. Solution to the problem of the parabola

In her email, Thais stresses the importance of group contributions to achieve the result. Although she draws her conclusion using a step-by-step construction, she makes it based on the discussions in the previous chats and on the interactions within the group and with the software.

As the work progresses, the group continues debating. The degree of difficulty seems to increase, particularly during the study of hyperbola, where one observes how the mini-cyclone takes a new direction, although at first the movements resemble those that had occurred during the study of the parabola. In this case, there is also a construction of hyperbola as a geometric locus, and a second section based on exercises possibly found in books. For this analysis we have selected some clips from the second part.

> Thais: Suggestion ... Girls, we need to find out what we have to do with those *a* and *b* $\left[\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\right]$ in the constructions ... Right? We can do this by using the capabilities of the software. GeoGebra is a simulator, so we can play with it. Modify the parameters and try to analyze what changes happen. Come on!!! Girls, I think is the easiest case to investigate, is the exercise i. [Determine the Center, the axes and focus of the hyperbola $\frac{(x-2)^2}{9^2} - \frac{(y-2)^2}{7^2} = 1$].
>
> Elza: Easiest?
>
> Thais: We have A(-7,2) B(11,2) F1(-9,6;2) F2(13,4;2). [These points were obtained on algebraic resolution made by Thais with pencil and paper and this is a knowledge that Thais already had]. Yes, Elza, because we have the two things: the algebraic and geometric elements. (Chat - 12/05/2011)

In the following figure, we share a representation of the problem of the hyperbola highlighted by Thais during the study of the parabola (see above excerpt), a representation displayed when the input field of the GeoGebra was used.

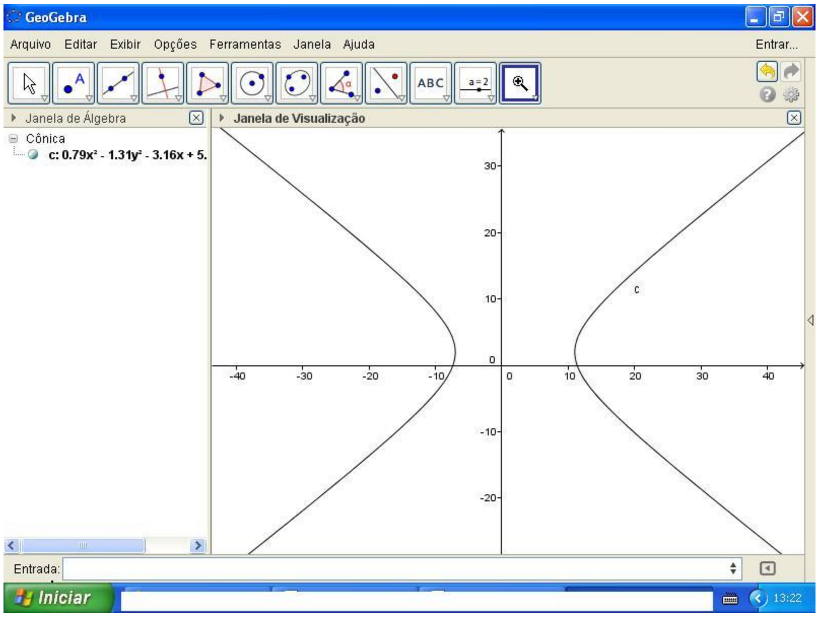

Figure 5. Solution with the use of the input field tool

Thais encourages her partners to try to understand the relationship between algebraic and geometric representations. Elza does not consider this an easy task, but seeks to follow along. During this collective and collaborative work, the process of an intershaping relationship (Borba, 1993, 1999) can be seen, particularly when assumptions are raised, tested, and proved or disproved. In addition, in some cases the software's feedback created new conjectures that were again tested until the group found a consensus from the mathematical point of view. These moves indicate that reorganizations of thought (Tikhomirov, 1981) are constant and are conditioned by the contributions of media in the process of searching for a solution.

During this process of "*thinking with*" the GeoGebra, the GeoGebra feedback from the tool drag may have influenced Thais' reasoning, which suggests that the asymptotes of the hyperbola had to be investigated. This concept needed to be revisited and remembered, because, apparently, none of the teachers were clear about it. At this point they had the idea to consult websites.

> Thais: You know what is the asymptote of the hyperbola? How can we find it here? Does the *b* $\left[\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\right]$ have relation with it? Yeah ... So my suspicion that we should investigate it.
>
> Bianca: Sure is ... If I understood what the asymptotes are. There are two, right, Thais?
>
> Elza: But the question is to find them. Any suggestions?
>
> Bianca: I'm going to get something to help us on the internet.
>
> Thais: Look at this site... http://www.algosobre.com.br/matematica/geometria-analitica-hiperbole.html. [Between this and the next intervention of Thais if passed 12 minutes]
>
> Bianca: Look at this site: http://alfaconnection.net/pag_avsm/geo0504.htm
>
> Thais: People ... Now it's easy. When we trace the asymptote, we can draw a perpendicular to the line y = d [Thais, based on what she searched on the internet-formulas and static constructions

> which allowed determine the asymptotes of the hyperbola, the Center and the focus-, refers to the construction shown in Figure 6, which was being prepared by the Group at that time]. Oops ... There's a mistake there. I'm going to write it all at once and tell how to find the focus using the tools of GeoGebra, and having the equation of the hyperbola. Bianca, the first step was to find the algorithm. Now we need to study it ... the saga is not yet over. I used what I found on the website "..." However, the why we will have to think ... (Chat - 12/05/2011)

The above excerpt indicates the questions' increasing degree of difficulty and a lack of time, which may have caused the group to abandon its initial desire to "not get theories," but instead, now, to look for help on the internet. In this case, the internet was playing the role of a community, because one of the roles of community is to navigate the system within the sociocultural context of those who share the same object (Engeström, 1999). In addition, it is also the community's function to mediate relationships between the subject and the object of the activity. In this regard, we note that the moment the group considered an approximation of the object, it was "studying the connections between mathematical concepts involved in the solution that was in the process of construction." Consequently, we interpret that the internet mediated the relationship of the subject to the object.

It is appropriate to note that the relationship of the subject and the community, according to the Activity Theory, is mediated by the rules. In this sense, the internet in the role of a community brought the system rules and social conventions that elevated math to a symbol of certainty which, until that moment, it had not been a part of.

From another perspective, we must also consider the existence of a relationship between community and object, a relationship mediated by the division or organization of "work" as we prefer to call it. In this case, we observe that internet access changed the forms of organization of work, because as the chat interactions declined in frequency, the work of the group went on to be more "individualized." Supported by the information from the internet, group members individually elaborated on the subject and tested their conjectures. Only then, following the individualized work, was there a kind of socializing and discussion of ideas. Thais' behavior, in particular, provides us with these indications.

In our analysis, we also identified transformations regarding reasons for the activity itself, which are closely linked to the relationship between community and object. According to Kaptelinin (2005), this is because "the object of the activity is determined by all effective motives" (p. 17). Thus, we observe that initially the teachers' motive was to build a solution to the problems about conic sections with GeoGebra in an online environment. However, when the internet played the role of the community, the motive changed, and a solution was quickly found.

While researching the internet's role of community, we realized a multivocality expansion of the system that encompassed not just the individuals who were part of it, but the multiple voices of those who were external but somehow related to it as well. The building of a website is permeated with different values, stories, conventions, placements, i.e., different experiences of its creators, and, when used as a source of research, it transmits all these variables into the system. These variables influence the mini-cyclone movements, because although the group had used the drag tool during the construction process, the final solution in this case was a static construction represented below in Figure 6.

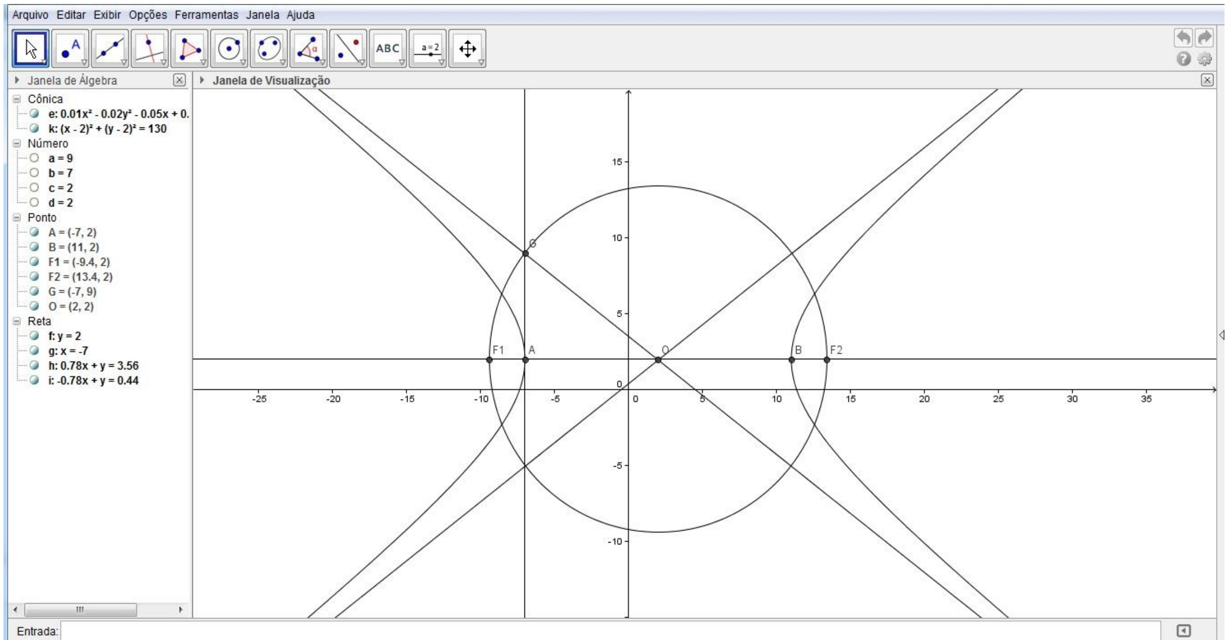

Figure 6. Final solution of the hyperbola

> We know equation $\frac{(x-c)^2}{a^2} - \frac{(y-d)^2}{b^2} = 1$, where point (c, d) indicates the center of our hyperbola. We draw line y=d. We find the intersection of the hyperbola with line y=d. Finding points A and B, which have distance *a* to the center *(c, d)*. In point A, we draw line t perpendicular to y=d. We draw the asymptotes $y - d = -\frac{b}{a}(x-c)$ and $y - d = \frac{b}{a}(a-c)$.
>
> We named G the intersection of an asymptote and of line t. There is the formation of a right triangle AOG, whose hypotenuse OG is the distance from the center to the focus. Then we simply use the tool that constructs a circumference, knowing the radius, with center in point O and radius distance [OG] and we have the two focuses determined. Thais (e-mail - 13/05/2011)

This solution results from the fruitful influence of the internet (information contained on the pages that had been consulted) in online mathematical knowledge production. This final solution can be understood as a relative slowdown in a mini-cyclone, because the desire to explore and build dynamic solutions with the software was abandoned. Another factor that should be considered is the time allowance (set in the rules, which, as we mentioned earlier, mediates the relationship of the subject with the community) because the desire to conclude the problems within the time period may have hindered the continuity of the work based on this experimental approach with technology.

Finally, as Thais emphasized, it is important to study, understand, and justify the solution found. This movement indicates that the mini-cyclone may have been nearing its end, because there was a search for arguments for the solution from the mathematical point of view.

6. Discussion of the results

We developed the following diagram (Fig. 7) to illustrate just how closely the movements realized by the activity system (mini-cyclone and intershaping relationship) are related, and at the same time, to illustrate the movements (or different functions) of the “actress” internet in the activity system as well.

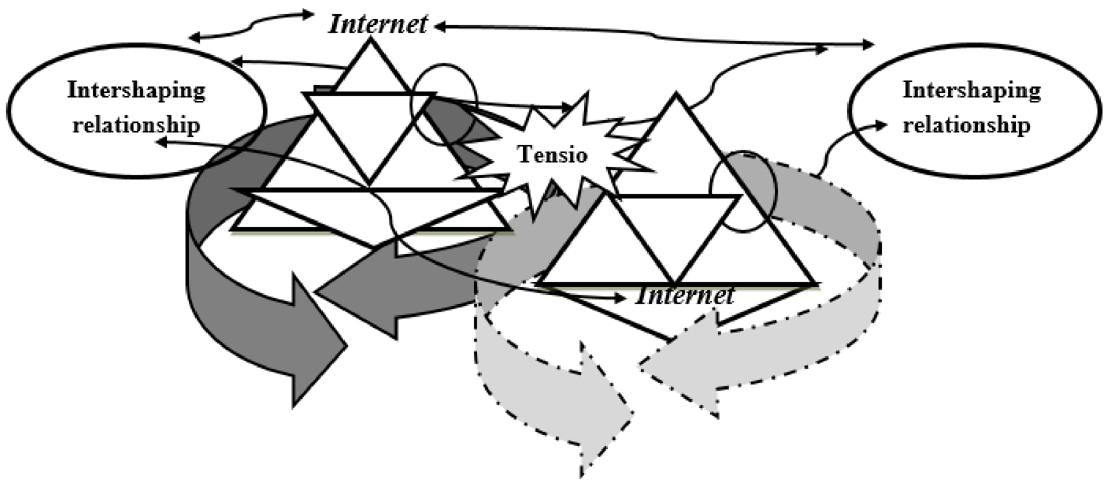

Figure 7. Diagram of movements of the internet, the mini-cyclone and the intershaping relationship

In Figure 7, each of the triangular representations corresponds to a particular moment in the development of the activity system. In the first on the left, the internet plays the role of an artifact and one sees the beginning of the mini-cyclone’s expansive transformation, represented by the two dark gray arrows on the left with a continuous contour. The mini-cyclone, discussed above, began with tension caused by the teachers’ difficulty in understanding the geometric representation of the parameters of the equation.

To the right and to the left of the figure we depict the role of the intershaping relationship(s) attached to an activity system, to a mini-cyclone, and to tensions. Here we want to highlight that in the evolution of an intershaping relationship, tensions arise that can lead to the beginning of mini-cyclones, which then contribute to the development of the system, this is, to the progression of the mini-cyclones. In addition, the presence of the intershaping relationship in two areas emphasizes that it is impossible to predict when such a relationship will occur or the direction a mini-cyclone will take.

The right triangular representation relates to the lighter gray arrows with dotted lines. This "multiplication" of the system shows the progress of the mini-cyclone. In this evolutionary process, we observed that the development of the mini-cyclone was influenced by the internet, because the ability to access information from the internet combined with the lack of time to find solutions to the problems transformed the motives of the activity system.

At the beginning of the study, the teachers' motive was not to reach a specific result, for example, a numerical one. Rather, the need that guided the work, the initial motive, was to create a solution to the mathematical problem within an online environment using software. There was, therefore, an inclination toward expansive transformation. However, when the teachers chose to use the internet as a search mechanism to revisit some mathematical concepts, we observed there was a change in the motive of the activity that, in our opinion, became this: to find a solution as quickly as possible. With this shift in motives, we observed a relative "stagnation" or slowed advance of the mini-cyclone, as compared to its previous momentum. To emphasize this process in the above diagram, we inserted arrows in dotted lines to represent the mini-cyclone and used a lightly shaded arrows to highlight the teachers' behavior the instant the internet came to play the role of a community.

To summarize, the internet played the role of an artifact, but as soon as the intershaping relationship process developed, tensions emerged and contributed to the development of the activity system. During this evolutionary process, the internet played the role of a community, but simultaneously also played the role of an artifact and contributed in changing the motive and developing a new system. Thus, there was an expansive transformation when the internet also became a community.

7. Final Considerations

In the conics study discussed above, the teachers initially produced dynamic solutions to utilize software to its fullest potential in order to understand the mathematical content, but only until they decided to "get help" on the internet. After this action, the constructions began to become static. This static aspect was similar to the ones found in the webpages they consulted.

We believe that these movements in the system suggest that the webpages (the source of the content consulted) transmitted some historically constructed norms to the system. These norms, which establish mathematics as exact, abstract, rigid and linear, interfered with the movements of the mini-cyclone's expansive transformations. This type of behavior suggests that internet has shaped the way the participant teachers produced mathematics.

Based on these considerations, we propose that the development of an activity system constituted an online learning environment that was influenced by social and cultural factors-rules, standards, ethical and moral values, etc.

From a theoretical point of view, we see that digital technologies, particularly the internet, contribute to possible expansions or new interpretations of theoretical perspectives involving the participation of non-human actors in the process of knowledge production. We base this analysis on the Activity Theory and on the humans-with-media construct. This construct allows for new perspectives on the Activity Theory, as it focuses on one kind of artifact: digital technologies.

The data analysis suggests that the “actress” internet can exist in different "vertexes" in the activity system diagram proposed by Engeström. In the case of this online course, we believe the internet was used as an artifact because the interactions occurred in synchronous or asynchronous moments as, for example, in chat rooms or in email exchanges. On the other hand, these same types of interactions, albeit at different times, might place the internet more appropriately in the community vertex of the diagram. Finally, it was possible to observe that the internet also influenced a change of motive of the activity. Based on the notion of humans-with-media, a similar argument can be made to place the internet in the subject vertex, as well.

We reinforce here the need to stimulate the development of a critical view regarding the information reproduced on the internet, including the mathematical ideas available online. We feel it would be wise to question them, criticize them, and (why not?) build or re-build them.

Returning to the question posed in the title of this article, we conclude there is no reversal of roles, but rather shared roles! This implies rule changes regarding mathematical production by non-human actors. It seems that the analysis we have developed “attaches new colors” to the statement that has been repeated over and over: the teacher no longer holds the status of being the sole knowledge transmitter. Internet plays such a role as well as an artifact, as a community, and as a subject. The rigid triangles usually presented in theoretical discussions have become dynamic. Whether this dynamicity is unique to digital artifacts (in particular to the internet, when one incorporates the collectives that produce knowledge) is a matter for another article, which can be written contrasting these data with the reflections of Aguilar and Kawasaki (2013). These authors discuss the alleged "rigidity" of the triangular representation, but with a special interest in the movements of the subjects (actors) of the activity.

However, we believe that this larger interaction of the humans-with-media construct and the third-generation Activity Theory can lead to the beginning of a fourth generation, where the anthropomorphizing of technologies can provide a dynamic model in a social environment.

These thoughts bring us to reflect on the human-with-media construct --about what it means to be human. Since human and non-human actors mingle when they play identical roles, there is no indication to distinguish the moment when one begins to take on the role of the other.

Acknowledgment

We thank the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior-CAPES by the financing of research that led to this article and National Council for Scientific and Technological Development (CNPq) - process 400590/2016-6 and 303326/2015-8.

References

Araújo, J. L., e Kawasaky, T. (2013). Movimento e rigidez de certo triângulo: um enfoque Histórico-Cultural de pesquisas em Educação Matemática. Em: C. R. Ferreira (Ed.), *Anais do Encontro Nacional de Educação Matemática: Retrospectivas e Perspectivas* (Vol 1, pp. 1-13). Curitiba, Brasil: Pontifícia Universidade Católica.

Baldwin, R. (1998). Technology's impact on faculty life and work. *New Directions for Teaching and Learning*, *77*(1), 7-21. doi: 10.1002/tl.7601.

Borba, M. C. (1993). *Students understanding of transformations of functions using multi-representational software.* (Tese de doutorado não publicada) - Faculty of graduate school of Cornel University, Ithaca, New York, United States.

Borba, M. C. (1999). Tecnologias Informáticas na Educação Matemática e Reorganização do Pensamento. Em: M. A. V. Bicudo (Ed.), *Pesquisa em Educação Matemática: Concepções e Perspectivas* (pp. 285 - 295). São Paulo, Brasil: Editora UNESP.

Borba, M. C. (2012). Humans-With-Media and continuing education for mathematics teachers in online environments. *ZDM Mathematics Education*, *44*(6), 1-14.

Borba, M. C., e Villarreal, M. V. (2005). *Humans-With-Media and the Reorganization of Mathematical Thinking*: *information and communication technologies, modeling, experimentation and visualization*. New York, United States: Springer.

Borba, M. C., Malheiros, A. P. S., e Amaral, R. B. (2011). *Educação a distância online.* (3ª ed.). Belo Horizonte, Brasil: Autêntica.

Borba, M. C., e Chiari, A. (2013). *Tecnologias Digitais e Educação Matemática.* São Paulo, Brasil: Livraria da Física.

Engeström, Y. (1987). *Learning by expanding: an activity-theoretical approach to developmental research*. [DX versão leitura]. Obtido de http://lchc.ucsd.edu/mca/Paper/Engestrom/expanding/toc.htm.

Engeström, Y. (1999). *Learning by expanding*: ten years after. [DX versão leitura]. Obtido de http://lchc.ucsd.edu/mca/Paper/Engestrom/expanding/toc.htm.

Engeström, Y. (2001). Expansive learning at work: toward an activity theoretical reconceptualization. *Journal of Education and Work*, *14* (1), 133-156.

Engeström, Y. (2002). Non Scolae Sed Vitae Discimus: como superar a encapsulação da aprendizagem escolar. Em: H. Daniels (Ed.) *Uma Introdução a Vygotsky* (pp. 175-197). São Paulo, Brasil: Edições Loyola.

Gracias, T. A. (2003). *A natureza da reorganização do pensamento em um curso a distância sobre Tendências em Educação Matemática* (Tese de doutorado não publicada). Universidade Estadual Paulista Júlio de Mesquita Filho. Rio Claro, Brasil.

Harasim, L. (1990). Online education: A new domain. In: R. Mason & A. Kaye (Eds.), *Mindwave: Communications, computers and distance education* (pp. 50-62). Exeter, England: Pergamon Press.

Jaccoud, M., e Mayer, R. A. (2008). A observação direta e a pesquisa qualitativa. Em: J. Poupart, J-P. Deslauriers, A. P. Pires, A. Lapparriére, R. Mayer, e L-H. Groulx (Eds.), *A pesquisa qualitativa: enfoques epistemológicos e metodológicos* (pp. 254-294). Petrópolis, Brasil: Vozes.

Kaptelinin, V. (2005). The object of activity: Making sense of the sense-maker. *Mind, Culture, and activity. An International Journal, 12*(1), 4-18.

Lévy, P. (1993). *As tecnologias da inteligência: o futuro do pensamento na era da informática*. Rio de Janeiro, Brasil: Editora 34.

Lincoln, Y., & Guba, E. (1985). *Naturalistic Inquiry.* Londres, England: Sage Publications.

Malheiros, A. P. S. (2008). *Educação Matemática Online: a elaboração de projetos de modelagem* (Tese de doutorado não publicada) Universidade Estadual Paulista Júlio de Mesquita Filho. Rio Claro, Brasil.

Mcdonald, J. (2002). Is "as good as face-to-face" as good as it get*s? Journal of Asynchronous Learning Network, 25*(10), 10-23.

Rosa, M., e Maltempi, M. V. (2010). A Construção do conhecimento matemático sobre integral: o movimento hipertextual em um curso utilizando o RPG online. Em: A. P. Jahn e N. S. G. Allevato (Eds.), *Tecnologias e educação matemática: ensino, aprendizagem e formação de professores* (pp. 25 - 44). Recife, Brasil: SBEM.

Santos, S. C. (2006). *A produção matemática em um ambiente virtual de aprendizagem: o caso da geometria euclidiana espacial* (Tese de mestrado não publicada). Universidade Estadual Paulista Júlio de Mesquita Filho. Rio Claro, Brasil.

Scagnoli, N. (2005). Impact of online Education on Traditional Campus-based Education. *International Journal of Instructional Technology & Distance Learning, 2*(10), 1-14.

Silva, M. (2003). *Educação Online.* São Paulo, Brasil: Loyola.

Souto, D. L. P. (2013). *Transformações Expansivas em um curso de Educação Matemática a Distância Online* (Tese de doutorado não publicada). Universidade Estadual Paulista Júlio de Mesquita Filho. Rio Claro, Brasil.

Souto, D. L. P., e Borba, M. C**.** (2013). Transformações expansivas em Sistemas de Atividade: o caso da produção matemática com a Internet. *Revista Perspectivas em Educação Matemática, 6*(1), 14-57.

Tikhomirov, O. K. (1981). The psychological consequences of the computerization. In: J. Werstch (Ed.), *The concept of activity in soviet psychology* (pp. 256 - *278). New York*, *United States: Sharp.*

Villarreal, M., e Borba, M.C. (2010). Collectives of humans-with-media in mathematics education: notebooks, blackboards, calculators, computers and ... notebooks throughout 100 years of ICMI. *ZDM Mathematics Educations*, *42*(1), 49-62. doi: 10.1007/s11858-009-0207-3.

Zulatto, R. B. A. (2007). *A natureza da Aprendizagem Matemática em um Ambiente online de Formação Continuada de Professores* (Tese de doutorado não publicada). Universidade Estadual Paulista Júlio de Mesquita Filho. Rio Claro, Brasil.