CHILDREN'S LENGTH ESTIMATION PERFORMANCE AND STRATEGIES IN STANDARD AND NON-STANDARD UNITS OF MEASUREMENT

DESEMPENHO E ESTRATÉGIAS DE CRIANÇAS PARA CÁLCULO DE COMPRIMENTO EM UNIDADES DE MEDIDA PADRÃO E NÃO PADRÃO

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ABSTRACT

This study aimed to examine elementary school children's performance and strategies when estimating linear measurements. 46 third-graders and 41 fifth-graders were asked to estimate the length of various objects in a series of tasks that looked at units of measurement and task context. The children's performance was quite poor and was influenced by age. Both age groups gave better length estimates when non-standard units of measurement were used rather than when centimeters (standard units) were used. Significant differences were revealed in estimating objects with different orientations, whereas no differences were found when estimating objects presented in different backgrounds or objects of three dimensions. Unit iteration and use of benchmarks were the most common strategies found, with differences in children's frequency use, however, depending on the presence of standard or non-standard units of measurement.

Key words: length estimation, measurement units, estimation strategies, elementary school.

RESUMO

Este estudo teve como objetivo analisar o desempenho e estratégias de crianças dos anos iniciais do ensino fundamental ao estimar medidas lineares. Pediu-se a 46 alunos do terceiro ano e 41 do quinto ano que estimassem o comprimento de vários objetos em uma série de tarefas que examinavam as unidades de medida e o contexto da tarefa. O desempenho das crianças foi bastante fraco e foi influenciado pela idade. Ambas as faixas etárias apresentaram melhores estimativas de comprimento quando unidades de medida não padronizadas foram usadas, em vez de quando foram utilizados centímetros (unidades padrão). Diferenças significativas foram reveladas na estimativa de objetos com diferentes orientações, ao passo que não foram encontradas diferenças na estimativa de objetos apresentados em diferentes origens ou objetos de três dimensões. A iteração de unidade e o uso de *benchmarks* (referências) foram as estratégias mais comuns encontradas, com diferenças na frequência de uso pelas crianças, no entanto, dependendo da presença de unidades de medida padrão ou não-padrão.

Palavras-chave: estimativa de comprimento, unidades de medida, estratégias de estimativa, anos iniciais do Ensino Fundamental.

1. Introduction

Over the last decades, mathematics education has shifted from the rote memorization of rules and formulas to the development of life skills. In this direction, estimation has been highlighted as a field of research and has received a lot of attention due to its practical nature and its importance in everyday life situations (Siegler & Booth, 2005). For example, everyone has to make decisions on a daily basis, regarding the products to buy according to their price, the clothes to wear according to the outside temperature or the waiting line to stand in, according to the number of people; these are all estimates.

Estimation is considered "the skill of making an educated guess as to the value of a distance, cost, size, etc., or arithmetic calculation" (Clayton, 1996, p. 87). The more often referred types of estimation in mathematics education literature are numerosity estimation (estimation of the number of objects), computational estimation (estimation of computations), measurement estimation (Hogan & Brezinski, 2003) and number line estimation (Siegler, & Booth, 2004), with the second type having received the biggest research attention of all (Sowder, 1992). Computational estimation is already included in the mathematics curriculum in many countries, whereas the other types of estimation need further investigation, for a better understanding of how people estimate. This paper focuses on measurement estimation.

Measurement estimation refers to the process of making a measurement for a particular object or task without using specific measurement tools (Bright, 1976, in Sowder, 1992. Hogan, & Brezinski, 2003). For example, estimating linear size involves measurement estimation in the sense of estimating the length of one-dimensional lines. Either standard quantitative units (such as a centimeter) or non-standard (novel) units that are usually derived from an individual's measurement experiences or frame of reference (such as body benchmarks) can be used as values for measurement estimation (Jones, Taylor, & Broadwell, 2009a; Joram, Gabriele, Bertheau, Gelman & Subrahmanyam, 2005). Even though findings concerning measurement estimation are very limited, it seems that difficulties in measurement estimation could be attributed to some of its particular aspects, such as its dependence upon accurate measurement competence (Sowder, 1992) or its being affected by the specific objects used when estimating (Joram, Subrahmanyam & Gelman, 1998). Additionally, because measurement is used to determine dimensions (length, width, height) and various units (e.g., area, volume and capacity, weight and mass, time) its estimation covers a quite heterogeneous content (Hogan & Brezinski, 2003). This study focuses on the case of length with great interest in estimating the length of different objects. In many countries, precise length measurement is systematically being taught at primary school as part of their mathematics curriculum. Whereas the importance of precise length measurements is recognized, there are not many research findings concerning the role of length estimation and how young children's competence is developed in this domain.

2. Theoretical and empirical background

2.1. Length estimation

In the last thirty years, mathematics educators in many countries showed a great interest in children's and adults' measurement ability and measurement estimation ability. Whether precise measurement computation should still keep its traditional leading role was a matter for debate, as its importance in everyday life diminished drastically since measurement estimation was revealed to be more meaningful, as measuring devices may not always be available or even necessary. Accordingly, in addition to the acquisition of routine computation expertise in many curriculums in different countries, great importance was attached to the acquisition of the ability to solve estimation tasks efficiently based on an individual repertoire of adaptively applicable strategies (see Joram et al., 1998; and Hogan & Brezinski, 2003, for an overview of the research interest from precise measurement computation to measurement estimation).

The ability to estimate length depends on understanding how continuous length measurement can be broken down into segments. As Joram et al. (1998) suggest, individuals who make length estimations need to count the number of smaller units into which an object is divided. For example, when estimating the length of a pen, an estimator may divide the total length of a pen into inches and count the number of inches. That division of an object into units is quite demanding and may explain why length estimation is difficult for many people (Joram et al., 2005). It has also been suggested that the ability to estimate length is related neither directly to number calculation (Dehaene, 1997) nor to general mathematical ability (Hogan & Brezinski, 2003). However, it appears to be closely related to experiences in length measurements and thus may be context bound (Harel & Sowder, 2005). For example, professionals, such as adults working in apparel design, make linear measurement estimations (e.g., estimating the diameter of a button or the length of a zipper) effectively (Workman & Ahn, 2011) and often invent novel tools to assist in these approximations (Jones & Taylor, 2009). Similarly, when a dermatologist explains to a patient about a mole that (s)he might have (e.g., saying that 'it is 6mm'), a more meaningful reference point (e.g., 'it is about the size of a pencil eraser') might be more helpful (Joram et al., 2005). Thus, estimation skills can be stimulated from experiences when establishing standard reference measures based on familiar objects.

Perceptual abilities such as spatial visualization may play an important role in length-measurement estimation. Per Joram et al. (1998), when estimating the length of an object, besides proficiency in unit iteration which has already been mentioned as necessary, the estimator must recall an image of the unit (if not present) or construct an image if the unit is present and repeatedly compare that image of the unit to the object being estimated. For instance, fourth-grade children were found to be able to mentally manipulate images and their spatial visualization skills were related to estimation and predicted success in estimation accuracy (Joram et al., 1998). Thus, skills involving mental visualization of a measuring tool are considered important.

2.2. Length estimation performance and strategies

Although there is a broad consensus among researchers on the importance of estimation performance as well as the strategies individuals use when estimating, empirical results indicate that students and adults do not always achieve the desired level of estimation ability and usually favor three main estimation strategies (Joram et al., 1998): a) *Unit iteration*, where the estimator mentally "measures" the to-be-estimated (TBE) object by iterating a mental image of a standard unit over it, b) *Decomposition/Recomposition*, where the estimator mentally decomposes the TBE object into smaller parts, estimates the length of each part and then adds or multiplies these lengths to reach the final result, and c) *Reference point/Benchmark* strategy, where the estimator uses a mental image of a non-standard unit (e.g., a pencil which is known to be 20 cm long) and iterates it over the TBE object. Indeed, the use of reference points or benchmarks (e.g. pencils, rubbers, width of one's thumb, etc.) as a measurement estimation strategy has been highlighted as more meaningful to the estimators than the use of standard units of measurement (Joram et al., 2005, Joram et al., 1998). Therefore, researchers focused on the use of the reference point strategy for measurement estimation and its relation to measurement estimation performance.

Because reference points allow individuals to model the process of unit translation, the main focus of using the reference point strategy is to develop more accurate representations of linear measurement units. The reference point instruction was supported by Joram and colleagues (2005) in their quasi-experimental design study with third-grade primary school students in the U.S.A. who were assigned to a reference point group - in which instruction was provided on use of the reference point strategy - and a comparison group. Results showed that the use of the reference point strategy was statistically associated with greater estimation accuracy: children who used the reference point strategy had enhanced representations of linear units that, in turn, led to more accurate estimates, compared to children in the comparison group. Using the body as a measurement tool was also revealed by Jones, Taylor and Broadwell (2009b), who reported that American visually-impaired secondary school students performed better than their sighted peers when estimating scales. These students tended to use the body as unit of measurement, in other words as a 'mental ruler', a term first described by Clements (1999). This allowed them to better visualize a measure and made them more accurate in estimation. In addition, these researchers saw measurement estimation as a direct teaching and learning link to formal measurement and implied that estimation ability may form the foundation of other mathematical concepts such as fractions and ratios (Joram et al., 2005) and proportional reasoning (Jones et al., 2009b). They proposed that good insight into measurement estimation performance also forms a basis for gaining insight into mathematics in a more general sense. Those internal connections between the various domains help children increasingly view mathematics as a coherent thread that runs through everyday life.

The issue of reference points and their use at the primary level was revealed by Buys and De Moor (2008), who supported that primary school children usually construct a 'system of personal reference measures' (p. 20), that is 'a series of measures obtained from everyday reality', to develop an understanding of a measurement system. For example, in estimating length, the width of a finger or a big step might be used as reference measures for the centimeter and the meter, respectively. For the number of times a natural measurement unit fits in or alongside a given object, a number is provided, which helps children gain better insight of the process of length estimation. These personal referents not only give meaning to the traditional units of measure - which they replace- but also help children develop a measure sense for length. They reflect the influence of both personal and shared experiences on the development of an estimator's length-estimation performance. It is this latter perspective that researchers draw upon when they propose the reference points strategy for teaching measurement estimation. For example, having conducted an interventional study, Hagena (2014) concluded that it is possible to foster students' linear measurement estimation performances through instruction on developing accurate benchmarks of linear measurement units and using these benchmarks for estimation in a short period.

In an effort to clarify the nature of the measurement units employed by high school students during a length estimation process, Gooya, Khosroshahi and Teppo (2011) conducted an exploratory study in Iran. When they asked 16-year-old girls to estimate the height of their school building, as well as the height of a pine tree outside their schoolyard, they identified three different kinds of 'individual frames of reference', as they called them. In particular, students were found to use mental images (e.g., images of a meter length), made comparisons with physically present objects (e.g., the length of a student's hand), and applied prior knowledge (e.g., they knew the fact that one-story building is about 3 m high). Interestingly, Gooya et al. suggested that students that tended to give good estimations also possessed good individual frames of reference and those who demonstrated an inability to employ such references usually gave unsuccessful estimations. Their findings indicated a complex relationship between the students' preference for specific reference points in an estimation activity and the physical context in which the estimation activity takes place: the context in which an estimation task is presented may influence the choice of referent that is selected. Thus, the variation in the referents that are available when estimating may lead individuals to use many different aspects of that context as non-standard units of measure.

The context of the estimation activities and its relation to the estimation performance also concerned Jones, Gardner, Taylor, Forrester and Andre (2012) in their study with American middle-school students who were asked to estimate the length of various objects in different contexts. Their results showed that students had difficulty estimating known objects with accuracy and that the context of the task influenced the students' linear estimation accuracy. Although the overall level of estimation accuracy was found to be low, students scored higher when they had to estimate with non-standard units of measurement, compared to when they were asked to estimate using metric units. Significant differences were observed in estimation accuracy for two- and three-dimensional estimation tasks, with students giving more accurate estimations when estimating the length of one side of a three-dimensional object. Authors also proposed that estimation accuracy for linear distances was correlated with logical thinking.

While previous research has mainly looked at adults' and middle-school children's length estimation performance and strategy use, issues concerning different types of units of length measurement and estimation performance at younger age ranges are less common. Furthermore, there is limited research that looks at factors that may contribute to more accurate length measurement estimation abilities (such as object orientation, spatial dimensionality) and the relationship of these factors to effective strategy use in primary school years.

2.3. Purpose of the study

Considering the literature outlined above, the aim of this study is to investigate the performance and strategies of primary school children when estimating linear measurements. The study was performed with third-grade and fifth-grade children who had not been taught about length measurement estimations in school. The following four specific research questions were investigated: a) How do 8- and 10-year-old children estimate the length of objects? What is the reasoning they follow?, b) Is children's length estimation performance affected by different types of measurement units (standard and non-standard units of measurement)?, c) Is children's length estimation performance affected by factors related to the task context (object orientation, spatial dimensionality, size of objects)?, and d) Is there a relationship between the length estimation performance of young children and the effective use of length estimation strategies?

Following the work of Jones et al. (2012) and Joram et al. (2005), it is hypothesized that the performance of children in length estimation situations with the use of non-standard measurement units will be better than in situations where they are asked to perform length estimation using standard measurement units. Furthermore, it is hypothesized that the children's procedures will differ, depending on issues concerning task context, because there may be particular factors that can be related to good length estimations more naturally than other factors. Although some research has dealt with the use of the reference point strategy for measurement estimation (Joram et al., 2005), there have been no comparisons between young children's use of such a strategy in length estimation situations in which different types of measurement units can be used. Yet in many studies (e.g., Joram et al., 2005, Jones et al., 2009a) the traditional teaching practice is to relate standard units of measurement to familiar objects, almost exclusively to teach measurement estimation to children. There seems to be an implicit assumption that this is the easiest situation for estimating the length of objects; however, there may also be other important components that need to be examined when estimating length with the use of standard and non-standard units of measurement.

3. Method

3.1. Participants

Forty-six third-grade children (26 boys and 20 girls, mean age: 8 years and 8 months) and forty-one fifth-grade children (24 boys and 17 girls, mean age: 10 years and 9 months) participated in the study. They were all recruited from state primary schools in urban areas of the city of Thessaloniki, Greece, after principals' permission and parental consent. They came from similar middle socio-economic status families and covered a range of academic performance levels in school mathematics. They were randomly selected from their class list and they had not been taught about length estimation in school[1], although the words 'about' and 'approximately' may have been familiar in other social settings.

3.2. Design

For the purpose of the study, data from eight length estimation tasks common across all participants (i.e., samples from two age groups) were completed. The tasks, which aimed to cover four main elements that may influence children's length estimation performance, were inspired by the research of Jones et al. (2012) and build on their research tool on length estimation. Each task consisted of two items, thus, 16 items (4 elements, two tasks for each element, two items in each task) were presented to all the participants. In each task item, children were asked to estimate the length of a particular object in two types of measurement units: centimeters (standard unit) and a novel unit (nonstandard unit). The novel units consisted of familiar objects such as pencils, straws and paperclips. Half of the objects shown in all tasks were short and the other half were long in length that ranged from 1 to 30 cm and from 65 to 100 cm for short and long items, respectively. Both standard and nonstandard units were shown to the participants at the beginning of each task. Table 1 shows a brief description of the tasks used in the present study.

Tasks 1 and 2 were used in order to examine the influence of *object orientation* on estimation performance: participants were presented with one wooden rod at a horizontal orientation (Task 1) and vertical orientation (Task 2) and they were asked to estimate its length in centimeters and pencils.

[1] In Greek mathematics curriculum, teaching length measurement and its units starts in Grade 2 with children being introduced only to standard measurement units, that is centimeters.

Table 1

Description of the tasks used in the present study

Category	Task Description	Material	Measurement Units Standard	Non-Standard
Object Orientation	Task 1. Horizontal orientation Task 2. Vertical orientation	Rod of 20 cm Rod of 90 cm Rod of 30 cm Rod of 100 cm	Centimeters	Pencils
Visual interference	Task 3. White background Task 4. Background with a complex pattern	Line of 15 cm Line of 80 cm Line of 25 cm Line of 85 cm	Centimeters	Straws
Spatial dimensionality	Task 5. Three-dimensional object Task 6. Two-dimensional image	Photo frame of 18cm Rope of 80 cm Rubber of 5cm Belt of 65 cm	Centimeters	Paperclips
Representation of standard measurement units	Task 7. Draw a line Task 8. Indicate an object	Line of 10 cm Line of 70 cm Object of 10 cm Object of 70 cm	Centimeters	

Tasks 3 and 4 were used in order to assess the influence of *visual interference* on estimation performance: participants were shown one black line drawn on a white background (Task 3) and on a background that had a complex pattern that made the visual isolation of the line difficult (Task 4). Then, they were asked to estimate the black line's length in centimeters and straws.

Tasks 5 and 6 were used in order to investigate the influence of *spatial dimensionality* on estimation performance: participants were presented with one real-world three-dimensional object (Task 5) and an image (two-dimensional) of one real-world object (Task 6). They were asked to estimate the length of one side of the three-dimensional object (Task 5) and the length of the two-dimensional image (Task 6) in centimeters and paperclips.

Tasks 7 and 8 were used in order to examine children's *representations of standard units of measurement*: participants were asked to draw a line of particular length (Task 7) and indicate a real-world object of particular length from the available present objects in the room (Task 8). Whereas the length of the items used in all previous tasks differed across tasks, in tasks 7 and 8 it was kept constant in order to examine whether children's representations of standard measurement units are affected by the type of object (line or real-world object).

To avoid order effects in children's responses, the order of presentation (tasks 1-8 and tasks 5-8 and 1-4) was varied systematically across tasks.

3.3. Procedure

All children were interviewed individually by one of the authors in a quiet place inside their school and their responses were recorded. Prior to beginning the tasks, all children were provided with the opportunity to familiarize themselves with the experimenter. They were assured that there is no 'right' or 'wrong' way to estimate the length of an object, and that their estimates would by no means be used for evaluation of any kind. Participation was completely voluntary and anonymity was guaranteed.

The instructions were presented orally. No feedback was provided about any of children's responses, though the experimenter periodically encouraged children by offering general praise for doing a good job. There was no restriction on the length of time that the children were given to respond. However, the procedure lasted roughly 20-25 minutes.

The authors initially administered the protocol jointly to develop consistency. Additionally, a subset of the children were selected at random and coded independently; the resulting inter-rater reliability was .86.

3.4. Data coding

Since the tasks used in this study aimed to gauge the children's estimation performance on length measurement, a lenient method of scoring -rather than a strict one- was used. Within this lenient scoring, an approximate but incorrect answer was considered as correct. Following the criteria used in some studies of adults' and children's estimation (e.g., Dowker, 1992. Dowker, 1997), reasonable estimates are defined as those that are within 30% of the correct answer. In this study, any reasonable estimate that allowed an estimation error of 30% was considered a successful response. For example, if an object's length was 100 cm, correct responses were found in the range of 70-130 cm. For the statistical analysis presented in this paper, 1 point was attributed if the child gave a correct response that falls within the range that was computed as described earlier. Unsuccessful responses were those whose estimation error exceeded 30% of the correct answer and, thus, were assigned 0 points.

In addition to their length estimates, participants were asked to justify their answers, independently of whether they were correct or not. These justifications revealed the strategies they employed and were classified on the basis of what reasoning they used in order to estimate the length of the object shown, giving seven distinct and ordered categories which are explained in a separate section below.

4. Results

4.1. Rates of correct responses

The children’s estimation performance was remarkably poor, with success rates of approximately 45% for third-graders and 53% for fifth-graders. However, this difference between the two age groups was found to be statistically significant (t=-2.357, df=85, p<.05). No statistical significant gender differences were observed in the children’s performance (t=.976, df=85, p=.332) with boys and girls presenting similar percentages of successful estimates. The order of task presentation did not affect the children’s estimation performance (t=1.826, df=85, p=.171): no significant differences were found between the scores of children who were initially presented with tasks 1-4 and later with tasks 5-8, compared to those who were initially presented with tasks 5-8 and later with tasks 1-4.

Length of objects. A three-way mixed model ANOVA was conducted to analyze the effects of age (Grade 3 and Grade 5) and gender (boys and girls) as the between-subjects factor, and length of the objects (short and long objects) as the within-subjects factor. The main term of object length was not significant (F(1,83)=3.172, p=.079), with all the participants presenting similar percentages of correct responses when estimating the length of short and long objects. There was a significant main effect of age (F(1,83)=3.079, p<.05), indicating that the fifth-graders performed significantly better than the third-graders. No significant gender effect was found (F(1,83)=.467, p=.496). Neither the interaction between age and length of the objects (F(1,83)=.067, p=.797) nor the one between gender and length of the objects (F(1,83)=.170, p=.681) nor the interaction between age, gender and length of the objects was significant (F(1,83)=1.028, p=.314), demonstrating that the children’s similar performance in estimating short and long objects was not affected by age and gender. This means that both age groups and both gender groups performed similarly in short and long objects. Figure 1 shows the interaction between age and length of the objects.

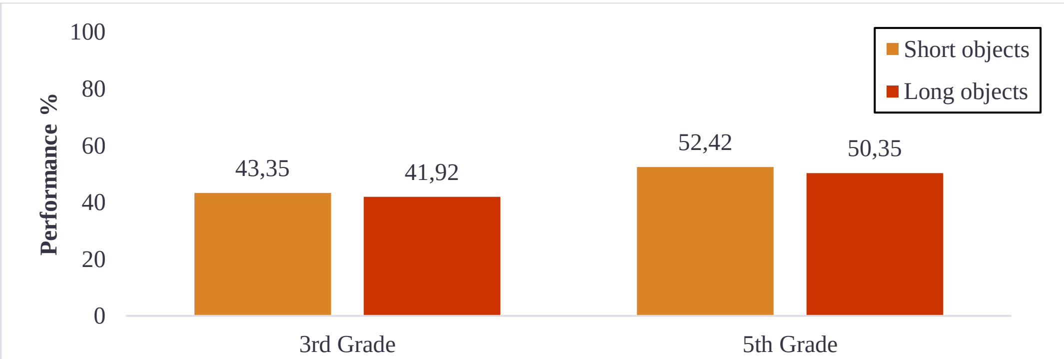


Fig. 1. Percentage of correct responses by age and length of objects

Types of measurement units. The number of correct responses was subjected to a mixed model Analysis of Variance in which age (Grade 3 and Grade 5) and gender (boys and girls) were the between-subjects factors and units of measurement (standard units and non-standard units) was the within-subjects factor. The analysis showed a significant main effect of units of measurement (F(1,83)=26.810, p<.001), indicating that all children’s length estimates were significantly better when non-standard units of measurement were used rather than when centimeters (standard units) were used. These differences were also confirmed for each age group separately (t=-2.375, df=45, p<.05

and t=-5.124, df=40, p<.001, for 3rd and 5th grade children, respectively) as well as for each gender group separately (t=-3.155, df=49, p<.01 and t=-4.085, df=36, p<.001, for boys and girls, respectively). The interaction between age and units of measurement was not significant (F(1,83)=2.620, p=.109), showing that the difference between standard and non-standard units of measurement was the same for both age groups. However, further analyses revealed that significant age differences in the children's performance were found when non-standard units were used (t=-2.658, df=85, p<.01, 51.25% compared to 65% for 3rd and 5th graders, respectively), but not when centimeters were applied (t=-0.309, df=85, p=.758, 38.58% compared to 40.25% for 3rd and 5th graders, respectively). The interaction between age and units of measurement is presented in Figure 2.

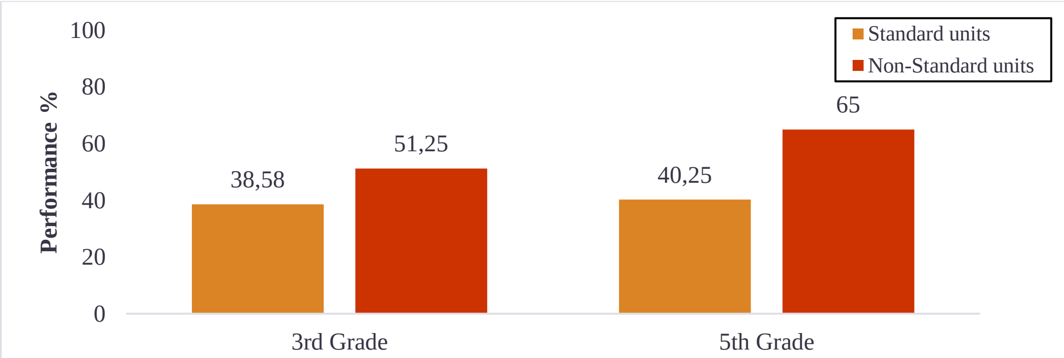


Fig. 2. Percentage of correct responses by age and units of measurement

The paired-samples t-test was performed to examine whether the use of the two types of measurement units had an effect on the children's estimation performance on problems with short objects and on problems with long objects. Significant differences were found between the scores of children when standard and non-standard units of measurement were used for problems asking for the length estimation of short objects (t=-9.027, df=86, p<.001), but not for problems involving the estimation of long objects (t=-.522, df=86, p=.603). Children performed significantly better in problems with short objects when non-standard units were used, showing that the use of non-standard units was favorable to problems with short objects only. Such a difference was not observed in problems with long objects, in which children performed similarly, regardless of whether they used standard or non-standard units of measurement. This is illustrated in Figure 3.

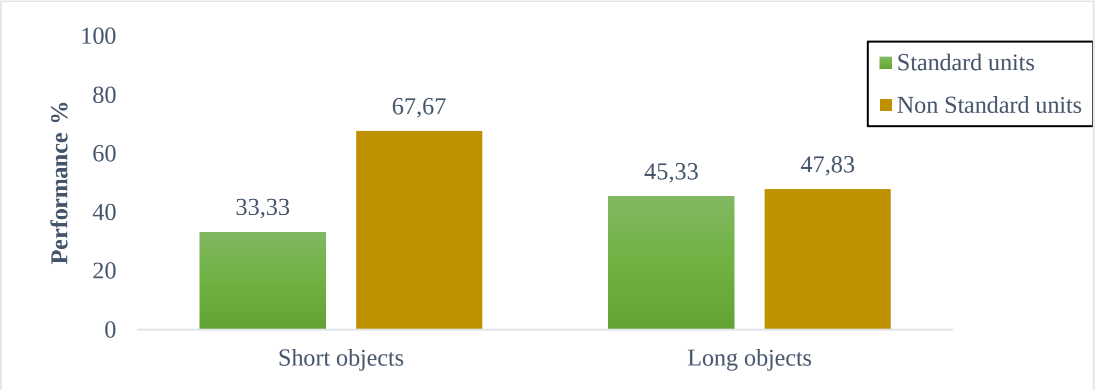


Fig. 3. Percentage of correct responses by units of measurement and length of objects

When further analyses were carried out for each age group separately, it was found that there were significant differences in the performance of children who used standard and non-standard units of measurement in problems with short objects (t=-5.291, df=45, p<.001 and t=-8.020, df=40, p<.001 for 3rd and 5th grade children, respectively). The two types of measurement units did not play any role in the children's performance in problems with long objects (t=.756, df=45, p=.454 and t=-1.663, df=40, p=.104 for 3rd and 5th grade children, respectively). However, when comparing children's performance in problems with short objects and problems with long objects in each type of measurement unit (Figure 3), it was found that children performed significantly better with short objects (67.67% compared to 47.83% in long objects) when their estimations were asked in non-standard measurement units (t=5.571, df=86, p<.001) and with long objects (45.33% compared to 33.33% in short objects) when centimeters were employed (t=-4.191, df=86, p<.001).

Categories of Tasks. The mean number of correct responses was subjected to a mixed Analysis of Variance in which age (2: Grade 3 and Grade 5 children) and gender (2: boys and girls) were the between-subjects factors and category of task (3: Object orientation, Visual interference and Spatial dimensionality[2]) was the within-subjects factor. The main term of category of task was significant (F(1,83)=12.582, p<.01), indicating that the children performed significantly better in the Object orientation tasks (approximately 56%) than in the Visual interference and Spatial dimensionality tasks (approximately 43% and 48%, respectively). Specifically, the children's performance in the Object orientation tasks was significantly better than in the Visual interference tasks (t=5.967, df=86, p<.001) and in the Spatial dimensionality tasks (t=3.558, df=86, p<.01). Moreover, the children's performance in the Spatial dimensionality tasks was significantly better than in the Visual interference tasks (t=-2.081, df=86, p<.05).

The interaction category of task by age was significant (F(1,83)=4.180, p<.05). Further analyses showed that the 3rd grade children found the Object orientation tasks significantly easier than the Visual interference and the Spatial dimensionality tasks (t=3.803, df=45, p<.001 and t=4.031, df=45, p<.001, respectively) with their rates of

[2] This analysis is carried out without data from children's performance in the fourth category of Tasks, that of Representation. The reason for this is that in this category only standard units of measurement were used and, thus, the maximum number of correct responses is not the same (mx=4) as in the other three categories of tasks in which standard and non-standard units of measurement were used (mx=8).

correct responses in the last two categories of tasks being very similar (t=.174, df=45, p=.863). On the contrary, the 5th grade children had their poorest performance in the Visual interference tasks compared to the other two categories of tasks (t=4.749, df=40, p<.001 and t=-3.438, df=40, p<.01, for Object orientation and Spatial dimensionality tasks, respectively) in which their performance was similar (t=.952, df=40, p=.347). The children's performance in the three categories of tasks by age is presented in Figure 4. The two-way interaction category of task by gender was not significant (F(1,83)=.508, p=.478). The three-way interaction category of task by age by gender was also not significant (F(1,83)=.114, p=.737).

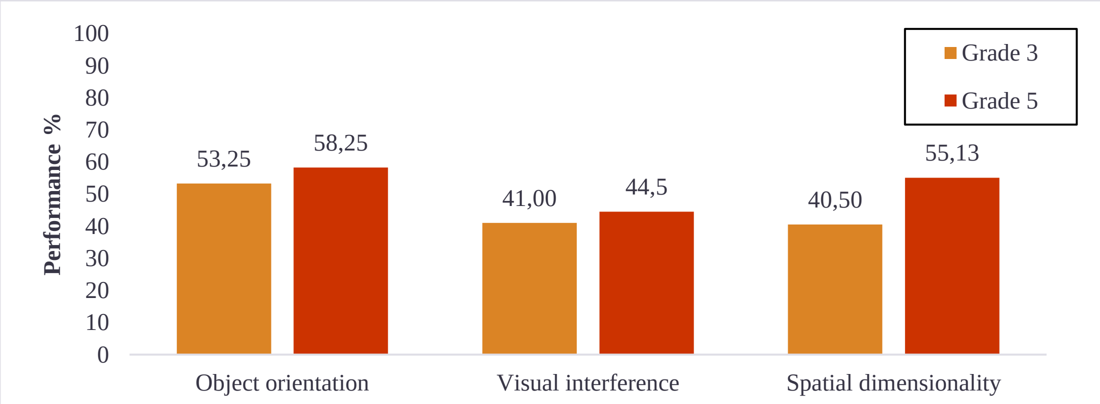


Fig. 4. Percentage of correct responses by age and categories of tasks

As mentioned earlier, the children's estimation performance across the four categories of tasks can only be examined with relation to the use of standard units of measurement. Paired-samples t-tests showed that there were not significant differences in the children's percentage of correct responses in the category of Representations tasks compared to the categories of the Object orientation tasks (t=1.926, df=86, p=.057), the Visual interference tasks (t=1.133, df=86, p=.260) and the Spatial dimensionality tasks (t=-.226, df=86, p=.822). In other words, the percentage of children who gave correct length estimations in the Representation of standard units of measurement did not differ significantly with the other three categories of tasks when length of the objects was asked in centimeters.

The children's performance in each of the first three categories of tasks was also analyzed in relation to the use of standard and non-standard units of measurement. It was found that asking for an object's length estimation in centimeters or in novel units of measurement influenced the children's performance for the categories of Object orientation and Spatial dimensionality (t=-6.021, df=86, p<.001 and t=-5.602, df=86, p<.001, respectively), whereas it did not affect the children's performance in the category of Visual interference (t=-,804, df=86, p=.424). Differences concerning the children's better estimation performance in novel units rather than in centimeters were also found when analyses were conducted separately for each age group. These differences are illustrated in Figures 5 and 6 for 3rd and 5th grade children, respectively.

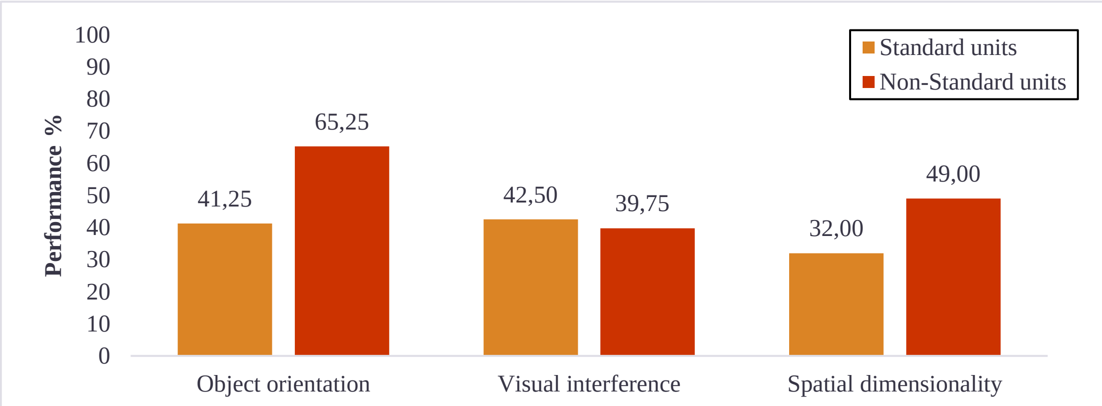


Fig. 5. Percentage of correct responses in each category of tasks by units of measurement for 3rd Grade children

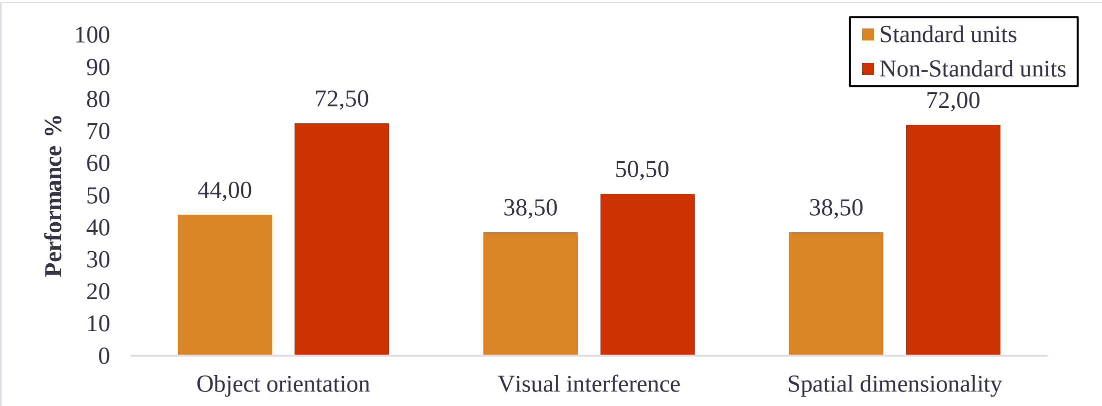
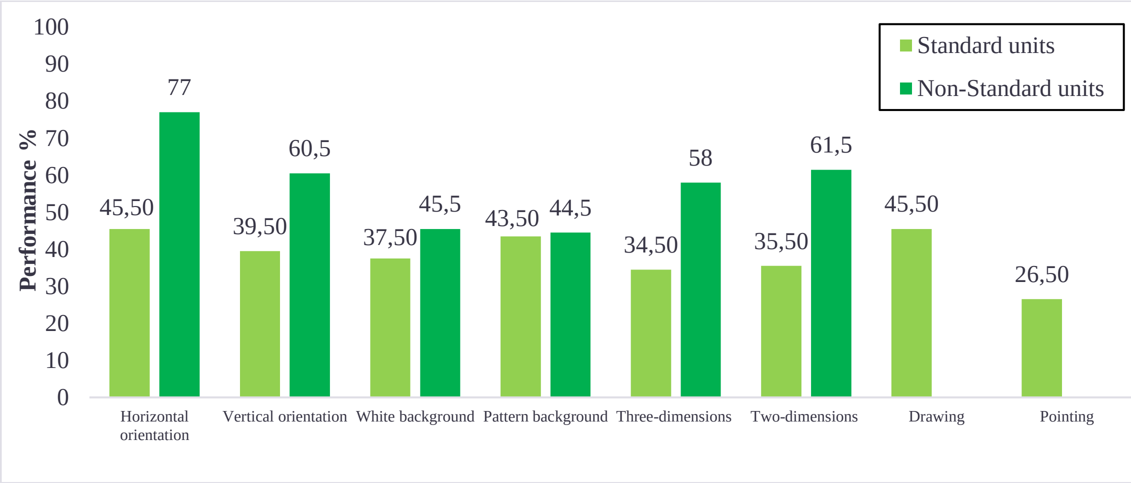


Fig. 6. Percentage of correct responses in each category of tasks by units of measurement for 5th Grade children

When analyses were carried out separately for each task within each category of tasks, better length estimations were mostly observed when the problems required the use of non-standard units of measurement. More specifically, children performed significantly better when estimating the length of an object in horizontal ($t=-6.030$, $df=86$, $p<.001$) as well as in vertical orientation ($t=-3.974$, $df=86$, $p<.001$) using non-standard units of measurement compared to using standard units of measurement. Similar differences were observed for estimating the length of three-dimensional objects ($t=-4.625$, $df=86$, $p<.001$) and two-dimensional images ($t=-4.777$, $df=86$, $p<.001$). No such differences were found when a white background ($t=-1.394$, $df=86$, $p=.167$) or a background with a complex pattern was used ($t=-.097$, $df=86$, $p=.923$). Figure 7 shows the percentage of correct length estimations in each task by units of measurement.



Object Orientation Visual Interference Spatial Dimensionality Representations

Fig. 7. Percentage of correct responses in each task by units of measurement

The type of orientation in the Object orientation category influenced the children's estimates when non-standard units were used (t=4.597, df=86, p<.001), with children performing significantly better when estimating the length of objects in horizontal rather than in vertical orientation (success rates of 77% and 60.5%, respectively).

Representation of Standard Measurement Units. Paired-samples t-test showed that all children provided significantly more accurate length estimations when they were asked to draw a line of particular length (45.5%) rather than when they were asked to indicate an object of particular length (26.5%) (t=3.965, df=86, p<.001). Similarly, drawing lines of given centimeters was found to be significantly easier than pointing to objects of particular length in centimeters for both 3rd and 5th grade children (t=2.904, df=45, p<.01 and t=2.740, df=40, p<.01, respectively). No other significant comparisons were found.

4.2. Children's justifications

All participants were asked to explain their reasoning, independently of whether they had produced a successful length estimate. Their justifications revealed their problem-solving strategies, which were classified into the following seven distinct and ordered categories:

1. *Idiosyncratic responses*. Children who gave responses that do not indicate any justified strategy (such as *'it seems so', 'I said it off the top of my head', 'I thought about it in my head')* and/or judged estimates by sight *('I saw it*', *'I used my eyes', 'I have a good eye*') were included in this type of justification. For example, a 3rd Grade boy estimated that the length of the rope in a picture (Task 5b) is about 20 cm and said '*I just looked at it and knew'*.
2. *Estimation based on the object's size.* This type of justification involves estimating an object's length based on its size (e.g., *'short', 'medium', 'long',*

'huge') without referring to any other characteristics. For example, a 5th Grade girl said that the line shown in Task 3a is about 10 cm, because '*it is medium'*. When asked to estimate the length of a rubber in paperclips (Task 6a), a 3rd Grade girl said: '*The rubber is about 2.5 paperclips long, because each paperclip is big and the rubber is big, too*'.

3. *Estimation based on previous knowledge and comparison.* This type of justification is offered by children who recall a past situation in which they made estimates, use information they had about the object they had estimated and compare it with the object they are now asked to estimate. For example, a 5th Grade girl said that the rod (in Task 2b) is about 100 cm, because '*I have seen pieces of wood like that in big stores and they were measured as being 100 cm*'. Similarly, when estimating the same rod in pencils a 3rd Grade boy said that it is about 12 pencils long and explained: '*At my dad's work, I've put pencils in a row and counted such pieces of wood*'.

4. *Use of benchmarks.* The children who used mental benchmarks or reference points (i.e. other objects or body parts) as important units for length estimation were classified as having used this type of justification. For example, when estimating the length of the rod (Task 2b) in centimeters, a 3rd Grade boy said: '*Spreading both my arms* (benchmark) *is 1 meter wide,…, I know, I have measured it. This rod is a bit longer, it's about 106 cm*'. When estimating the rod (Task 2a) in pencils, a 3rd Grade girl said: '*The rod I saw earlier* (she is referring to the rod shown in Task 1a) *is half of this rod in size* (benchmark), *that was approximately 1 pencil long, so this one is 2 pencils long*'.

5. *Unit iteration.* The children who showed this justification identified a unit they knew (e.g., 30 centimeters on a ruler) and then mentally applied that unit to the length of the object they were asked to estimate (e.g., counted per 30 cm). For example, a 3rd Grade girl said that the rod (in Task 1b) is about 70 cm, because '*up to here it's 30 cm long, another 30 cm is up to here and 10 cm more*'. With regards to length estimates using non-standard units, only children whose eye movements showed that they did repeat the non-standard unit mentally to the length of the object were included in this type of justification. For example, when a 3rd Grade boy was asked to estimate the length of the line (Task 4b) in straws, he was classified as having used this type of justification: '*it is approximately 6 straws, I know it, I just put them in a row'*.

6. *Use of subdivisions.* This type of strategy indicates the children's ability to break the TBE object mentally to be estimated down in smaller parts, estimate the length of each part and then count the estimated lengths of the parts to estimate the length of the whole object. The children who used this type of strategy found it easier to estimate shorter parts of a more manageable length than to estimate the whole length as one. For example, to estimate the length of the photo frame (Task 5a) in centimeters, a 3rd Grade boy divided the length of the photo frame in two segments and then estimated the length of one of the segments: '*To the middle of the photo frame is about 5 cm, so the whole is 10 cm long*'. Similarly, when estimating the length of the line (Task 4a) in straws, a 5th Grade girl said that it is approximately 3 straws long and explained: '*Half of the line is 1.5 straw long, so it is 3 straws for the whole line*'.

7. *Computations.* This type of justification was only found when length estimates were asked in non-standard units. Children who gave this type of strategy estimated both the length of the object shown and the length of the non-

standard unit in centimeters and then divided the two lengths in order to find the number of the non-standard units they are going to use. For example, a 5th Grade boy found that the belt (in Task 6b) is approximately 10 paper clips long after having produced the following computations: '*The belt is approximately 50 cm long and each paperclip is approximately 5 cm long. I divide 50 with 5, it's 10 paperclips long*'.

4.3. Children's use of the justifications

Table 2 shows the mean number of the children's responses in the seven types of justifications with respect to the units of measurement used (standard and non-standard units of measurement). In the problems asking for length estimates in standard units of measurement, the children used all types of justifications: although they offered the unit iteration type of justification (mn=4.25) mainly, their estimates were also based on the use of benchmarks (mn=2.94), the object's size (mn=1.63), the use of subdivisions (mn=.59) and their past knowledge (mn=.34). In contrast, when the problems asked for non-standard units of measurement, the mean number of children who gave the iteration type of justification was significantly higher (mn=7.71), with the other types of justifications almost being non-existent, except for the idiosyncratic responses that were the second most common type of justification (mn=2.39). All comparisons made for the children's use of each type of justification across the two types of unit measurement were significantly different. The only pair that did not present a significant difference was the use of idiosyncratic responses in problems asking for standard units of measurement and problems requiring non-standard units of measurement (t=-.358, df=86, p=.721). In other words, this type of justification was the only one in which the type of measurement units did not seem to make a difference to the children's reasoning. Overall, the analysis of the children's justifications indicates that a wide variety of strategies were revealed when estimates were required in standard units of measurement, whereas the children's strategy use in estimates with non-standard units was quite unilateral.

A more in-depth analysis of the children's justifications showed that there were differences between age groups, concerning the types of problem-solving strategies used. Almost half of the 3rd graders justified their answers using unit iteration (45%), whereas there were only a few who reasoned based on the object's size (16%) or on factors which did not show any logical explanations (justification 1, 25%). The 5th graders differed in their justifications from the younger children. A few reasoned by developing and using benchmarks for their length estimates (22%), whereas over 55% of them demonstrated the unit iteration strategy. It seems that the third-graders used a range of strategies, whereas the fifth-graders based their estimates on fewer and more sophisticated strategies, mainly using the unit iteration and the use of benchmark strategy.

Table 2

Children's mean strategy use (and standard deviations) (max=12)[3] with standard and non-standard units of measurement

Strategy	Units of Measurement: Standard units	Units of Measurement: Non-Standard units	*Significance*
Idiosyncratic responses	2.25 (3.56)	2.39 (3.09)	*p=.721*
Object size	1.63 (3.24)	.77 (2.13)	*p<.01*
Previous knowledge	.34 (.816)	.01 (.10)	*p<.01*
Use of benchmarks	2.94 (3.27)	.59 (1.26)	*p<.001*
Unit iteration	4,25 (4.18)	7.71 (3.62)	*p<.001*
Use of subdivisions	.59 (1.49)	.24 (.73)	*p<.05*
Computations	N/A	.29 (.92)	N/A

Significant age differences were observed in the types of justifications referring to the use of idiosyncratic responses, object size and the use of benchmarks. More specifically, estimates based on idiosyncratic responses and object size were used significantly more often by the 3rd Grade children than by the 5th Grade children ($F(1, 86)=5.804$, $p<.05$ and $F(1, 86)=8.233$, $p<.01$ for strategies 1 and 2, respectively). The use of benchmarks seems to be characteristic of the oldest age group $F(1, 86)=19.402$, $p<.001$). These differences were also observed when comparisons were separately made for problems where standard and non-standard units of measurement were used: the first two types of justifications were more frequently used among the 8-year-olds than the 10-year-olds ($p<.01$), whereas the use of the more advanced types of reasoning increased with age ($p<.05$), both for standard and non-standard units of measurement problems. Figures 8 and 9 present the percentage of responses per age group involving the different types of strategies for standard and non-standard units of measurement problems.

4.4. Estimation performance and use of strategies

Finally, Pearson's correlations were used to explore a relationship between the children's length estimation accuracy and their reasoning, as indicated in the types of justifications they used. For the problems requiring the children to estimate in standard units of measurement, no correlations were found at all, meaning that all types of the children's strategies were observed both with successful and unsuccessful length estimations.

[3] Comparisons between standard and non-standard units were feasible only for tasks 1-6 (2 items in each task), as tasks 7 and 8 involved standard units only.

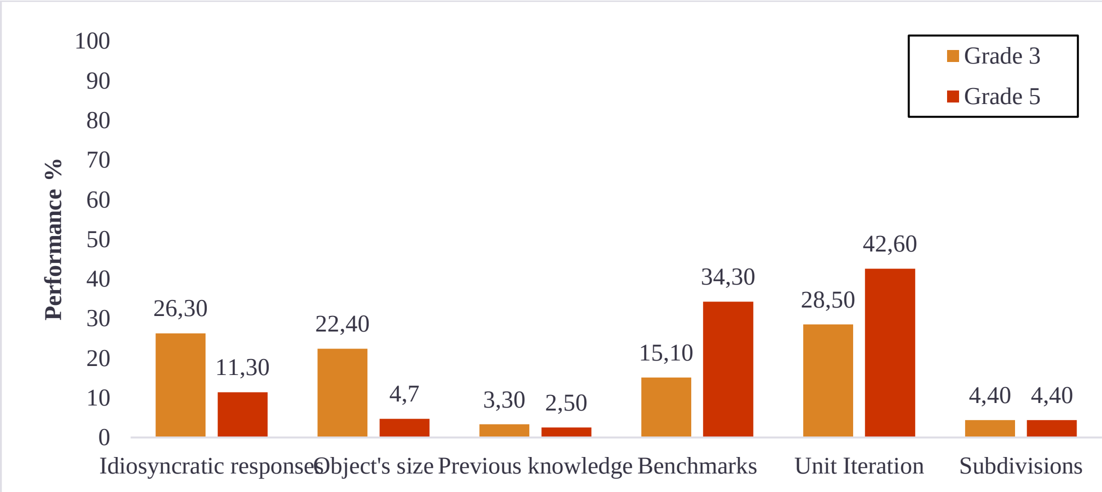


Fig. 8. Percentage of responses in each type of strategy by age in standard units of measurement problems

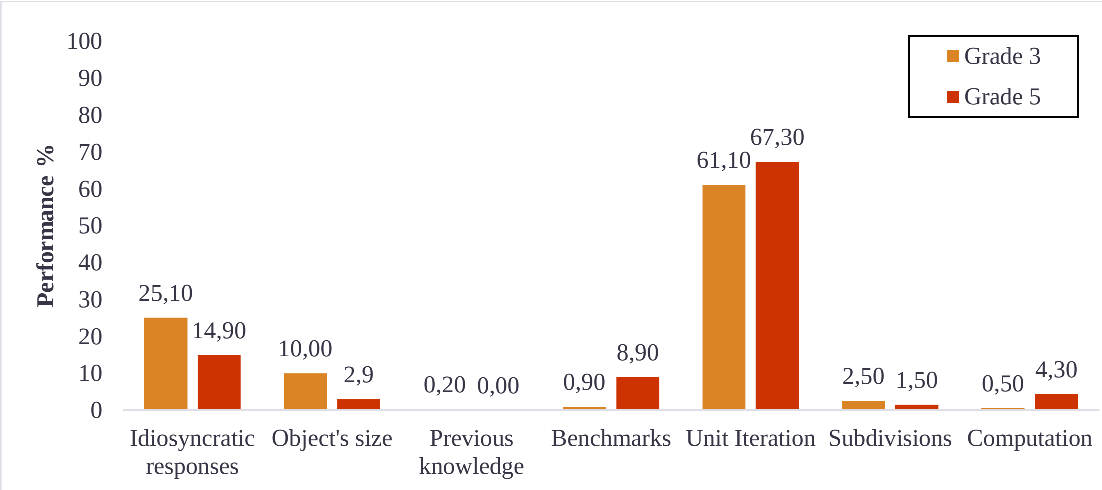


Fig. 9. Percentage of responses in each type of strategy by age in non-standard units of measurement problems

For the problems requiring the children to estimate in non-standard units of measurement, idiosyncratic responses were significantly negatively correlated with success rates (r= -0.583, p<.001), whereas both the use of benchmarks and the iteration unit type of strategy were significantly positively correlated with estimation accuracy (r= 0.220, p<.01 and r=0.555, p<.001, respectively). It seems that when children used these last two types of strategies, they tended to give correct length estimates. On the contrary, the more often they gave idiosyncratic responses, the less successful they were in their estimates.

Interestingly, the positive correlation in the children's use of iteration unit type of strategy was confirmed with regards both to successful length estimation accuracy in problems referring to short objects (r= 0.314, p<.01) and problems referring to long objects (r= 0.594, p<.001). The use of benchmarks was significantly correlated to correct responses in problems referring only to short objects ($r= 0.249$, $p<.05$) whose length was estimated in non-standard measurement units.

5. Discussion

In general, estimation performance was quite poor for both third- and fifth-graders whose rates of success were approximately 45% and 53%, respectively. This finding is in agreement with previous research reviewed by Joram et al. (1998). However, this study identified a complex interaction between performance and age: although fifth-graders did demonstrate better performance than the younger children, this was observed only for estimates made in non-standard units. On the contrary, the two age groups had similar performances when centimeters were applied. This finding is rather frustrating, implying an inadequate instruction of metric units during primary school that does not lead to the improvement of students' knowledge through the years.

Estimation performance was dependent upon the type of measurement units: the children's length estimation performance was better when non-standard units were used, in comparison to the metric units, a finding that has also been highlighted by Jones et al. (2012). In their study, the researchers suggested that this might happen because people in the U.S.A. have less experience with the metric system. This proposition, however, may not explain the difference observed in this study, whose participants were only familiar with the use of metric units. Additionally, the relative small size of centimeters that demands more iteration of the units in order to cover, and mentally "measure", an object compared to non-standard units might explain the students' poor estimations.

The children's length estimation performance was affected by factors related to the task context. Whereas objects presented in horizontal orientation led children to more accurate length estimations compared to objects presented in vertical orientation, neither three-dimensional objects and two-dimensional images nor objects shown in a white background or in a background with a complex pattern were found to cause differences in the children's estimations. This leads us to consider that object orientation plays a significant role in the children's ability to estimate effectively, a finding that does not agree with other studies (Jones et al., 2012). Considering that length estimation might require visual and spatial skills (Jones et al., 2012), it might be the case though that, if these skills are more encouraged in mathematics courses, children's length estimation performance might improve. Differences between the children's performance in the categories of tasks were even greater depending on the units of measurement used: third and fifth grade children approached 65% and 72% of accurate length estimations, respectively, in the Object orientation tasks when non-standard units of measurement were used. Although participants were more familiar with measurements in horizontal orientation, it is not known why this familiarity was not reflected in estimations using centimeters.

Drawing lines of particular length was found to be significantly easier than pointing to objects of particular length in centimeters. This could be interpreted as a sign of relative facility in producing something new rather than finding the appropriate object among those available. These results, which are in line with Joram et al.'s findings (2005), suggest that the representations of measurement units held by the estimators play a major role during the process of estimation and that, maybe, one of the factors that contribute to the enhancement of estimation performance with age is the improvement of an individual's representations, as (s)he gains more experiences from everyday life objects and measurements. However, future research on this issue may help clarify the influence of the context on estimation performance.

Estimation performance was also related to the length of objects that were used (short or long objects). When children used non-standard units to make their estimates, they were better at short objects, but when they used centimeters, they were better at long objects. Regarding the use of non-standard units of measurement, it seems rather natural for length estimates of short objects to be favored, because of the few iterations of the non-standard unit that were needed to cover the length of the object (i.e. to mentally measure an object of 30 cm in pencils of 17 cm). On the contrary, the fact that the children's performance in estimating the length of short objects did not differ from their performance with long objects when centimeters were used was rather surprising, because of the small relative size of the centimeter in comparison to the length of an object of, for example, 65-100 centimeters. One could attribute this finding to the length of the short objects (5-30 cm) that allowed an estimation error of few centimeters in absolute value. For example, for an object of 10cm, the biggest estimation error allowed (30%) corresponded to 3cm and the correct answers were found in the range of 7-13cm, whereas for a long object of 100cm the biggest estimation error corresponded to 30cm and the correct answers were found in the range of 70-130cm. Thus, although the estimation error was the same in rate (30%) for all the objects used, it is possible that children struggled to give successful estimates, within a narrow range of absolute values. For this reason, it is suggested that future studies use objects of various lengths and longer "short objects", such as objects of 50-100cm, especially if the participants are young children. In another analysis, though, we could hypothesize that this finding may be related to specific cognitive processes related to the children's representations of numbers that lead to better estimates as the length of the objects increases. This paradox was not reported before, thus future studies could further investigate it.

Strategy use on length estimations was found to grow with children's age, since the older children used more complex strategies than the younger ones. A primary indication to this result was found in Forrester, Latham and Shire's study (1990) in which differences in estimation strategies according to the participants' age group were reported. It is unknown whether the age-related differences in the present study are due to developmental factors or experience. Regarding the occurrence of strategies, unit iteration was the most common one, a finding that has also been identified by older studies (Joram et al., 1998). Overall, about 35% of the items in which standard units of measurement were used and about 65% of the items in which non-standard units of measurement were used were processed by the aid of this specific strategy. It seems that the use of novel units gives good opportunities for children to learn an effective strategy use, a finding that is in line with theoretical assumptions (Joram et al., 1998). Regarding the use of typical units, the proportion is lower but, a wide variety of strategies appeared instead. Children from both age groups were found more flexible using different strategies, including the use of benchmarks, the use of subdivisions and the unit iteration strategy.

The use of benchmarks and the iteration unit type of strategies appeared as the most successful strategies for estimates in non-standard units, whereas all strategies were found to correlate for both more or less accurate length estimations when standard units of measurement were used. These findings suggest that young children could benefit from the instruction of reference points, in order to create powerful imagery that would be used during the estimation, as well as from the unit iteration strategy, which is intuitively being preferred by estimators. The issue of measuring length in measure systems that are regularly used in everyday life was raised in the past (Joram et al., 2005. Buys & De Moor, 2008) indicating that the emphasis does not necessarily have to be on practicing with a measuring instrument, but more on learning the principles on which this measuring is based as well as on becoming familiar with important aspects of measurement, such as estimating and interpreting results. This could be better achieved if classroom is seen as an ecological space in which classroom language use (of both children and teacher) interplays with the development of measurement estimation (Towers & Hunter, 2010). Future studies examining performance and strategies of both children and adults would provide an insight on how estimators choose their strategies. Indeed, it would be very interesting to explore whether adults would rely more on a specific strategy such as the benchmark strategy because of their accumulated experience and more representations of real-world objects whose length is known.

6. Conclusion

There is evidence that the situation in which the concept of length estimation takes place influences children's performance in length estimation tasks. The levels of success in children's performance on length estimation situations and their use of strategies support the idea that young children have some informal knowledge of the logic of length estimation, i.e., children have some knowledge of the logic of estimating objects lengths that was developed in their everyday life, without instruction in school. Children's performance in length estimation situations with the use of non-standard measurement units is better than in situations where length estimation is required with use of standard measurement units. Nevertheless, traditional teaching practices use typical metric units to introduce the concept of estimation. Thus, maybe we should rethink the best situation for introducing children to length measurement estimation in the classroom.

Relating units of measurement estimation to familiar objects have proven helpful in leading to accurate estimations. One way for children to develop estimation skills is to apply these skills to meaningful estimations. It was also found that the children mainly based their estimations on the use of reference points and the unit iteration strategy, which appeared to lead to more successful estimates. Those two findings might support the notion that without the ability to make the connection between measures obtained from everyday reality and their application to measurement situations, success in estimating the length of objects is limited. Measurement estimation needs a repertoire of a wide range of everyday measurement referents. Children will then be able to utilize perceptions to make judgments about relative size without using tools. Thus, measurement estimation as a part of measurement instruction from the beginning of school would be beneficial for young children.

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