CONTRIBUTIONS OF THE CULTURAL-HISTORICAL THEORY TO MATHEMATICS TEACHING IN THE EARLY YEARS OF SCHOOL

CONTRIBUIÇÕES DA TEORIA HISTÓRICO-CULTURAL PARA O ENSINO DA MATEMÁTICA NOS ANOS INICIAIS

Joselia Euzebio da Rosa
joselia.euzebio@yahoo.com.br

Universidade do Sul de Santa Catarina - Brazil

Ana Paula da Silva Galdino
anagaldino_1@hotmail.com

Colégio Dehon/UNISUL - Brazil

Luciane Pandini Simiano
lucianepandini@gmail.com

Universidade do Sul de Santa Catarina - Brazil

ABSTRACT

This text aims to highlight the principles of the Cultural-Historical Theory to the structure of Mathematics teaching. As an argumentative appeal, a virtual story based on the Teaching Guiding Activity proposed by Moura and followers (1996; 2010) was elaborated. The mathematical concepts covered were: quantity, unit, counting, number, addition and multiplication. This conceptual framework was discussed in the context of arithmetic, algebraic and geometric significances. In this system, possibilities of objectification were presented in the structure of principles of the Cultural-Historical Theory teaching. The results reveal indicators that envision the possibility of contributing to the development of the basis for the formation of mathematical theoretical thinking in the transition period between the play activity to the study activity.

Keywords: Cultural-Historical Theory. Mathematics Education. Teaching Guiding Activity. Davýdov and his colleagues' proposition.

RESUMO

O presente texto tem por foco evidenciar os princípios da Teoria Histórico-Cultural para a organização do ensino de Matemática. Como recurso argumentativo, elaborou-se uma história virtual com base na Atividade Orientadora de Ensino proposta por Moura e seguidores (1996; 2010). Os conceitos matemáticos contemplados foram: grandeza, unidade, contagem, número, adição e multiplicação. Tal sistema conceitual foi abordado no contexto das significações aritméticas, algébricas e geométricas. Nele apresentaram-se possibilidades de objetivação, no modo de organização do ensino, dos princípios da Teoria Histórico-Cultural. Os resultados desvelam indicadores que vislumbram a possibilidade de contribuir para o desenvolvimento das bases para a formação do pensamento teórico matemático no período de transição da atividade do jogo para a de estudo.

Palavras-chave: Teoria Histórico-Cultural. Educação Matemática. Atividade Orientadora de Ensino. Proposição de Davýdov e colaboradores.

1. Introduction

Recent research on Mathematics Education[1] has questioned the current contents and methods used in Basic Education in Brazil. Such questions are generated from the results presented by students in scientific research (Galdino, 2016; Rosa; Flores, 2015) and official assessments, such as the 2014 National Literacy Assessment (ANA). According to Renato Janine Ribeiro, Minister of Education, from April to September 2015, the major problem for children in the 3rd year of Elementary School is Mathematics, an area of study in which 57% of them demonstrated an inadequate level of learning (Agência Brasil, 2015).

The Ministry of Education (MEC) divided the Mathematics exam results into four levels, the first two levels were considered inadequate. With regard to this division, Renato Janine Ribeiro states that in the "[...] first level, the child has only the knowledge acquired from home. In the second level, he or she is able to solve operations, but very simple ones" (Agência Brasil, 2015). Therefore, ANA results published in September 2015 reinforce what some researchers from Brazil already claim (Galdino, 2016; Rosa, 2012; Brunelli, 2012; Hobold, 2014 among others): the majority of Brazilian students remain within the limits of empirical knowledge.

When analyzing these results, we questioned ourselves about Mathematics in early childhood. To answer the questions that move us into this research activity, the aim of this article is to reflect upon the principles of the Cultural-Historical Activity Theory, particularly upon the dialectical thinking, and "express them in the ‘teaching materials development technology'" (Davídov, 1988, p. 108, our translation, emphasis in original). The initial question is: what are the possibilities of objectification of the fundamentals of the Cultural-Historical Theory in the structure of Mathematics teaching in the first years of study activity? For this purpose, we assumed two fundamental aspects highlighted by Moura (2007): Mathematics, as a cultural and symbolic tool; and childhood, as a cultural-historical state of being of the individual who learns (MOURA, 2007).

Based on the Cultural-Historical Theory, we sought support from the studies concerning Davydov’s proposition for teaching Mathematics, in the work of some researchers, such as Galdino (2016), Crestani (2016), Freitas (2016), Moya (2015), Silveira (2015), Rosa (2015), Mattos (2015), Cunha (2014), Hobold (2014), Rosa (2012), among others. Also in the

[1] Among which stand out the investigations carried out by researchers of the Studies and Research Group on Educational Activity (GEPAPe - USP), Theory of Developmental Teaching in Mathematics Education Group (TEDMAT - UNISUL) and Research Group in Mathematics Education: a Cultural-Historical Approach (GPEMAHC - UNESC).

Teaching Guiding Activity (TGA) proposed by Moura and his followers (Moura, 1996; Moura Et Al. 2010; Araújo, 2003; Moretti, 2007; Serrão, 2006; Moraes, 2008, Cedro, 2008; Lopes, 2004; Panossian, 2014; Nascimento, 2014; among others). Although it is possible to observe an expansion of studies and research that is based on the principles of the Cultural-Historical Theory in Mathematics teaching, there are still few studies that articulate the TGA with Davydov's proposition. In this article, we considered Davydov's proposition and the AOE as a unity, as a possibility of objectification of the Cultural-Historical Theory fundamentals in the way Mathematics teaching is structured. Along with the concepts of teaching and education, it is convenient to consider a broader concept - the concept of appropriation, once "the process of appropriation leads the individual to reproduction, in their own activity, of historically formed human capacities" (Davídov, 1988, p. 56, our translation).

Clarifying the concepts of education and teaching requires considering that both of them are, therefore, appropriation and reproduction of historically and socially formed human productions (Davýdov, 1982). The TGA proposes, to the appropriation process of concepts from the initial learning situation, that "the need that led humanity to the development of the concept" be clarified (MOURA et al. 2010, p. 223), similarly to what may have happened in a certain historical moment. Davídov (1988) points out that "it is essential that children know about the problems encountered when man had to solve similar tasks for the first time, in the process of appropriation of the new acting procedure" (Davidov, 1988, p. 189, our translation).

The TGA, “as mediator, leads to the psychic (mental) development of the subjects who perform it” (Moura et al., 2010, p. 108). According to Moura (1996), the initial learning situations can occur through different methodological resources: children's games, virtual story of concept and emerging everyday situations. The relation between mental development and education implies, however, in understanding the role of some activities that humans develop.

In every leading activity the corresponding psychological configurations are considered, the succession of which forms a unit of the child’s mental development. Among the main activities analyzed by Davídov (1988), such as play, learning, work, activity and so on, we will focus on two of them: the play activity and the study activity[2], as we understand them as essential to pedagogical action in Early Childhood Education, as a transitional period from the first to the second. In the author’s word,

> [...] the play activity is most characteristic for children between three and six years old. In its completion, imagination and symbolic function it develops in the child the orientation in the general sense of human relationships, the ability to separate among them the aspects of subordination and direction; [...] the study activity develops in children aged six to ten. On its base, awareness and theoretical thinking arise in children, the corresponding skills (reflection, analysis, mental planning) are developed and also the needs and purposes of study (Davídov, 1988, p. 74, our translation, emphasis in original).

When analyzing the main activities, we could adopt, as a basis for this study, the study activity alone, for we consider it as the most typical of children in the first years of the schooling period, to which we will focus on this article. However, we chose to reflect upon the transition between the play activity and the study activity. Because,

[2] The term study activity should not be taken as a synonym for learning. "The study activity has a specific content and a special structure and it is necessary to differentiate it from other types of leading activities" (Davídov, 1988, p. 159, our translation).

> [...] although a leading activity, in particular, is characteristic of a period of development associated with certain age group, that does not mean that other types of activities are absent or are 'introduced by force' in a certain age group. It is known, for example, that playing is the leading activity for preschool aged children. But, it is also found in children study and work elements in the preschool period of their lives. However, these elements do not determine the character of the basic psychological changes in a certain age: the different aspects of these psychological changes depend more than ever on the nature of the play (Davídov, 1988, p 71-72, our translation, emphasis in original).

From this perspective, it is necessary to develop watchful eyes for the transition from the play activity to the study activity, trying to learn a little more about the children in the early years of the school period, in addition to areas of education and the teaching of mathematics.

2. Transition from the play activity to the study activity: the virtual story about "lady cockroach"

We understand the virtual story of concept as one of the elements that make the transition from the play activity to the study activity possible, through the playful plot, the essence of the concept and its historical necessity. Thus, reflections on content and teaching method, presented in the course of this study, are directed to the transitional period between play and learning. Those were carried out in association with researchers from TEDMAT[3] research group, starting with the development of a virtual story. The plot context lies in the folktale "LADY COCKROACH'S WEDDING" (Machado, 2004), in which the main character, Lady Cockroach, sends a letter asking children for help:

Illustration 1- Lady Cockroach's Wedding

LADY COCKROACH'S WEDDING

Hello, kids.

Lady Cockroach is the one who writes here, who has a ribbon in her hair and money in her box. You may have heard my story out there and now you know that after many suitors, I will marry Lord Dormouse. We will gather all animals for a big party. To do so, we rented the prettiest party room of our small town Pinheiral and a nearby town band will liven up the party.

As for the room decor, we like roses a lot, so each table will have a vase filled with them. As our friends have many children, sons and daughters-in-law and grandchildren, there will be many tables all over the room.

[3] Theory of Developmental Teaching in Mathematics Education Research Group based at UNISUL, created and coordinated by Prof. Dr. Joselia Euzebio da Rosa. This group is part of the *gepapean* contact unit (GEPAPe / USP – Portuguese acronym for the group's name) of Santa Catarina with GPEMAHC (UNESC - Portuguese acronyms), created and coordinated by Prof. Dr. Ademir Damazio.

But I'm having trouble figuring out how many roses we will need for the decoration. My fiancé is a little clumsy with everything. Although he does not make too much noise like the other suitors, he talks too much. So when I try to think about the amount of roses that we will need, he keeps distracting me, talking about the wedding cake, sweets, cheese, feijoada (black bean meal) ... and that distracts me.

So, I ask for your help: how many roses will we need to decorate all the tables?

I'd like to ask you to send me an explanation on how to solve this problem because I need to pass on the instructions to my friends who are helping me at the party.

Thanks in advance,

Lady Cockroach.

Source: Galdino, 2016.

The first version of the previous letter was presented in Galdino's research (2016), referring to the concept of multiplication. In order to generate reflections on Mathematics in the early years of schooling, we opted to make a few adjustments in the story previously elaborated. It should be noted that we considered the letter a virtual story because, due to its playful plot, it places the child before a situation problem similar to the one experienced by man in history (Moura; Lanner De Moura, 1998).

For playful we understand a situation where "the violation of the logic of actions and rules is repulsed" (Elkonin, 1998, p. 299). According to Davídov (1988, p. 80, our translation), the characteristic feature of the play consists in the possibility for the child to "perform an action in the absence of the conditions to truly get results once their reason does not consist in achieving an outcome, but lies in the very process of fulfilling the action." This lack of coincidence between action (for example, helping solving a problem) and its operations (for example, sending a letter to a cockroach) leads the child to fulfill a playful action in an imagined situation.

So, with the story, we aim to initiate the process of resolution, through a playful action, as in Lady Cockroach's story. Thus,

> [...] in the learning process, as a leading activity in the early school age, students not only reproduce knowledge and skills related to the fundamentals of the forms of social consciousness [...], but also the historically emerged capacities, that are on the basis of theoretical consciousness and thinking, reflection, analysis and thought experiment (Davídov, 1988, p 81-82, our translation.).

It should be noted that we understand the early school age as a preparation for learning as of human development, in accordance with Leontiev (1983), to whom every child is at birth a candidate for humanity. This means "allowing the appropriation and incorporation of the social experience and not only the accumulation of knowledge and skills, but the development of, specifically human, qualities and capacities" (Nascimento; Araujo, 2010, p. 117).

The basis for qualitative advances in the development of thinking should include reflection, analysis and thought experiment, which are characteristic of theoretical thinking (Davídov, 1988), as this step aims to develop the basis for this type of thinking, especially during the transition period between the play activity and the study activity.

During this period, in relation to students' reflection, the teacher has the role of guiding and directing such reflection, so that they are not limited to searching for answers, but to the preparation of questions and hypotheses (Rosa, 2012). Therefore, it is important to develop, among others, the investigative action, materialized in the elaboration of questions directed to both teacher and classmates that will contribute to the structure, at a later stage, of the study activity (Rosa, 2012). Furthermore, according to Rubinstein (1979), the pedagogical process as the activity of the teacher builds the child's character in development, as the teacher guides the activity of the child.

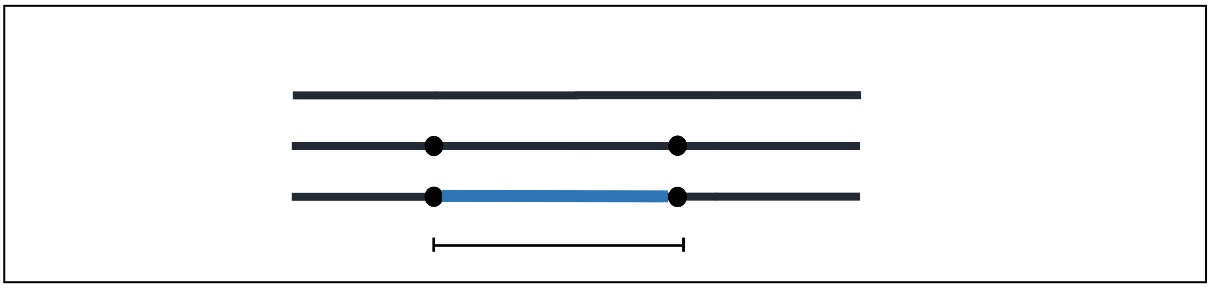
According to the fundamentals of the Cultural-Historical Theory, the appropriation of the conceptual framework is based on an oriented movement from the general to the particular (Davýdov, 1988). This movement occurs, from the beginning, by means of the universal relation itself (Rosa, 2015). In this regard, the resolution of the story presented previously originates from the general aspect of the concept. The amount of vases or roses is not given. The survey of hypothesis for particular situations becomes the first step towards the development of new questions, since the letter is presented in its general character; there is neither a specific amount of vases nor the amount of roses in each vase.

The teacher guides the children to reflect upon the problem involving Lady Cockroach (What is the total amount of roses that she will need?) and they reflect upon how to solve it without having the information on the arithmetical value of the quantities involved. Therefore, it is necessary to analyze the relation between the quantities, more specifically the relation between each rose and the total amount of them. Which one will be larger? Which one will be smaller? Are they the same? The findings may be recorded using line segments. But what are line segments?

To answer that question, we suggest the teacher the following action: first, ask for suggestions on how to draw a straight line on a poster attached to the classroom wall. How to draw a straight line? The logically correct answer refers to the use of an object with straight sides or a ruler (Rosa, 2012).

Then, the teacher asks a child to mark two points in each of these straight lines. In addition, the teacher also asks the child to highlight with a different color the part between the two points and explains "that the highlighted part is called segment. That's why, sometimes, the ends of the segments are marked with , as if it was the cutting line" (Rosa, 2012, p 88-89.), as illustrated below (Illustration 2):

Illustration 1 – Lines, points and segment



Source: Our illustration based on Rosa (2012).

The introduction of these geometric elements in Davydov’s proposal, includes "[...] the interrelation and connections between elements like point, line segment and straight line inserted into a system [...], once it is consisted of infinite segments, each one of them, no matter how small, is consisted of infinite points" (Rosa, 2012, p. 90).

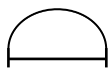
The question that arises after drawing the segment is that: how to represent, using line segments, the relationship between each rose individually and the total amount of them?

Illustration 2 – Scheme of the inequality relation between quantities

Source: Our illustration, 2016.

In other words, which one of the previous segments (Illustration 3) represents the unit (one rose)? And which one represents the total amount of units (all roses)? Based on the reflections already undertaken, it is possible to conclude that the first segment represents the whole (total amount of roses); and the second, the unit (each rose). Thus, the smaller segment, from one end to the other, is a unit, as indicated by the arc (Illustration 4).

Illustration 3 – Introduction of the arc



Source: Our illustration, 2016.

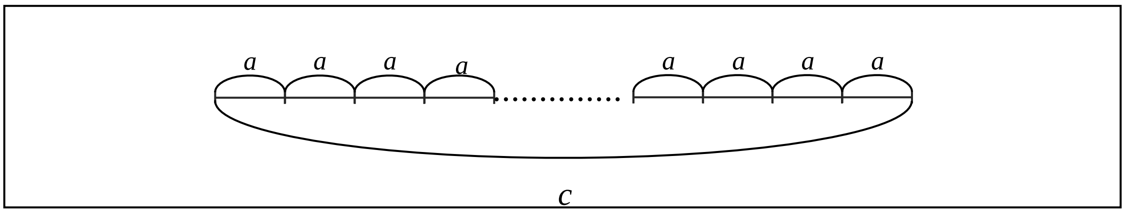
Based on illustration 4 we have, so far, according to Rosa (2012), the geometric representation of the unit and the whole. However, in Mathematics, when we do not know the arithmetic value of a quantity we want to measure or count, when they are within the general, we can represent it by a letter, or through algebraic significances (Rosa, 2012). "Algebra as a mathematical knowledge [...] is essentially characterized by the pursuit of the **quantitative relation among the variable quantities, in general**, which it is its essential theoretical relation or cell" (Panossian, 2014, p. 107, emphasis in original).

Therefore, the letters represent values that can be varied in the movement between general and particular. Accordingly, as an example, let us consider a rose as a unit to count *one by one*.

It will be represented by the letter *a*. Besides, we have another unknown value, the whole, which will be represented by the letter *c*. With these general values, represented algebraically, it is possible to make some reflections. For example, which is one greater, *a* (unit) or *c* (whole)? Or are they the same? Or is the whole less than the unit? The mathematically expected responses are: *a* is less than *c* ($a < c$) and *c* is greater than *a* ($c > a$), once there will be more than a rose in the hall. As for the less than (<) and greater than signs (>), the wide side always points to the larger number.

So far, the discussions were held in the general context and represented geometrically and algebraically, but the question arises: how can the arithmetic representation be revealed in this context? It is up to the teacher to make questions such as, for example, how many times is *a* repeated in *c*? Or, in the context of the story in question, how many roses will Lady Cockroach need? (Illustration 5).

Illustration 4 – Geometric and algebraic representation of the amount of roses

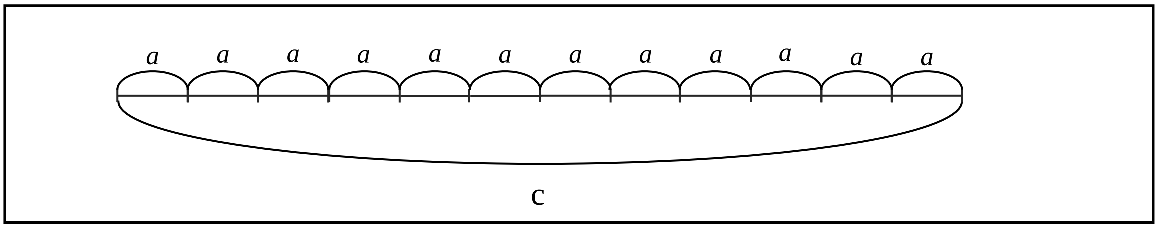


Source: Our illustration based on Rosa (2015).

Based on the scheme presented visually, through algebraic and geometric symbols (Rosa, 2015), can we determine the amount of roses that Lady Cockroach will need? Not yet, because we do not know how many times *a* is repeated in *c*. It is an unknown value. Thus, $a + a + a + a + a + a + a + a + a + a + a + a + ... = c$. How many times is *a* repeated? As it is an unknown value, we will represent it as *y*. Thus we have that $a \times y = c$.

We still do not know how many roses she will put in each vase or how many vases there will be. These values are essential for us to solve the problem experienced by Lady Cockroach. Assuming that Lady Cockroach will use the amount of roses presented on the scheme in illustration 6:

Illustration 5 – Assumption for an amount of roses



Source: Our illustration based on Davydov's proposition, 2016.

In the previous scheme, how many times is the amount *a* repeated? In response, we have: $a + a + a + a + a + a + a + a + a + a + a + a = 12a$. That is, the number *a* is repeated twelve times. The result (12), for this particular situation, emerges from the universal relation of the number concept consisted of a quantity to be measured or counted, a unit of measurement and the number of times the unit is repeated in the whole (Rosa, 2012).

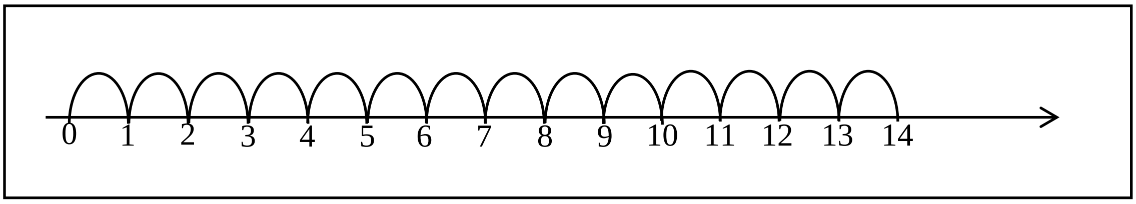
As for the previous scheme (Illustration 6), how is it possible to represent results greater than 12? Is it possible to represent, for example, 14 roses? No, because it is limited to 12 units, that is why it is necessary to build a new scheme from the scheme already built to represent the new amount (14).

Let us begin with the drawing of a straight line: the suggestion is that, over this line, we indicate the amount of roses that Lady Cockroach will need. It is necessary to choose one of the points within the straight line to represent the starting point of the counting procedure, when we have not counted any rose yet. Besides, there is another need: which number can we use to represent that starting point? Historically, mankind has determined that this number is zero. And how can we indicate in which direction the counting procedure will take place? Using the arrow located on the right end of the origin of the line (Rosa, 2012).

Then, which number can represent one rose? In accordance with the humanly standardized numerical sequence, the required number is 1. Therefore, we will draw an arc for each unit (each rose). Will this counting procedure have continuity with what number? With number two. That is because *one* plus *one* equals *two*.

This process of including new numbers will depend on the particular situation established with the children. To build a diagram representing 14 roses, for example, we proceed with this process until the final point consists of this quantity, using the following reasoning: 2 + 1 = 3, 3 + 1 = 4, 4 + 1 = 5 ... (Illustration 7).

Illustration 6 – Number line



Source: Our illustration, 2016.

This scheme (Illustration 7), in Mathematics, is known as the number line. Thus, the concept of number is expressed "as the algebraic relation between a larger quantity and a chosen unit (Galperin, Zaporozhets; Elkonin, 1987, p.306). It is necessary, in the introduction process of the number line, that the perspective of infinite is present, in other words, that the child understands that numbers do not end at 14, once every number has a successor, for example, 14 + 1 = 15 15 + 1 = 16, and so on. Each and every whole number (*a*) will always have a successor number to it ($a + 1$). It should be emphasized that the number is the result of a counting procedure, and the concrete expression of the number, the number line, is not an act by which we capture in elementary and primarily sensory form, but it is mediated by the essential and universal relationship of the concept (Rosa, 2012).

On the other hand, in the implementation process of empirical knowledge, it does not occur, it is limited to "selecting illustrations, examples that fit in the corresponding class of objects" (Davídov, 1988, p. 154-155). In the analysis about teaching, when he was in Russia, Davýdov (1982) verified the emphasis on the development of empirical thinking, "in the formation process of the concept of number, each number was presented to the student with the idea of direct relation with the amount of objects" (Rosa, 2012, p. 24). Thus, number one was represented by an object, number two, by two objects, and so on, as currently prevails in most Brazilian propositions (Hobold, 2014; Silveira, 2015; Galdino, 2016). In the example in question, number one would be directly related to one rose; number two, with two roses, and so on. There are no mediating elements, geometrically given, between the object that needs to be counted and its arithmetic expression. Is number one just one object? Does the child need to go to school to learn this numeral-object representation?

To answer these questions, first, it is necessary to emphasize that, in educational institutions (Early Childhood Education centers, schools, etc.), children must learn what is not accessible to them at home, outside home and when playing with friends (Davydov, 1982; Davídov, 1988). Therefore, the required understanding of the concept of number, primarily, is that the number one is not just one rose: for example, it may also refer to one liter, one dollar coin, one meter, etc., depending on the quantity (volume, monetary value, length, discrete quantity...) and the chosen unit. In this sense, the question is: which one is greater, number 1 or number 3? The logically correct answer is that it depends on the quantity and unit in question. For example, if we have inside a container 1 liter of any liquid, and inside another container, 3 milliliters of the same liquid. In which container will there be more liquid, in the first one or in the second one? To answer this question, it is not enough to just look at the numbers alone (1 and 3), because, from this analysis, we would mistakenly conclude that 3 > 1. However, when we focus on the unit of measurement, we will note that the first container has more liquid. Still using the same numbers, let us take another example, comparing the monetary value taken as a unit of measurement: dollar: Which one is greater, 1 dollar or 3 cents? Number three, in this case, represents three cents in one dollar, therefore, it is a smaller amount than the one represented by the number one. Unlike what happens in the context of the virtual story in question, in which 3 roses are more than 1 rose.

Such reflections unleash mental processes on the concept of number, in which the unit is not limited to a rose. In addition, the reflection on the number line enables the representation of different amounts of various magnitudes. Including those generated from the unit subdivision (Rosa, 2012).

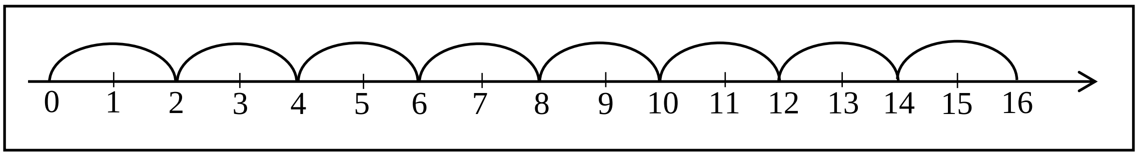
Another aspect limiting the development of theoretical thinking to occur, when we associate the number to the amount of corresponding discrete objects, refers to the understanding of the infinite character of the number. This is because the symbols have a concept of such large numbers that would hardly be created by direct observation of quantities (Aleksandrov, 1976). When we hang posters on the walls of Early Childhood Education or the early years of Elementary School classrooms that represent number one: a duck, number two: two cars, number three: three bears, and so on, how would we represent number 1,236?

Although larger numbers are not introduced in Early Childhood Education or in the early years of Elementary School, we believe, based on Mathematics Education research from the perspective of the Historical-Cultural Theory (Rosa, 2012; Hobold, 2014; Silveira, 2015;

Galdino, 2016 among others), that the introduction of the concept of number exclusively by means of counting the collections of objects hinders its theoretical understanding. There is also a need for mediating elements.

Continuing the reflections on the virtual story in question, let us assume that Lady Cockroach will put two roses in each vase and that there will be eight vases in total.[4] To accelerate the counting, we can adopt as a unit the amount of roses by vases (two). We will do the calculation with the help of the number line, and each group will be represented by an arc. The result will be the geometric point in which the last arc reaches:

Illustration 7 – Calculation of the number line



Source: Our illustration, 2016.

Thus we have 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 2 x 8 (two taken eight times) = 16. In accordance with the calculation result obtained using the number line, in this situation, Lady Cockroach will need sixteen roses to decorate the party hall. It would be in this context of reflections in which the multiplication table of number two would be systematized.

Therefore, the mathematical concepts are addressed not in isolation but in the context of a conceptual system that involves, based on the study of the relations between quantities, the reflections on counting, addition and multiplication in the interrelation of geometric, algebraic and arithmetic significances.

It is worth mentioning that there is some resistance regarding the algebraic systematization as presented throughout this paper. However, it is important to consider this broader conceptual framework, in its different meanings in the reduction movement from the concrete to the abstract and from the abstract to the concrete thought.

We considered as a concrete starting point the initial situation experienced by Lady Cockroach: from that we revealed the elements that comprise the universal relation (unit, the amount of times the unit is repeated in the whole and the whole). From this relation, abstracted and modeled by geometric and algebraic elements, it was possible to solve the initial problem of the virtual story through hypotheses of particular situations. This suggestion originates from Davydov's understanding that children, during the transitional period from the play activity to the study activity,

> [...] initially master the general procedure of a particular tasks solution. The homework solution is important not only in the particular given case, but for all cases of the same type. Here the students thinking moves from the general to the particular [and singular] (Davídov, 1988, p. 179, our translation)

[4] The amounts shown are merely an example; each teacher may adapt it as the children's potential development area.

In summary, the conceptual movement created from the reflections on the letter was oriented from the general to the particular and specific. The initial problem, which includes the conceptual system in its general aspect, is the starting point for the various particular and specific situations that may arise, "the transition from the general to the particular [and specific] is carried out not only fulfilling the contents of the initial abstractions, but also replacing the symbols expressed as letters with concrete numerical symbols" (Davídov, 1988, p. 215, our translation).

Moreover, after the movement of reduction from the concrete (starting point) to the abstract, it is also necessary the ascent from the abstract to the concrete (Davídov, 1988), because, in this theoretical thinking movement of the phenomenon comprehension, the concrete appears twice, "as the starting point of contemplation and representation, further elaborated on the concept and as a mental outcome of the forms of abstractions" (Davídov, 1988, p. 150, our translation). Concrete is "a process of synthesis, of synthetic inference; starting from the initial abstraction, it develops all the concrete diversity of the phenomenon" (Ilienkov, 2006, p. 172-173).

The learning actions presented throughout this article, as possibilities for resolution of the virtual history: *Lady Cockroach's Wedding*, are directed to the disclosure of the conditions of the emergence of the conceptual framework, "of which are in process of appropriation (for what and how to separate its content, why and where this is fixed, in which particular [and individuals] cases it manifests later)" (Davídov, 1988, p. 183-184, our translation).

The specific situations, concrete final point, focus on the different possibilities to reply to Lady Cockroach through letters or even drawings. This diversity is possible only because the concrete that we have taken as a starting point was presented in its general character and not in its specific. Thus, as a result,

> [...] as a specific content of the theoretical concept, the objective relation of the universal and the specific appears [...]. In that concept, different from the empirical, it is not included something that is equal in each object of a class, but rather the interrelations of isolated objects within the whole, within the system of their training (Davídov, 1988, p. 126, our translation).

Therefore, developing a concept about a phenomenon or object means to reproduce it mentally, build it; in other words, it consists in revealing its essence (Davídov, 1988). The learning of scientific concepts and the development of theoretical thinking, which are the main purposes of the instruction activity, only occur through an oriented, organized and systematized process, through analytical procedures that exceed the limits of empirical experiences.

3. Final considerations

This text was prepared in order to reflect upon the possibilities of objectification of the fundamentals of The Cultural-Historical Theory in the way Mathematics teaching is structured in the transition period between the play activity and the study activity.

We may be questioned: after all, is the objectification of these assumptions possible in the classroom? The answer lies in the results of 25 years of Davýdov and his colleagues' research. They say it is possible, indeed. In addition, the authors explain that children develop the foundation for theoretical thinking during this transition phase.

It is worth noting that the Russian education reality, when Davýdov elaborated his proposal, was similar to our current reality (Rosa, 2012; Brunelli, 2012; Hobold, 2014; Galdino, 2016). In addition, the results of our research in the classroom indicate that the possibilities do exist (Rosa; Flores, 2015).

We emphasize that, if the basis of human development occurs through appropriation and reproduction of knowledge historically produced by humanity, we have, as a fundamental premise, the structure of content and method that provides the basis of theoretical concepts, in its current stage of development, which exceed the limits of the prevailing everyday experience in educational propositions in Brazil.

We believe that the fundamentals of the Cultural-Historical Theory can contribute to rethink of Mathematics teaching. For this purpose, we are conducting investigations that focuses on the interaction between Davýdov's proposition and the Teaching Guiding Activity (Crestani, 2016; Galdino, 2016). These are initial studies, but they indicate great possibilities of changing the Brazilian Mathematics Education, which is currently reaching poor results.

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