TEACHER CANDIDATES' ONLINE MATH JOURNALS: A SEARCH FOR PEDAGOGICAL SURPRISE

PERIÓDICOS ONLINE DE MATEMÁTICA DE FUTUROS PROFESSORES: EM BUSCA DE UMA SURPRESA PEDAGÓGICA

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ABSTRACT
Surprise and insight are an integral part of doing mathematics. However, surprise does not appear to be on the radar of most mathematics curriculum documents. In this paper, we present an analysis of TCs' online journals and their associated online discussions from a K-6 mathematics teacher education blended course. This online component of an otherwise face-to-face course also included readings and viewings of documentaries from classroom-based research, along with mathematician interviews, animations, and other support material (available at researchideas.ca/wmt), which connected to, and extended face-to-face course activities. We address the question: How did this limited online experience affect TCs' thinking about mathematics teaching and learning? Participants were 168 K-6 TCs, distributed among six sections of a mandatory mathematics methods course. We employed a case study approach and qualitative content analysis of TC discussions of journals and related online resources, and we identified six themes: (1) low floor, high ceiling approach; (2) contrast with personal math learning experience; (3) visual and concrete representations; (4) real world contexts; (5) aesthetic math experience; and (6) sharing math experiences.

Keywords: pedagogical surprise, pedagogical insight, mathematical surprise, teacher education, online education.

RESUMO
Surpresa e percepção são parte integrante do fazer matemático. No entanto, a surpresa não parece estar no escopo da maioria dos documentos curriculares da matemática. Neste artigo, apresentamos uma análise dos periódicos on-line de futuros professores e suas discussões on-line sobre o assunto a partir de um curso de formação de professores de matemática para o ensino fundamental (K-6). Este componente on-line de um curso de

outra forma presencial também incluiu leituras e visualizações de documentários de pesquisas baseadas em sala de aula, juntamente com entrevistas com matemáticos, animações e outros materiais de apoio (disponível em researchideas.ca/wmt), relacionados com, e estendidos às atividades presenciais do curso. Nós abordamos a questão: como essa experiência limitada on-line afetou o pensamento dos futuros professores quanto ao ensino e aprendizagem de matemática? Participaram 168 futuros professores dos primeiros anos do ensino fundamental (K-6), distribuídos em seis seções de um curso obrigatório de metodologia de matemática. Utilizamos uma abordagem de estudo de caso e análise qualitativa de conteúdo de discussões de periódicos e recursos relacionados online de futuros professores, e identificamos seis temas: (1) abordagem de piso baixo e alto teto (low floor, high ceiling); (2) contraste com a experiência pessoal de aprendizagem de matemática; (3) representações visuais e concretas; (4) contextos do mundo real; (5) experiência estética de matemática; e (6) compartilhamento de experiências matemáticas.

Palavras-chave: surpresa pedagógica, percepção pedagógica, surpresa matemática, formação de professores, educação on-line.

1. Introduction

Surprise is the mechanism that allows human beings to notice a discrepancy between previous knowledge and beliefs, and the world as it is (Meyer, Reisenzein, & Schützwohl, 1997). Surprise as an emotion is closely related to the concept of cognitive conflict, as surprise is considered a resulting emotional state of such conflict, triggered by unexpected events or information (Pekrun, 2012). In the learning process, conflict and the subsequent feeling of surprise are considered fundamental motivators, because they stimulate curiosity and exploration to regain a state of balance (Macedo, Reisenzein & Cardoso, 2012). Piaget saw cognitive conflict, which he also referred to as disequilibrium, as necessary for mathematical development (Lefrancois, 1995).

In mathematics education, several studies have focused on the importance of surprise to motivate and elicit curiosity towards mathematical concepts. For example, Movshovitz-Hadar (1994) and Watson and Mason (2007) believe that mathematics is full of surprises. Paradoxes, one form of surprise, have played an important role in the development of mathematical ideas (Kleiner and Movshovitz-Hadar, 1994). Also, cognitive conflict, associated with surprise, plays an important role in mathematics learning (Zaslavsky, 2005) as it combines the confusion and frustration of being stuck with the excitement of new discovery, which is part of the experience of mathematicians (Burton, 1999). In sum, surprise and insight are an integral part of doing mathematics (Sinclair & Watson, 2001); they give rise to curiosity and lead to the search and exploration of mathematical concepts that resolve a frustrating state of conflict.

We are particularly interested in cognitive conflict that results in conceptual insight through surprise, which offers new and unexpected connections and relationships and helps us see big ideas in new light. An example of surprising insight into a big idea of mathematics would be seeing that "parallel lines", which students learn to be straight and never meeting, can actually meet, as they do on a sphere. An example of surprising insight into a big pedagogical idea would be that the often-assumed stages of mathematical development identified by Piaget do not necessarily hold true if we can effectively create the low floor, high ceiling learning environments defined by Papert (1980).

However, surprise does not appear to be on the radar of most mathematics curriculum documents. Looking at mathematics curriculum documents for K-8 in a small sample of Canadian provinces and US states, we found only one mention of surprise, in the British Columbia curriculum in the context of Grade 5 student journals: "reflect on their learning, identify new ideas, areas of confusion or difficulty, surprises, misconceptions in their prior knowledge, etc." (p.282).

Over the last 10 years, the first author, working with colleagues in Canada and in Brazil, has been developing and researching approaches to mathematics education with an explicit focus on students (a) experiencing mathematical surprises, and (b) being able to share these surprises with family, friends and the wider community (Gadanidis & Borba, 2008; Gadanidis, Borba, Hughes & Lacerda, 2016a; Gadanidis & Hughes, 2011; Gadanidis, 2012; Gadanidis, Hughes, Minniti & White, 2016). As research dissemination, documentaries of this work, along with a wide variety of teaching and learning resources, in the form of online modules on a variety of mathematical topics, are being shared publicly online researchideas.ca/wmt.

This material has been integrated into our K-6 mathematics teacher education program in three important ways: (1) as the core math-for-teachers activities that K-6 TCs experience in their face-to-face course; (2) as online instructional modules, that extend the face-to-face experience through related readings, classroom documentaries, interviews of mathematicians engaging with the same topics, animations, and pedagogical support material; and (3) an online journal assignment where TCs share and discuss their reflections on the experiences with this online resource.

In this paper, we present an analysis of TCs' online journals and their associated online discussions. Through this analysis, we address the question: What appears to be the impact on TC thinking about mathematics teaching and learning?

2. Theoretical Framework

Our theoretical framework for this paper draws on three different ideas: (1) surprise as a pedagogical tool; (2) integration as a design principle for blended courses; and (3) new media as disruptive actors in teaching and learning. We describe these below, and then revisit them in the concluding section of our paper.

Surprise as a pedagogical tool. Our focus on surprise stems from Boorstin's (1990) work on what makes movies work, which parallels Norman's (2004) "reflective" criteria for "emotional" design of everyday things. Boorstin (1990) describes the “joy of seeing the new and the wonderful” (p. 12), which corresponds to Rodd’s (2003) call for experiencing “awe and wonder” in mathematics education. Boorstin says that audience “demands surprise – so long as surprise comes with a rational explanation” (p. 13). “For the writer, this means constantly creating expectations that (for the right kind of reasons) aren’t quite fulfilled" (p. 50).

Pervasive course outcomes as a principle of blended course design. Blended courses in teacher education can take a variety of forms. Collis and Jung (2003) explain that, on the one hand, technology can be included either for TCs to learn how to use it in the classroom, or only as the medium for training, and on the other hand, technology can be included either as the core or as the complement of curriculum content. According to Jung (2005) many teacher education programs include blended courses in their training process. Garrison and Vaughan (2008) state that when done well, combining the properties and possibilities of face-to-face and online learning go beyond the capabilities of each separately. Means, Bakia & Murphy (2014) conducted a meta-analysis on blended learning research and found blended instruction more effective than either purely face-to-face instruction or purely online instruction. Aycock, Garnham & Katela (2002), Futch (2005) and Sands (2002) suggest that this improved effectiveness is more likely to occur when instructors use common, pervasive course outcomes across face-to-face and online components, along with assignments that link the two components.

New media as a disruptive actor. The study adopts a sociocultural perspective based on Vygotsky's (1978) view that knowledge is constructed in interactions with others. By "others", we also refer to digital artefacts that permeate our new media culture. We view the online artefacts that TCs engaged with (such as readings, classroom documentaries, mathematician interviews, and so forth) as actors in the TCs' learning milieu, whose affordances affect their thinking about mathematics and pedagogy. Levy (1997) sees technological artefacts not simply as tools used for human intentions, but rather as integral components of the cognitive ecology that forms when humans collaborate in a technology immersive environment. Humans-with-media form a collective where new media also serve to disrupt and reorganize human thinking (Borba & Villareal, 2005). Specifically, in our study we want to be mindful of how TCs think with online tasks and resources and experiences with tasks in their face-to-face teacher education classrooms.

3. Research Method

3.1 Context

Participants were 168 K-6 TCs who agreed to participate in the research, out of a total of 187, distributed among six sections of their mandatory mathematics methods course. The course had a total duration of 17 weeks, using a face-to-face delivery mode, where TCs engaged hands-on with mathematics activities and coupled with an online learning management platform where instructors posted course schedules, assignments, weekly tasks, and course resources, and where they set up online discussion groups.

One of the course assignments involved keeping a journal on online readings and resources and classroom experience, based on reflection questions identified by the instructor. TCs shared their journals in their online discussion groups and commented on the ideas shared by their peers, in groups of 4 to 6 students. Typical instructions given by the teachers were to write an original comment and reply to two or three posts by their colleagues. The following prompt was given to TCs at the beginning of the course:

Each week you will be required to complete readings and activities posted in Sakai [the online learning system], then post your thoughts, and respond to 2-3 colleagues in the forum discussion. Also, each week 1 person in your discussion group will be assigned to share a summary of your group's discussion in class.

There were ten journal tasks assigned and six of these were focused on readings, resources and activities that were designed to elicit mathematical surprise, and our analysis is based on these six journal tasks (listed below).

- Week 1 - Being a Mathematician
- Week 5 - Growing Patterns
- Week 9 - Infinity and Beyond
- Week 10 - Math for Teaching
- Week 13 - Parallel Lines
- Week 16-17 - Great Math Stories

The modules included a wide variety of teaching and learning resources, documentaries of teachers' work in real classrooms, interviews with mathematicians and many other extensions such as handouts, math songs and interactive simulations.

3.2 Method

Our study employs a case study approach, looking at the case of TCs' online experience, which is a component of a primarily face-to-face course. A case study is suitable for collecting in-depth stories of teaching and learning and studying a 'bounded system' (that is, the thoughts and actions of participants of a particular education setting) so as to understand it as it functions under natural conditions (Stake, 2000a, 2000b; Yin, 1994). For Stake (1995), case study research is "the study of the particularity and complexity of a single case, coming to understand its activity within important circumstances" (p.xi). Our case focused on a part of the teacher education course, namely the online component, and was situated within the wider context of the blended course. Given our goal of common goals across the components of the blended course, the boundaries between the online component and the face-to-face component were not clearly defined, as Yin (1984) suggests should be the case. The online components of all six-course sections of K-6 TCs were treated as single case, as they all participated in the same online environment with the same tasks to complete.

We used qualitative content analysis (Berg, 2004) to look for pedagogical surprise in TC journals and related discussions. First, we extracted all data from the online forums, organized by weekly topics, we deleted any postings made by TCs not participating in the study, and we also deleted any identifiers (such as names). Then for each week, we used a manual content analysis by reading all the discussions and identifying the main themes of discussion, such as, integrating math with other subjects, using real life examples, the role of parents in math learning, and so forth. Finally, with the aid of a Qualitative Data Analysis (QDA) software - named QDA Lite - and our own summaries of data, we identified a total of six themes where students expressed pedagogical surprise: (1) low floor, high ceiling approach; (2) personal experience with math as compared to their teacher education experience; (3) using manipulatives and visual learning; (4) real life examples and connections; (5) integrating art and stories into math; and (6) sharing at home.

The QDA software also allowed us to notice the frequency for which each theme appeared in each week, and to identify relevant quotes in the students' journals based on the appearance of several keywords within the same paragraph.

3.2 Analysis and discussion

3.2.1 Low floor, high ceiling approach

We seek to "engage students with activities that have a low mathematical floor, allowing engagement with minimal prerequisite mathematical knowledge; and a high mathematical ceiling, so that concepts and relationships may be extended to more complex connections and more varied representations" (Gadanidis, 2012, p. 21). The idea of a low floor and a high ceiling comes from the work of Papert (1980), in his design of Logo so that very young children can engage with it while also not being restricted in terms of the complexity of the ideas they can investigate.

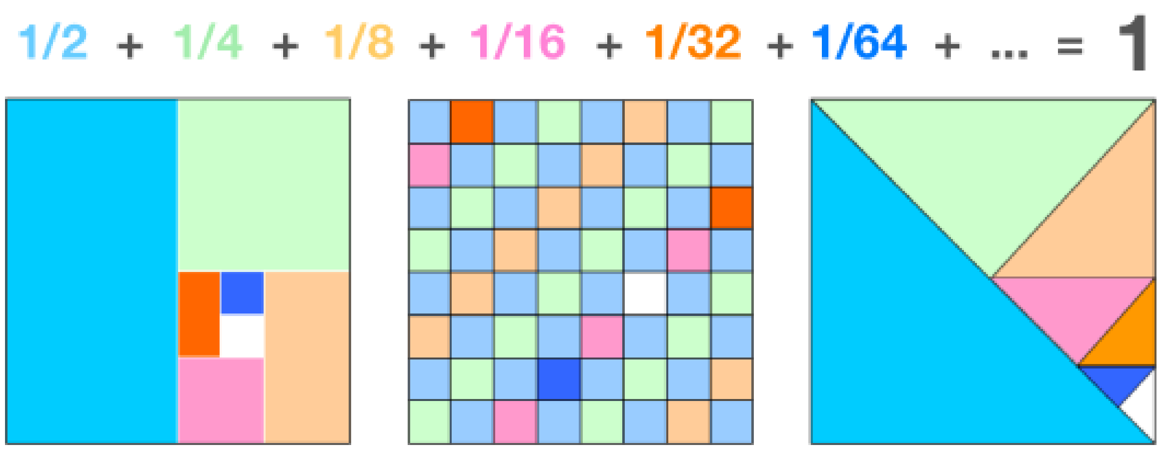

Figure 1. An infinite number of fractions in a single square

An example of a low floor, high ceiling from our work in classrooms is a grade three activity where students explore patterns in the area representations of fractions 1/2, 1/4, 1/8, 1/16 and so forth by shading 16x16 square grids. Students then cut out the shaded parts and join them together to form a new shape. If the shading, cutting out and joining continues forever, how big would the new shape become? Students discover that all the shaded parts fit in one of the original 16x16 squares, and that "I can hold infinity in my hand!" They then explore different ways of shading to represent these fractions, as shown in the illustration in *Figure 1*. Thus, students start with the low floor of shading fractions while engaging with the higher ceiling associated with infinity and limit, which is a topic typically addressed in the study of Calculus.

This low floor, high ceiling concept was introduced to TCs in the first week of the course, through the article "Why can't I be a mathematician?" (Gadanidis, 2012), which also includes the above "infinity in my hand" example. During this week, the topic "Low floor, high ceiling" was a common discussion theme, with a frequency of appearance of over 300 mentions, as shown in *Figure 2*.

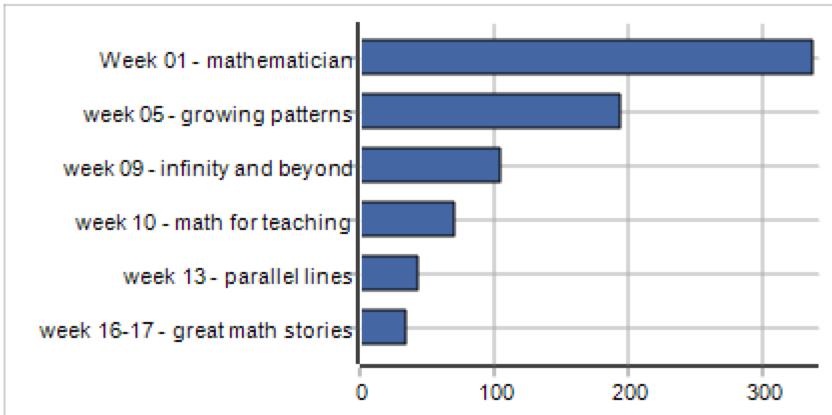

Figure 2. Frequencies per week for the theme "low-floor, high-ceiling"

In week 1, TCs commented mostly about how interesting it was to present young children with abstract concepts, such as infinity, through simple and relatable activities. A frequently commented insight was the necessity of challenging young students' minds. TCs expressed eagerness to try this type of activities during their teaching practice. For example, one student commented:

> *A theme that really stood out for me in this article was the low-floor/high-ceiling concept. This concept of presenting abstract mathematical thoughts to younger children is extremely interesting. […] Upon first glance, you would never think that a student in the third grade would be able to come up with something so advanced. […] It is important to always challenge your students, and give them the opportunity to shine as mathematicians.*

For the other weeks, the low-floor high-ceiling theme continued to appear as TCs reflected on the modules on the site researchideas.ca/wmt, where more low floor, high ceiling examples are offered. This is one indicator that TCs had developed a robust enough conceptualization of the low floor, high ceiling idea to be able to recognize it in other activities.

In week 5, TCs commented specifically about how the low-floor high-ceiling approach was used by Dr. Lindi Wahl in a series of video interviews (see researchideas.ca/wmt/c2b5.html), where she discussed the topic of growing patterns. A common insight TCs expressed was how the lesson evolved from using blocks to represent growing patterns to the use of algebraic expressions of linear and exponential growth. One student commented: "In watching the video from Lindi Wahl, I was struck by the simplicity and yet complexity of the math she was presenting; in essence, a 'low floor, high ceiling' approach."

In a similar fashion, week 13 also had TCs commenting on how the low-floor high-ceiling approach was applied in the module on parallel lines. It is important to note that some of the TCs were learning the math concepts themselves while at the same time they were learning how to teach the concepts. Regarding this aspect, one TC commented:

> *Connecting it back to the low floor, high ceiling approach, I am reminded that we should not place limits on what we believe students are able to understand. I enjoy how these modules are explained in easy to understand ways as it helps me to grasp some of the concepts that I don't remember how they work and also shows me how to easily explain them to my students.*

In week 9, TCs discussed the module "Infinity and Beyond" available at researchideas.ca/wmt/c1b0.html, where the concept of infinity is explored through the area representation of fractions (as shown in Figure 1). During the first week of the course, TCs read about the implementation of this activity. However, during week 9 they could see the full set of activities, including classroom documentaries where grades 3-4 students represented the fractions in a square and discussed the sum of the fractions and the concept of infinity. TCs gained insight on how this type of activities work for young students, as they understand complex concepts such as infinity and how it can fit in a square. For example, one TC commented: "This demonstrates the effectiveness of the low floor high ceiling approach. If we teach these complex concepts to younger grades, something wonderful happens, they understand it!"

In week 10, students were presented with a model of math-for-teachers that consists in preparing teachers by engaging them with the same experiences they will share with the students, that is, to see math from the students' perspectives and to also personally experience a model of teaching that they may use in their own classrooms. TCs showed a lot of interest in this concept, and some of them related it with the necessary abilities a teacher needs to have to design low-floor high-ceiling activities. For example, a student commented:

> *I agree that teachers need to see things from their students' perspectives, especially to be able to anticipate student responses. However, I still believe that it is important for teachers to have a different quality of knowledge than their students, especially to incorporate low floor/high ceiling approaches in the math lesson. I think that helps with providing extensions after the lesson, and really helps foster functional thinking in students.*

Finally, during the last weeks of the course (16-17), TCs learned about how to create mathematical experiences worth sharing, that is, experiences that offer the pleasure of mathematical surprise and insight that students can bring home to share with family and friends. They connected this to the low-floor high-ceiling approach by identifying it as an important element of creating great math stories. As one TC commented: "By providing students with inquiry based lesson, giving them open ended questions that allow for low floor-high ceiling thinking, and showing students that math can be creative by incorporating poems, songs, and stories math can be engaging and exciting."

The insights shared by TCs in their journals align with Ginsburg's (2002) thoughts when he wrote about the challenge of creating activities for children that offer the opportunity to "achieve the fulfillment and enjoyment of their intellectual interest" (p. 7). TCs also showed understanding of the fact that "children possess greater competence and interest in mathematics than we ordinarily recognize" (Ginsburg, 2002, p. 7).

By acknowledging this quality in younger children, TCs recognized the need to incorporate a more flexible approach in curriculum, introducing topics from higher grades, because "small ideas do not offer math surprise and insight. To solve this problem, we teach math content in a context of big math ideas, often from higher grades, like 'area representations of fractions' in the context of 'infinity and limit'" (Gadanidis, 2014, p. 40).

3.2.2 Contrast with personal math learning experience

Biographical learning is an educational approach in which learners engage in reflective activities about their own experiences and the history of their lives (Christensen, 2012). This method has been largely used in teacher education and teacher identity (e.g. Goodson, 2003). According to Christensen (2012) "insights in written or told life histories allow for an understanding of a person's way of structuring his or her life. They also provide a connected picture of the different practicalities and limitations which structure a person's life" (p. 459).

Because teachers' previous experiences and beliefs related to math affect how they teach (DeCorte, 1996; Ferguson, 2008), the course attempted to engage TCs with activities where they can experience different views of mathematics and mathematicians, and to model different pedagogical approaches in the online activities and documentaries. For this reason, the course provided not only the opportunity for TCs to share and reflect upon their own experiences and beliefs about math, but also to engage with learning experiences that challenged these previous notions through the pleasure of mathematical surprise and insight (Gadanidis, 2012).

An example on how the online modules motivated TCs reflection on their personal experience is shown in Figure 3. This figure displays a screenshot of a video from week 13 which directly compares the experience of a young girl that studies parallel lines through the straightforward definition of "they are straight and never meet" (in the left side of the screen), and a girl who learns the same concept through a riddle and explores how parallel lines can meet in 3D surface such as a sphere. This type of material compels TCs to question and reflect about their own experience learning math.

Figure 3. A video comparing two parallel universes of math learning

During the first week of the course, TCs were asked to describe in their journals what their personal experience had been with math. So, for week 1 this was a very common topic, with over 1000 in frequency of appearance, as shown in Figure 4. However, for the rest of the weeks, TCs would continue to comment in their journals about how they wished their math learning experiences had been different. Many of these comments were based on comparisons to the activities in the modules, reflecting upon the use of manipulatives, visuals, art and songs that they believe would have helped them in their understanding of math.

Below, we present sample comments for some of the weeks:

> *I believe that if math had been taught to me through stories or songs, I may have felt less intimidated by mathematical concepts. I remember elementary school math as sitting at a desk and filling out multiplication sheets, and these students are experiencing math in a much different way. While I did not enjoy math as a child, I did enjoy music and language arts, and blending the subjects together allows each student to feel comfortable within the learning environment and enjoy these mathematical experiences. (Comment on week 1)*
>
> *Fractions were definitely something that was complicated for me as well during school, I wish I had had this type of instruction. I feel like I would have grasped the concepts far better. (Comment on week 9)*
>
> *When I was in elementary school I remember no one wanting to provide their answer. In relation to my experiences I definitely think it is the change of focus from the right answer in the past to now focusing on the process that has really changed the mindset and excitement to show their solutions. (Comment on week 16-17)*

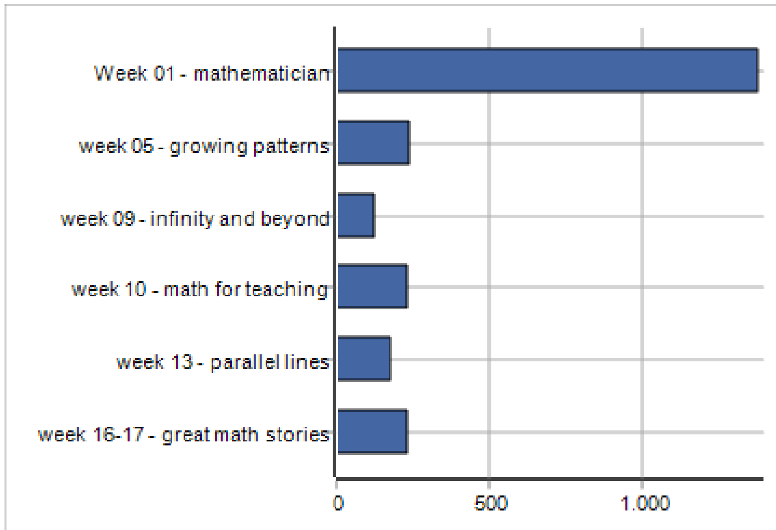

Figure 4. Frequencies per week for the theme "Personal experience with math"

Engaging TCs to share and reflect upon their past experiences with math is an important start to their teacher education. As suggested by Guillaume and Kirtman (2010), "helping teachers to surface and analyze their mathematics stories can serve as a powerful starting point for enriching their understanding of what mathematics can be and do" (p. 140). By going through the activities in the math-for-teachers course, and comparing them to their own experience, TCs had the opportunity to rethink their previous and perhaps more limited understanding of math. The modeling of mathematics pedagogy in the online activities and documentaries offered artefacts for comparison with past experiences, for questioning what they may have assumed to be mathematics teaching and learning, and for experiencing pedagogical and mathematical surprises and insights.

3.2.3 Visual and concrete representations

According to Hardy and Koerber (2012), "visual representations are depictions that use space to represent nonspatial concepts, offering a wide range of possibilities to display qualitative and quantitative information in scientific contexts" (p. 2926). They facilitate knowledge construction and have been largely employed in educational contexts, as Eisner (2002) explains "curriculum designers need not to use verbal forms of expression as the only means of presenting ideas to students" (p.148). This thought is also true when trying to develop conceptual understanding in mathematics. Borba and Villarreal (2005) agree that in mathematics an algebraic approach should not be the only way of presenting ideas to students. The use of manipulatives, which are an important form of visual learning and modeling, is essential to help students visualize abstract concepts in mathematics (Gravemeijer, Lehrer, van Oers & Verschaffel, 2002).

Throughout the course, the use of manipulatives and diverse ways of presenting the content was a much-emphasized topic. For example, one of the ways in which the growing patterns were represented in week 5 was by using blocks (see Figure 5), and the modules on week 13 explained the concept of parallel lines by using a globe and string.

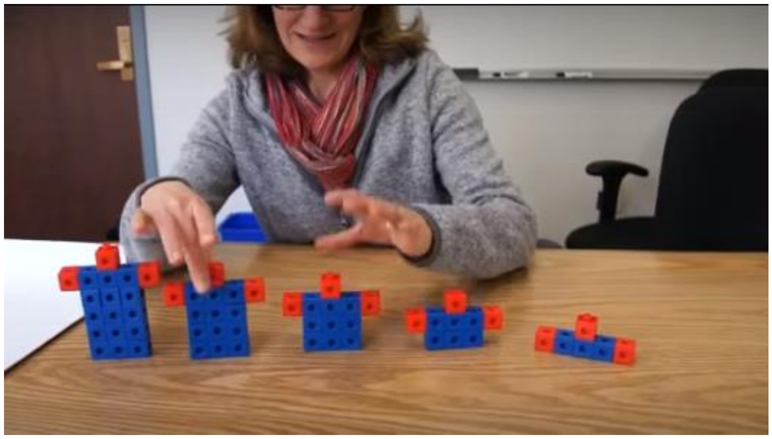

Figure 5. Video screenshot showing the use of blocks to explain a growing pattern.

TCs responded to this type of materials in their journals, writing about the importance of manipulatives and visualization in math learning. They also commented on how the visual elements in the modules helped them grasp the concepts. It is worth highlighting that among the studied weeks, the one with the most comments on the use of manipulatives and visual learning was week 5, as shown in Figure 6.

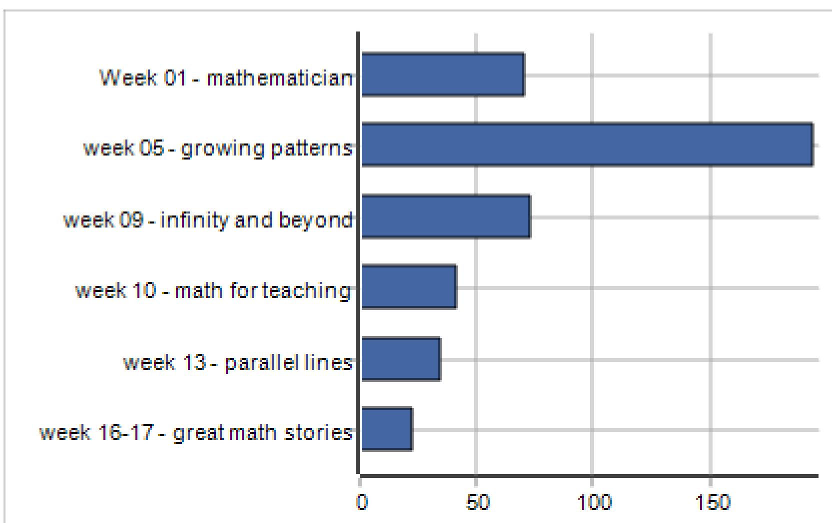

Figure 6. Frequencies per week for the theme "Visual and concrete representations."

TCs reflected on varied forms of representation as an important way of attending to students' individual needs and interests, as portrayed in this comment from week 5:

> *I liked that both videos described a similar concept but used different visual aids and manipulatives. I think that using different modalities like this is very important in the classroom because one method (ex. bingo dabbers) may resonate with some students, whereas another method (ex. the graph on the computer) may resonate with others.*

Also, they wrote about making abstract concepts more tangible and relatable using manipulatives and visuals, as shown in the example comments below:

> *After watching the interview with mathematician Graham Denham it became clear to me that fractions can be understood clearly when they are represented in a visual way […] Through using manipulatives and visuals, fractions are not so ambiguous to learners when they can see what goes into those numbers and how they are formulated. (Comment from week 9)*
> *I enjoyed watching the interview with Dr. Megumi Harada, as I found the manipulatives she used helpful when explaining the concept of parallel lines. I found that by using string, it was easy to visualize the concepts she was teaching. Furthermore, I liked how she used string because it is a common object. (Comment from week 13)*

TCs also commented on the important role that manipulatives play in constructivist and inquiry-based learning, as they allow students to discover concepts and relations on their own. For example, one TC commented on week 16-17:

> *I've realized that in order to really experience math, the use of manipulatives really help to keep students engaged and construct their own learning. No longer do students need to be lectured about the rules involved in math because they discover it for themselves.*

The insights expressed by TCs in their journals show that they are developing an interest in and a conceptualization of the use of concrete and visual tools and models in math instruction. The way TCs describe their experience with manipulatives align with Papert's (1993) description of manipulatives as "objects to think with" (p. 11). Additionally, TCs reflected on how in math learning, manipulatives and visualization provide "a bridge from the concrete to the abstract, which, in turn, promotes greater conceptual understandings" (Marley & Carbonneau, 2014, p. 1). Also, they promote "positive attitudes towards mathematics since they supposedly provide 'concrete experiences' that focus attention and increase motivation" (Durmus & Karakirik, 2006, p. 117). Finally, some TCs commented on how they liked the use of common objects, such as strings and chocolate, as manipulatives. The use of these common objects relates to the topic of real life examples and connections, which is further discussed below.

3.2.4 Real world contexts

According to Podolski (2012), the functional context approach "stresses the importance of making learning relevant to the personal experience of learners and the context of their activity" (p. 1328). As a theory, the functional context is framed within the broader concept of socio-constructivist learning (Vygotsky, 1978). Based on this concept, education has emphasized in elements such as providing usefulness for real-world applications in course content, considering the previous knowledge of participants, and using contexts, tasks and materials as close to the real life as possible.

In the context of mathematics education, "helping children to believe their world is filled with mathematical ideas is essential to helping them become confident problem solvers and mathematical thinkers" (McVarish, 2008, p. 10). For this reason, the use of real life examples in mathematics is a way to make concepts more relatable and less abstract. This theme was explored by TCs throughout the entire course, especially in the first two weeks of their journals, as shown in Figure 5.

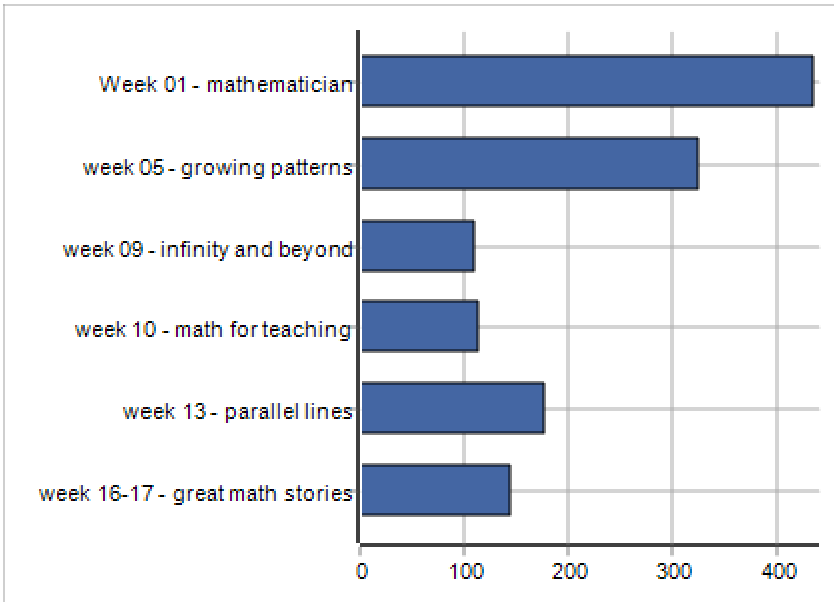

Figure 7. Frequencies per week for the theme "Real life examples and connections"

One concrete example on how the functional context was included in the online modules are the interviews with mathematicians. These are series of videos where practicing mathematicians explain mathematical concepts and provide examples of how they use that specific concept in their work. For instance, in the week 5 interview, Dr. Lindi Wahl explained how the growing patterns related to her work with the patterns of bacteria growth.

TCs identified the real-life connections presented in the modules as an important element that helped them understand the concepts. They translated this to their future teaching experience, and commented on how they, as teachers, will need to connect math to real life examples. Below are some representative comments:

> *I also found it beneficial when Lindi made a real life connection to bacteria/money as the concept began to make more sense. (Comment from week 5)*
> *Something as simple as relating fractions to walking out the door or even relating fractions to a well known fairy tale allows students to fully grasp these concepts and recall them in future math lessons. From my experience, fractions are not an easy concept to understand, but creating lessons and activities that allow students to relate these ideas to real world experiences will overall lead them to succeed with the difficult concept. (Comment from week 9)*
> *I loved how the video "parallel lines" gave real-life connections [...] showing how parallel lines can be found in your home, school etc. I think this would be an excellent way to start a lesson on parallel lines, in which the teacher could prompt students' prior knowledge of parallel lines with the objects around the classroom. (Comment from week 13)*

Through these math surprises and stories students are able to connect math to real life experience, overall making them meaningful. (Comment from week 16-17)

The insights TCs shared in their journals show understanding of how “familiar contexts provide wonderful opportunities to motivate math learning that are based on students’ perspectives of what is important enough in their own lives to be worthy of mathematics effort” (Willis, 2010). Also related to this topic, TCs commented on relating the content to fairy tales, such as the story of Rapunzel, which connects to children’s interests, but also relates to the use of stories and art as will be discussed below.

3.2.5 Aesthetic experience in math learning

Aesthetic experiences are those that help learners achieve conceptual understanding by bringing forth emotions comparable to those experienced when admiring a piece of great art (Dewey, 1934). According to Girod (2012), “philosophers have long recognized that science and art share some underlying values such as parsimony, form, symmetry, pattern, and unity” (p. 2972). In education, there is extensive literature to promote the linkage between science and art to improve conceptual understanding, meaningful learning, and positive attitudes toward science (e.g. Girod, Twyman, & Wojcikiewicz, 2010; Pugh, Linnenbrink-Garcia, Koskey, Stewart & Manzey, 2010)

In mathematics pedagogy, the issue of creating aesthetic experienced for students is not commonly explored because "it may not always be easy, but the mathematical beauty that results gives so much pleasure" (Gadanidis, 2012, p. 26).

With the purpose of creating aesthetic experiences, the online activities during the math-for-teachers course offered opportunities for TCs to explore art and storytelling in mathematics. For example, the online modules included artistic representations (such as the one in figure 8), songs and stories about the topic. The “Infinity in your hand” section (researchideas.ca/wmt/c1b2.html) opens with an excerpt of the poem Auguries of Innocence by William Blake:

To see a world in a grain of sand,
And a heaven in a wild flower,
Hold infinity in the palm of your hand,
And eternity in an hour.

The theme of integration between math and art was emphasized in the first and last week of the course (as shown in *Figure 9*) because in these weeks TCs read articles that addressed the specific topic of integrating art and stories into math. In the remaining weeks, TCs had the opportunity to see these activities put into practice, and commented on how this would help students feel more engaged in learning mathematics. Some students also related this topic to including a cross-curricular approach to mathematics, which connects math with other subject and the real world.

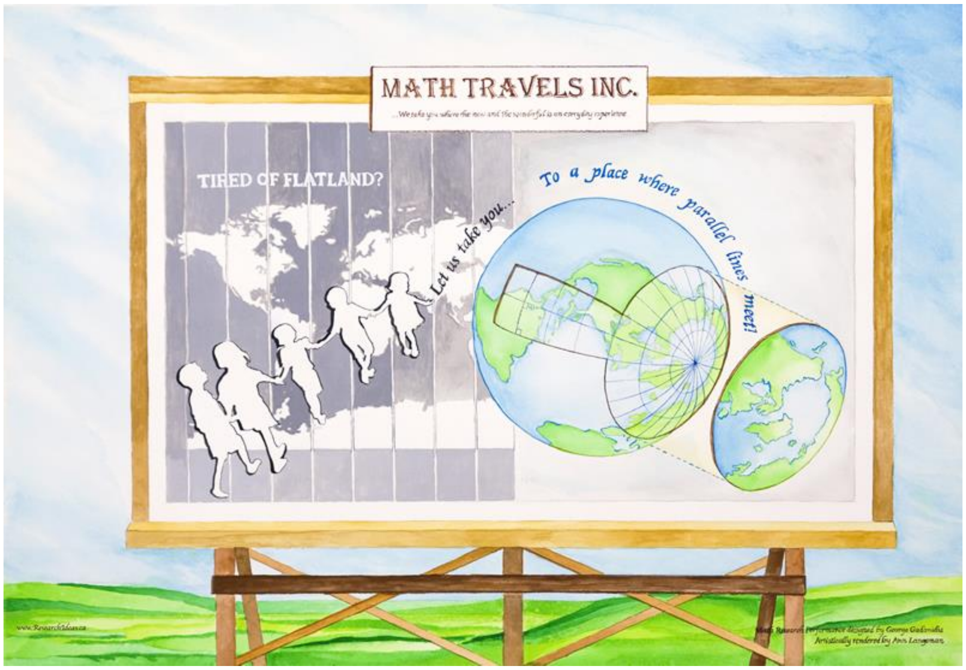

Figure 8. Artistic representation of "Where parallel lines meet." Artistically rendered by Ann Langeman (Faculty of Education, UWO). Designed by George Gadanidis.

Below we present sample comments related to students' engagement and motivation through the arts:

> *While reading the article, a theme that caught my interest is the art-based communication in learning math. By engaging in a wider audience for mathematics through art-based communication skills, I feel this would take away the fear or mathematics for students who have a hard time with the subject. (Comment from week 1)*
>
> *This lesson could appeal to students who enjoy art because of the emphasis on visual representation. Furthermore, the song that was created by students was a great integration of music. (Comment from week 5)*
>
> *Starting the lesson with the first quatrain of William Blake's "Auguries of Innocence" was a great idea to engage students through cross-curricular content, which is extended by having students write songs on the subject and make art from their fractions. (Comment from week 9)*

Many students also related this topic to including a cross-curricular approach to mathematics, which connects math with other subject and the real world, as portrayed in this comment from week 13:

> *I thought there was a great cross curricular approach as well in the sense that it incorporated geography, mapping, literacy and art into the mathematics lesson. This is definitely something I would want to do in my own classroom.*

A popular theme among TCs' discussions was the idea of using different forms of art to diversify assessment in mathematics education. As an example, one TC commented on week 16-17:

> *I like this idea because it is inclusive for all student, as some student might be stronger in the arts than in math. Also students might become more engaged in the math activity because they get to work in groups and because they are having fun while doing math. Using a song or performance as a summative project is a great way to include non-test assessments.*

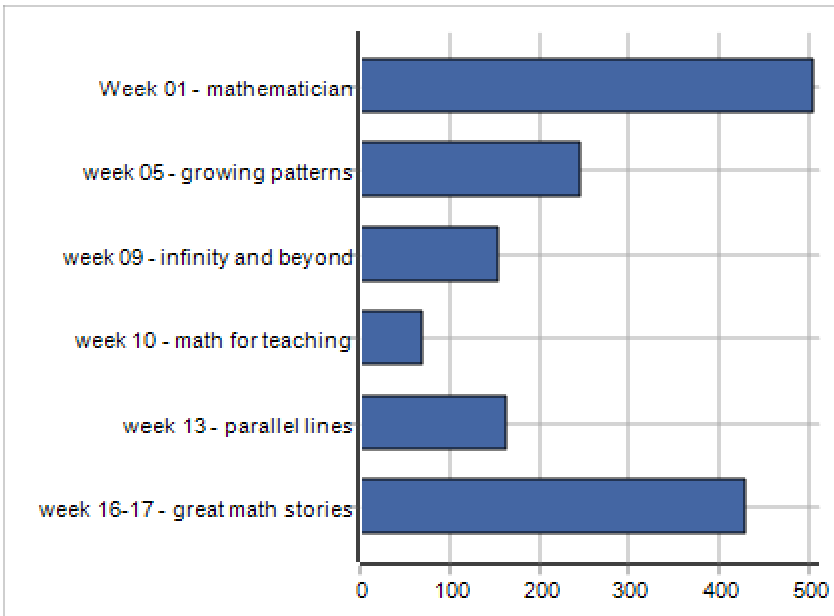

Figure 9. Frequencies per week for the theme "Integrating art and stories into math"

TCs' insights are consistent with findings by Holtzman and Susholtz (2011) that the integration of math and art "encourages multiple perspectives and avenues of access, supporting engagement and understanding. It deepens and personalized the learning experience" (p. 2). Also, when TCs comment on the creation of songs and different ways of sharing math knowledge, they are addressing to Holtzman and Susholtz's (2011) comparison of math and art, stating that "effective communication is essential in both fields, as part of the process and as a reason to share the work with others" (p. 2). This idea of sharing with a larger audience is further explored in the next section.

3.2.6 Sharing math experiences

According to Dewey (1899), "interest in conversation, or communication; in inquiry or finding out things; in making things, or construction; and in artistic expression [are the] natural resources, the uninvested capital, upon which depends the active growth of the child" (pp. 47–48). Dunst (2006) explains that participating in interesting learning experiences increases competence, motivation, and personal well-being.

Based on these premises, during the course TCs were exposed to the creation of math experiences that were so exciting, due to the incorporation of mathematical surprise, that students would be motivated to take them home to share with friends and family. This experience of sharing generates a larger audience and gives students a sense of purpose and ownership, which leads to more meaningful learning.

One example of how TCs were exposed to this content through the modules can be seen in figure 3, in the comparison of the two parallel universes, the first where the young girl just recites the definition she learned in math class, and the second where she tells her mom a riddle. Furthermore, all modules ended with the question "What did you do in math today?" and prompted students to think about stories that might catch the interest of their audience. For instance, the Infinity and Beyond module ends with this prompt:

> *If someone asked you "What do you know about infinity?", what story might you share to capture their imagination and surprise them mathematically?*

The topic of sharing at home was more popular in the last week of the course (Figure 6), as they read an article that was directly related to telling great math stories. However, they were presented with this idea from the first week of the course and commented on sharing at home during each of the online activities.

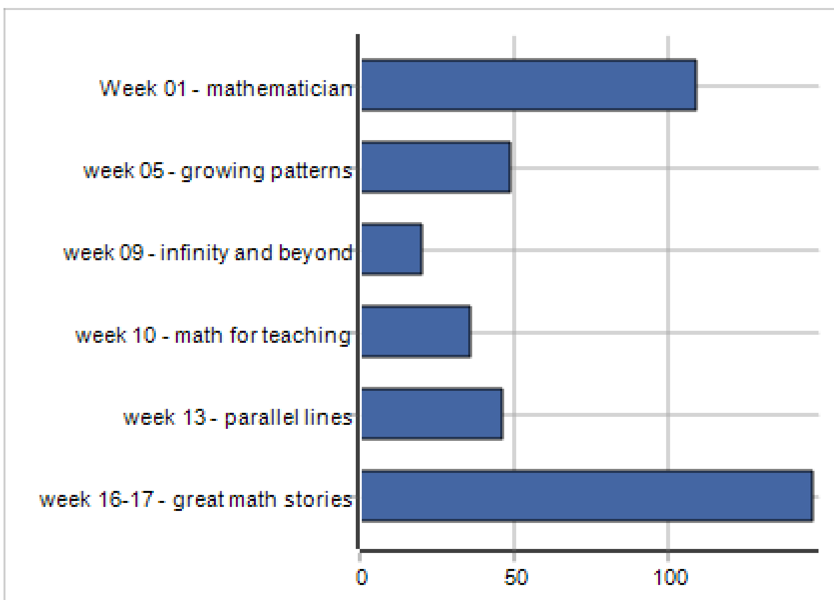

Figure 10. Frequencies per week for the theme "Sharing at home"

Below are sample comments for some of the weeks:

> *Students need to share positive, fun, enlightening, good mathematic stories with family and peers to change individuals' impressions of mathematics.* (*Comment from week 1)*

I think that children would feel excited to go home and share what they learned with their parents, which in turn will make them excited to learn certain concepts. (Comment from week 5)

Having families on board with teaching is a bonus. I think it's great to be able to engage a child so much that they go home and tell their parents or siblings about what they learned in mathematics that day. To me, the best teachers are the ones who can get students genuinely excited to learn and teach their family members something new. (Comment from week 13)

I think being able to share what you've done with an audience not only makes a subject more exciting, but it also gives you a sense of ownership to your learning and something to be proud of. You are able to share with others and perhaps learn something new from your audience or feel purpose as you share your learning with an audience. (Comment from week 16-17)

The insights shared by TCs show an understanding of the importance of parents in mathematics learning, as they "can act as 'math allies' if they find ways to integrate real-world math into their child's hobbies and interests" (Willis, 2010, p. 11). Also, they reflected on the sense of ownership students get when sharing mathematics "as a narrative process where the learners have agentic control over authorship" (Burton, 1999, p. 31).

4. Conclusion

The online component discussed in this paper was a relatively small part of the K-6 mathematics teacher education course. The main part of the course consisted of face-to-face classes. The online component had two parts: (1) an online resource to support and extend ideas in the face-to-face experience, and (2) an online assignment where TCs shared and discussed their journals of their experiences with the online resource. So, in this blended course, technology was used as facilitating media for extensions of course content and experiences and as a networking method among students.

The analysis in the preceding section shows that TC journals and online discussions brought to light several pedagogical surprises and insights that correspond to course goals: (1) TCs made connections to the idea of a low floor and a high ceiling (Papert, 1980), which continued to appear in their online discussions as they reflected on the modules on the site researchideas.ca/wmt, indicating that they had developed a robust enough conceptualization of the low floor, high ceiling idea to be able to recognize it in other activities and other settings. A low floor, high ceiling is a key design element in the mathematics activities used in face-to-face TC classrooms, in our research classrooms, and in the online documentaries. TCs expressed pedagogical surprise that such an approach was effective with young children. (2) Course experiences prompted TCs to make comparisons and contrasts with their own school mathematics learning. Engaging TCs to share and reflect upon their past experiences with math is an important start to their teacher education. (3) TCs noticed that visual elements in the modules helped them grasp the concepts and they wrote about making abstract concepts more tangible and relatable for their own students using manipulatives and visuals. (4) TCs drew insights from documentaries and interviews about the importance of creating meaningful contexts for learning, through real-life connections, familiar settings, and children's literature. (5) TCs discussed the importance of cross-curricular connections, with special emphasis on art and other aesthetic components. (6) TCs took up the idea of mathematics as a "good story" to share with others, especially in the content of making home connections where children share their learning with family and friends.

Although we cannot make strong conclusions about the effectiveness of our course design, we can say that the pedagogical insights that TCs shared and discussed were closely aligned to our course goals. Below we revisit the three ideas from our theoretical framework, to illustrate how our course design may have facilitated our positive, yet tentative, findings.

Surprise as a pedagogical tool. Surprise connected to the TC course experience in three ways: (1) we engaged TCs with face-to-face math-for-teachers activities that elicited mathematical surprises, such as "parallel lines can meet" and "you can hold infinity in your hand"; (2) we offered an online resource where they could see these "surprises" through a variety of perspectives, such as classroom documentaries, related readings, and the eyes of mathematicians; and (3) we offered TCs opportunities to share and discuss their pedagogical surprises and insights in an online forum, by sharing and discussing their journals of their experience with the online resource.

Integration as blended course design principle. What is unique in our teacher education program is that both the math activities that TCs experienced in the face-to-face component and the readings and documentaries they accessed and discussed online came from our research classrooms, and thus potentially complemented each other in powerful ways. Designing our face-to-face and online components to meet common goals (Aycock, Garnham & Katela, 2002; Futch, 2005; Sands, 2002) have increased the pedagogical impact on TCs.

New media as a disruptive actor. The online resource we developed extended TCs' face-to-face course activities where they engaged with mathematics experiences designed to offer mathematical insight and surprise. This may have facilitated pedagogical ideas from the online resource to come to life—to become more robust and prominent actors that TCs thought with—in their online journals and pedagogical discussions.

Looking ahead. The experiences developed through the online journal assignment, and through the course are a starting point "to involve preservice teachers in doing mathematics—mathematics where they have to attend deeply, mathematics that offers the potential of experiencing the pleasure of mathematical insight, mathematics that engages their imagination" (Gadanidis & Namukasa, 2007). There is a potential that through their own feelings of pedagogical surprise and insight, TCs will feel the need to explore big math ideas in their classrooms by: (1) using low floor, high ceiling approach; (2) reflecting on their personal experience with mathematics and try to overcome their preconceived notions; (3) providing students with rich activities using visual and concrete representations; (4) connecting math to students' real world context; (5) integrating the arts in their mathematics teaching; and, (6) creating experiences that are worth sharing beyond the classroom. Will some of these elements be translated into TCs' classroom practices?

The next step in our research is to follow up with a sample of these TCs as they engage with mathematics teaching and learning in their practicum experiences and in their own classrooms. We are also considering the ways in which the online discussion can be organized and supported differently to enhance the experience. For example, some of our ongoing research involves the use of collaborative mind-mapping instead of discussion forums. And, we are looking at students' expression of mathematical insight and surprise through a computational thinking course for math teachers. This research is being developed with the same set of participants, and we hope that all the experiences combined result in a reinforcement of the ideas portrayed in this paper to ultimately effect a change in the way they teach.

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