CHANGING TEACHERS' MATHEMATICAL KNOWLEDGE DURING THEIR TEACHING ACTIVITY

A MUDANÇA DO CONHECIMENTO MATEMÁTICO DO PROFESSOR EM ATIVIDADE DE ENSINO

Wellington Lima Cedro
wcedro@ufg.br

Universidade Federal de Goiás - Brazil

ABSTRACT

The aim of the current study is to discuss the mathematical knowledge changing process in future Math teachers. Based on the theoretical framework provided by the cultural-historical approach of Psychology and, mainly, of the Activity Theory, the study highlights the mathematical knowledge of teachers from the movement that inserts them in the educational activity. The formative experiment was herein used as methodological procedure. The experiment was set and developed with Mathematics pre-service teachers from a Brazilian public university, throughout a school year. The results indicate the need of organizing the teachers' education process as a way to allow teachers to experience the teaching activity, as well as to reason about it and to become aware of their actions in order to overcome the humans' alienation from themselves. Thus, we believe in the possibility of having a transformation process through the teaching activity in Math teachers. Therefore, it is possible to change their mathematical knowledge.

Keywords: Mathematical knowledge; Teacher's education; Teaching activities; Cultural-historical theory; Formative experiment.

RESUMO

Este artigo tem como objetivo discutir o processo de mudança do conhecimento matemático de futuros professores desta disciplina. Tendo como base os aportes teóricos fornecidos pela abordagem histórico-cultural da psicologia e, em particular, da Teoria da Atividade enfoca-se o conhecimento matemático dos docentes a partir do movimento de inserção do professor em atividade de ensino. O procedimento metodológico utilizado nesta investigação foi o experimento formativo. Este experimento foi organizado e desenvolvido durante um ano letivo com estagiários do curso de licenciatura em Matemática de uma universidade pública brasileira. Os resultados obtidos pela pesquisa indicam para a necessidade de uma organização de processo de formação do docente que permita não somente a vivência da atividade de ensino, mas a reflexão e a tomada de consciência das suas ações possibilitando a superação da alienação do sujeito humano frente a si mesmo. Com isso acreditamos na possibilidade do processo de transformação da docência em atividade para o professor de Matemática. Com isso temos a possibilidade de mudança do conhecimento matemático do professor.

Palavras-chaves: Conhecimento matemático; Formação de professores; Atividade de ensino; Teoria histórico-cultural; experimento formativo.

1. Introduction

The mathematical knowledge is one of the main aspects in the education process of Math teachers. Overall, most formative approaches used in the Licentiate framework lose their effectiveness because they are not able to change the mathematical knowledge of teachers. Thus, it is worth asking ourselves: How can we ask teachers to change their teaching practices if the mathematical knowledge perspective they take is based on a utilitarian and reductionist view of Mathematics?

The current study presents the results of a survey (Cedro, 2008) conducted with future Math teachers in a Brazilian higher education institution, focused on understanding the mathematical knowledge changing process in initial-education teachers.

Many studies have focused on teachers' mathematical knowledge over the past 30 years. It is worth highlighting the studies conducted by Ball (1998), Ponte (1998), Blanco and Contreras (2002), Garcia and Sanchez (2002), Garcia (2003), Moreira and David (2005), Santos (2005), Ball, Thames and Phelps (2008), Ball and Hill (2009) and Duarte, Oliveira and Pinto (2010), who focused on the knowledge of mathematical contents. Thus, the current study contributes to such debate by approaching teachers' mathematical knowledge according to the theoretical contributions of authors linked to the so-called historical-cultural theory.

The current study is organized as follows: firstly, we discuss the Math teachers' education process. Next, we present a proposal to organize the Mathematics teaching process according to the Teaching-Orienteering Activity (Moura, 1996) and briefly discuss its formation. Subsequently, we present the methodological aspects, as well as the concepts of the future teachers who participated in the study, in order to analyze their mathematical knowledge changing process. Finally, we present our final considerations.

2. Educating the teacher in question

Currently, the teachers' education process is not a necessary condition to empower individuals in education and, consequently, to value the universal human wealth. On the contrary, one can see that education eventually becomes an element that decisively contributes to the reproductivist model proposed by modernity. Thus, it is possible to understand that there is not an education process, but a semi- or pseudo-education one (Maar, 1992).

The challenge lies on how to provide the necessary and sufficient education to individuals so that they can make a qualitative leap from their individual conceptions about the world to conceptions that reflect the most advanced universal knowledge gotten by humankind. Such situation becomes more delicate when we learn that education is one of the ways to accomplish such humanization process. According to Canário (2001), the school is currently going through a "mutation" process, which leads us to question its organization, i.e., the ways of thinking about school institutions.

Thus, the school issue may be summarized as follows:

> According to the historical configuration we are aware of, the school, on the one hand, is obsolete and anachronistic. On the other hand, according to the viewpoint of those who work in the school, it suffers from a meaning deficit and is marked by a social legitimacy deficit, since it does the opposite of what it says (it reproduces and accentuates inequalities, and creates forms of relative exclusion) (Canário, 2001, p. 10).

It is possible to notice the alienation process in the pedagogical work within such school changing scenario: the individuals are expropriated from the control over their work and it has consequences, i.e., the human labor becomes a commodity and such transformation is the core of the historical transformations that allowed building the industrial capitalism. The process of separating humans from their work makes such work harrowing and prevents it from being experienced as an expression of the self. This issue, which gives structure to the debate about the *old* and the *new* social issue, also constitutes the key issue about the school and the nature of the school work. Overcoming the school work alienation is the condition for the real production of meaning, regardless of its "instrumentality" in the access to material and symbolic goods (Canário, 2001).

Thus, the challenge the research community faces is: how can they develop research instruments able to reveal the elements that constitute teachers' professional development and education in order to overcome the school work alienation? Several concepts and metaphors (reflective teacher, researcher teacher, situated practice, etc.), which seek to understand teachers' education, have emerged within the educational environment as a way to respond to such challenge (Ferreira, 2003).

According to our viewpoint, the debate about teachers' education should focus on understanding its object. Thus, it is firstly necessary to define the teacher's activity object as the delimiter of parameters and variables that allow understanding the teachers' education and, consequently, their professional development. The understanding of teachers' object is related to the needs of individuals involved in the educational activity. Such activity is understood according to the claim, which states school as the place where constitutive meanings are produced and exchanged in order to give meaning to the actions of all individuals involved in educational activities, i.e., the school is understood as a learning space (Cedro, 2004).

Therefore, the teachers' activity object is the concrete character of their need. We herein understand the object based on the Leontievian perspective (Leontiev, 1978), according to which the reason is the object that responds to a particular need and that encourages and directs the activity of the individual.

Since teachers primarily work with concepts, it is possible to say that "these professionals create meaning for what is taught and use words as their main instrument" (Moura, 2004, p.258). This cultural-historical perspective of the individual - in this particular case, the teachers - implies to understand that they "build themselves by producing their objects and that, by producing their objects, they also produce their meanings" (Moura, 2004, p.260).

Thus, the teachers' activity is permeated by the relation between meaning and significance, since "whenever an individual performs an activity, he/she needs to

understand it as something that will meet his/her needs. Such activity must have a personal meaning because, somehow, it is triggered by a reason that moved or can move the individual"(Moura, 2004, p.259)

3. Teaching Mathematics: The teaching-orienteering activity

For some time, the scientific community of Mathematics educators (Kilpatrick, 1994) has discussed and presented teaching organization models as an attempt to overcome the traditional view, which is guided by the repetition and memorization of certain procedures, as well as to overcome the passivity against the mathematical knowledge. However, few objective results have been observed for many reasons. We have noticed that we still apply an education process that seems increasingly isolated from other actions we take on daily basis. Thus, it is imperative to find new ways of organizing the learning spaces by taking into account the essential role played by the teaching activity as the core element in the organization of Mathematics.

Such conviction leads us to propose a teaching organization way that has the aim to promote the humanizing education of the individual and that is outlined according to the *teaching-orienteering activity* (Moura, 1996). We believe that such form of organization will enable all individuals involved in the process to take ownership of the conceptual nexuses[1] that allow the full development of their human condition.

According to Moura (1996, 2000, 2001), the *teaching-orienteering activity* is structured in such a way to allow individuals to interact mediated by a content and to negotiate meanings in order to collectively solve a problem situation. The orienteering character dues to the fact that such activity defines the fundamental elements of the educational activity and respects the dynamics of interactions that emerge in the learning space and that not always achieve the results targeted by the teacher.

The teaching-orienteering activity structure is based on the unity between the logical and the historical aspects of the concept (Lanner de Moura & Sousa, 2005). This statement implies the need of not understanding the genesis movement of the concept as just another element in the History of Mathematics, at the risk of converting such History into a mathematical content itself. Thus, the historical dimension of the mathematical knowledge is understood within the social and cultural process of the concept. According to such perspective, understanding the genesis of the concept means perceiving that it is part of the history, in which men and women sought and developed solutions for certain problems in order to meet their real needs.

The objective nature of the logical and historical aspects of the concept is found in the problem situation that triggers the educational process. Such problems must embed the essence of the concept. According to Moretti (2007), it implies understanding that the History of Mathematics, which involves the triggering problem, is not the factual history, but the history impregnated in the concept. Conceiving the problem situation in

[1] Conceptual nexus is the link between ways of thinking the concept, which do not necessarily meet the different languages of such concept (Sousa, Panossian & Cedro, 2014).

this way opens a range of possibilities for its materialization, i.e., the problem situation may either take the form of a virtual history (Moura, 1992) or that of a game, of a playful activity or of a contextualized problem. Thus, what prevails is the teacher's intent.

> By using it [the problem situation] as a teaching resource, the teacher's intent is that the concept to be taught becomes a cognitive or material necessity to the students so that the actions they take in the search for the solution to the problem must comply with the reason that leads them to act. Therefore, they can, in fact, be in activity (Moretti, 2007, p. 99).

Thus, the teacher should create conditions for individuals to interact motivated by the attempt to address a particular problem in order to enable an uninterrupted flow in the shared solution elaboration process, which may encompass either isolated individuals, small groups or the entire class. Such statement reflects the need of developing a teaching organization model to allow the actual development of individuals. According to Vygotsky (2007, p.103), "the properly organized learning results in mental development and sets in motion a variety of developmental processes that would be impossible apart from learning". Only through this properly organized learning can the individual be aware of and internalize a variety of developmental processes, which just emerge through the cooperation and collaboration with other partners and in a particular context and space. According to the Vygotskyan terms, there is the emergence of the zone of proximal development.

> It is the distance between the actual developmental level, as determined by independent problem solving, and the level of potential development, as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 2007, p.97).

By analyzing Vygotsky's definition, it is possible to understand the actual developmental level as the one that comprises the existence of a proper knowledge by the individual, and it indicates that such individual is able to independently perform a given task. On the other hand, the potential development level corresponds to processes under formation. Therefore, the space between these two levels, which Vygotsky called zone of proximal development, appears as the field of possibilities that allow learning the scientific knowledge (Araújo, 2003), as well as taking ownership of the characteristic traits of human beings. Thus, teachers must guide the actions in the classroom in a manner suitable to the individual's appropriate development level. They must use mediation instruments, such as the teaching-orienteering activity, in order to enable the development of their mental functions.

In short, with respect to the main features of the teaching-orienteering activity, it is possible to say that:

> The activity [...] belongs to the individuals; it is a problem that triggers the search for solutions, allows individuals to improve their knowledge through analysis and synthesis processes, as well as to develop their ability to deal with other knowledge based on the knowledge they acquire as they develop their ability to solve problems (Moura, 2000, p.35).

Such statement allows to conclude that the teaching-orienteering activity is headed by the intentionality of individuals, by the development of conceptual knowledge nexuses

that, once mediated by different forms of language, enable the appropriation of theoretical knowledge and the consequent formation of theoretical thoughts. Therefore, this cycle has a dual character. Such double movement of the teaching activity transforms students by inserting them in a meaning-exchange process. It also enables teachers to create instruments to favor the learning process, to review the educational aims, as well as the teaching content and strategies by continuously assessing their work. Thus, the teaching activity assumes the role of organizing and educating element in individuals' learning, i.e., the activity is an education element for both the student and the teacher.

By taking the teaching-orienteering activity as basis, it is possible to give a new meaning to learning spaces, which may be understood as *"the place where individuals learn guided by the intentional action of those who teach"* (Cedro, 2004, p. 34, emphasis added by the author). Thus, they become spaces characterized by critical view, discovery and social practices.

The teaching organization through the orienteering activities gives to individuals the opportunity to critically and systematically analyze their practices, as well as their internal findings. Thus, it establishes a context that favors the emergence of critical remarks, since individuals learn and take ownership of their activities while they develop them (Engestrom, 2002).

According to such position, the learning spaces become places wherein the social practice constitutes an essential and necessary element for the individuals' development, since these spaces are true practice communities (Lave & Wenger, 1991). All participants fully develop the core tasks of the activity in such spaces and there are abundant horizontal interactions between such participants, especially interactions mediated by stories, by problem situations and by the search for solutions.

Finally, the learning spaces are characterized by the formation of theoretical concepts based on the ascension process from the abstract to the concrete sphere, which is an essentially-genetic strategy used to discover and reproduce the original conditions of the concepts to be assimilated (Davydov, 1982; 1988; 1999). Such process allows individuals to develop their ability to deal with problems in a theoretical way and to reflect about their thinking. Thus, it is possible to overcome the schooling processes that just contribute to the acquisition of special skills and knowledge, as well as enable the general mental development of individuals.

The elements that feature the learning spaces decisively enable the teaching to expand individuals' potential, i.e., to develop their personality as a whole; they also provide conditions for individuals to discover and establish their creative potential; allow all participants to effectively become learning activity individuals; lead to the understanding that the authentic learning and teaching occur when all individuals share their actions; and assume that the teaching procedures and techniques must meet the individuals' diversity and specificities. Thus, they do not allow the methods to become uniform and strict.

4. The Mathematics teachers' education and the formative experiment

Choosing the teaching activity as the basic element in the educational activity implies being aware that it requires an effort to understand the teaching and learning processes, as well as all the issues related to the spaces where these processes are developed.

Thus, it is possible to understand that teaching is the activity of those who build symbolic instruments in order to allow perceiving the reality or acting on it within a particular cultural universe. It makes the teachers' activity highly complex, because they deal with the concept in its most abstract aspect, namely: the word. The aforementioned studies showed that the individuals develop a process to give meaning to the actions they perform, whether they are students or teachers. However, for it to occur, it is necessary to organize the activity within a structure that allows inserting the individuals in an interaction process, thus enabling them to share meanings. The pedagogical practice learning is constituted in this activity's structuring movement. Such movement is equivalent to the development processes that any individual experiences when he/she faces the need of performing any activity in which certain instruments should be used, and which is defined by certain modes of actions. Thus, whenever the individuals use these instruments, they also develop skills to use them, give plasticity to their movements, improve them, and become more effective in their work. This very same process is necessary for the teacher to understand the concepts as instruments that may allow learners to access and appropriate the theoretical knowledge.

Having in mind the concern to offer theoretical and practical elements to help the teaching-learning process, we defined our proposal for the academic and scientific education of teachers. The first hypothesis of the current proposal is that the education processes must enable not only the acknowledgement and understanding of the work, historical, cultural and social realities of the teaching practice, but also give individuals the possibility to transform them and to exercise the condition of subject of their own knowledge.

According to this perspective, the academic and scientific education processes - the so-called initial education - form the basis for the appropriation of the content inherent to the teaching activity. The Mathematics teacher's education model we herein propose is not based on any theoretical, complex archetype marked by innovation. On the contrary, as we have previously indicated in the title of the current study, it is mostly focused on changing the way of walking *the path* already known by many professionals. In other words, we use the *old* theoretical and methodological instruments to understand the reality and to insert individuals in a theoretical, political, social and professional practice (Serrão, 2005).

Thus, the teachers' education presupposes their professional development and it totally escapes from the professionalism traps mentioned by Contreras (2002). However, it directly links such development to the appropriation of the teaching activity meaning, which allows teachers to broadly and unconditionally develop their human condition. This statement leads us to the following understanding of what a Mathematics teacher is.

> [...] the individual who masters the content, and who, above all, has the strategic vision of his/her action in the school teaching project in which Mathematics has a certain cultural and educational value. Thus, by carrying it out, he/she improves him/herself through new teacher's qualities while building, along with the students, a Mathematics able to humanize their worlds (Moura, 2000, p.126).

Thus, we developed our teacher's education proposal based on the well-known "2 + 2" curriculum model (the curriculum in this model is divided in such a way that the first two years of the course are intended to teach the basic knowledge from the scientific field and the other two years concern pedagogical disciplines - in the case of the licentiate degree - or more specific disciplines - in the case of bachelor's degree) and on the supervised practice. The proposal was implemented as a formative experiment, which associated the theoretical precepts of the historical-cultural approach, as well as those of the activity theory, with school education. Hedegaard (2002, p.214) indicated that, according to Vygotsky, "the experiment [...] is the embodiment of the claim that the formative genetic method is a research method necessary to investigate the formulation and the development of conscious aspects of the human relation with the world".

The herein performed formative experiment comprised three Mathematics pre-service teachers (Donizete, Laurinda and Tereza) from a Brazilian federal university, throughout an entire school year. The aim of the experiment was to observe the teaching-learning process and, consequently, acknowledge the actions that revealed the transformations or quality changes in the reasons of these individuals. Thus, in an attempt to unveil the phenomenon in question, we gathered data, evidences and information on the subject at a given time and space, as well as accumulated theoretical knowledge about it, in order to understand it.

The three participants presented very different features regarding a number of important aspects such as teaching experience, the time of experience, and the teaching activity concurrently performed with the activities in the undergraduate course. A brief description of the three participants is given below. Further details will be presented in the next section, wherein we will focus on the personal path of each one of them.

Tereza was a 25-year-old single student who was transferred from another campus of the same university. She did not work and shared an apartment located in one of the upscale neighborhoods of the city with a friend. Her teaching experience was restricted to her activity as a computer teacher.

Laurinda was a 19-year-old single student who was also transferred from another public university located in the state's countryside. She lived with her parents in a neighborhood near the campus. Her experience as a teacher came from her participation as a pre-service teacher in a young people and adult education project for employees of a large company in the food industry.

Finally, our last participant, who will be herein called Donizete, was a 32-year-old married man. He lived in a house of his own, which was located in a neighborhood near the campus. Donizete had degree in Physics and worked in a small private school as Math teacher, in the second phase of the Elementary School. During the course of the experiment, he resigned from the institution and started teaching Physics in public schools, first temporarily, and then as a regular teacher hired through public tender.

In order to carry out the supervised education activities, Laurinda and Tereza formed a duo and developed their activities, during the morning shift, in the sixth grade of the Elementary School, in the University campus school. On the other hand, Donizete performed his supervised education activities alone, during the night shift, in the second year of a public High School. This campus school was located in the vicinity of the University campus.

5. Summary of the pre-service teachers' activity before the development of education actions

The elements composing the activity system in the current section (subject, instruments, object and result, rules, community and work division) were used to organize data derived from the professional path of each pre-service teacher. Such organization was used to identify the main contradictions and similarities between their conceptions and perceptions about their activity (Table 1). Thus, their conceptions concerning the role played by the teacher, the teaching and learning of Mathematics, as well as the mathematical knowledge are used as elements in the overall featuring of the future teachers' activity.

Table 1 – The synthesis of the pre-service teachers' conceptions

	Donizete	Tereza	Laurinda
What are the mathematical knowledge features?	Tool. Human creation. Preparing for life.	Vital. Useful. Mechanical.	Indispensable for mankind. Logical and universal. Basilar.
How should the teaching and learning of Mathematics be?	Content assimilation. Valuing the context. Valuing the dialogue.	Valuing the playful. Establishing connections.	Being able to motivate. Being interesting. Arousing curiosity. Marked by action.
How should the Mathematics teacher act and be?	Introducing the content. Interacting. Fulfilling his/her task: to teach.	Maintaining a good relationship. Being a mediator. Being dynamic.	Creating an informal environment. Having an interdisciplinary perspective. Being creative.

Source: the author

By observing the professional path of the pre-service teachers, it became clear that the option for teaching was marked by family influence (Tardiff, 2002), by personal experiences at school (Fontana, 2003), by the social and personal acknowledgement or

by financial rewards (Richardson & Watt, 2005). It is worth highlighting the similarities and differences in the beliefs, visions, conceptions and preferences assumed by the future teachers with respect to mathematical knowledge, to its teaching and learning, and to the role played by the Math teacher. A common point in the speeches of all pre-service teachers was that none of them assumed the political nature of the teaching activity (Ribeiro, 1995).

In addition to these similarities and differences between the conceptions of the future teachers, it is also worth emphasizing the issue concerning the degree of awareness about the relation between their beliefs and the actions arising from such beliefs. According to Lanner de Moura (1998), "looking at things and at life is not enough; we need to feel and think about them in order to know them". Thus, the established truths are fallible and can be discussed and reconsidered. Therefore, people must seek to know the natural and social realities they live in and build the truth based on such realities.

Such factor explains the difficulty they all had to answer the questions that were mainly related to the teaching and learning of mathematical knowledge. Although we noticed the presence of terms referring to a perspective that escapes the traditional Mathematics teaching approach, they are apparently in the empiricism sphere, since they are meaningless and fail to transform the teacher's activity into something meaningful for those involved in the educational process. According to this perspective, the mathematical knowledge is characterized by the staticity, accuracy, formal logic and infallibility that dominated the lessons. According to Fasheh (1988), this type of Mathematics teaching approach can only lead to passive, stiff, shy and alienated students. Thus, the pre-service teachers found themselves facing dilemmas inherent to the pedagogical activity, such as the relation between the specific requirements of the students and the program content to be taught (Zabalza, 1994).

Based on the future teachers' concepts, we summarized the pre-service teachers' view of the activity in a diagram (Figure 1), which represents their activity system prior to the formative experiment. It also presents a brief discussion of the main aspects of such system.

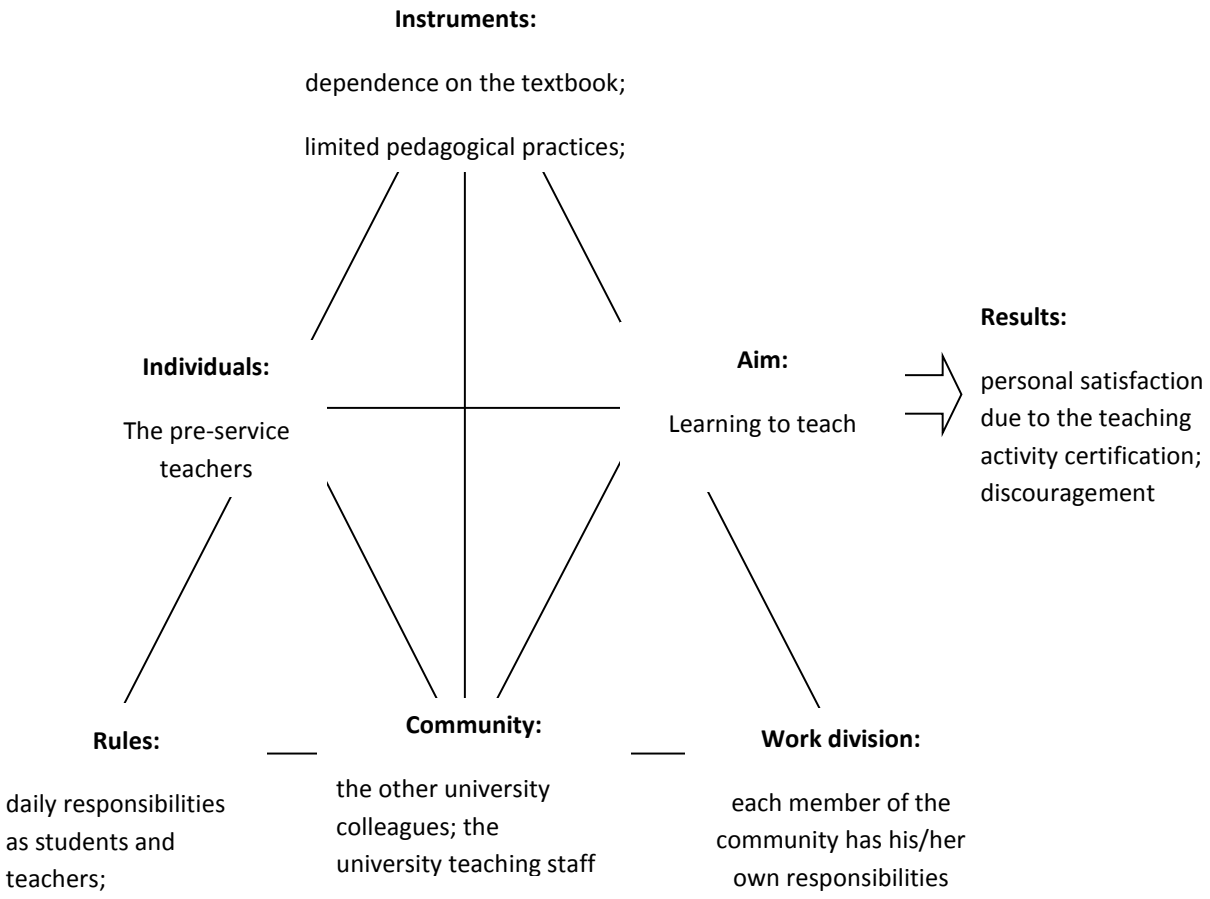

Figure 1 – The pre-service teachers' activity before the education experiment using the diagram proposed by Engestrom (2002)
Source: the author

The individuals in this system are the pre-service teachers: Donizete, Laurinda and Tereza. As we have previously reported, they developed the supervised education activities and wished to complete the teaching-learning process; thus, learning to teach was the object of their activity. However, in our viewpoint, they did not have the necessary resources to enable the actual appropriation of the teaching activity. As for the pre-service teachers' professional path, we noticed that the instruments they used to perform their activities were limited and totally dominated by the common sense and by empirical knowledge. These resources were represented by instruments and included, among other things, a limited view of the teaching practices, which were essentially dominated by the use of textbooks. The rules that guided the pre-service teachers' activity resulted from their responsibilities as university students (Donizete, Laurinda and Tereza) and as teachers (only Donizete), and from the goals set by each of them. Their daily responsibilities as students included, among other things, developing a Course Conclusion Paper (CCP) based on their experience during the supervised education. The personal goals of each of them were linked to their personal aspirations regarding the completion of the Mathematics degree course and to the actions taken during the course of the supervised education.

The community with which the pre-service teachers should fulfill their daily responsibilities, and which they belonged to, was the university group, most specifically, their colleagues in the Mathematics course and the professors of the institution. However, their colleagues, and even some professors, did not all value, appreciate and respect their efforts towards learning to teach. Thus, it was possible seeing that the work division within that community was individually done. The actions were not shared at any time, each one performed the tasks that were within his/her competence without worrying about the actions of others.

The conditions inherent to the pre-service teachers' activity lead to some contradictions, such as the tension between the implementation of the teaching-learning process and the limitation of instruments; the conflict that results from the daily performance of multiple activities, which competes with the personal goals of each pre-service teacher; and the main contradiction between the need of carrying out supervised education activities and the possibility of developing actions that do not satisfy the individual. For example, Donizete already works as a teacher; thus, carrying out the supervised education activities is an additional *burden* in his daily routine.

These contradictions ultimately lead future teachers to face contradictory situations in their daily practice. Such situations decisively help to accentuate the alienation towards the meaning of teaching. Consequently, the dissociation between the reasons and the object of the individuals' activity leads to an unsuccessful teaching-learning process.

According to our viewpoint, this situation can only be supplanted if the individuals confront the contradictions and try to overcome them. Thus, the results of the pre-service teachers' activity, which could be discouraging in some situations, because they are superficial, or bring an ephemeral satisfaction, due to the teaching certification in other situations, may turn out to be really significant for the individuals. It happens as they start a real teaching-learning process that allows them to unconditionally take ownership of the human condition. The next section will address such process and show how our teaching organization mode allows changing the meaning that Mathematics teachers in education attribute to mathematical knowledge.

6. A new mathematical knowledge perspective

The current section focuses on the relations the pre-service teachers established between the mathematical knowledge and their ways of teaching and learning. It outlines how the conceptions and perceptions of pre-service teachers have undergone qualitative changes and had their content restructured in a movement that reflects the scientific knowledge formation process, and consequently, that of the scientific and theoretical thinking. Such phenomenon will be presented based on the teaching-learning process pre-service teachers have experienced when they were faced with the need of organizing the Mathematics teaching process in order to go beyond the mere repetition and memorization of procedures.

The situation the future teachers had to face required much more than the knowledge acquired in their previous experiences. At the beginning of their journey, these

knowledge and expertise worked as basis for them to establish what should or should not be done in a Mathematics class so that the students could learn the discipline in an appropriate manner. However, the herein selected episodes allowed seeing that by playing the role of the individual who should organize the teaching, i.e., the role of teacher, the pre-service teachers found that their knowledge about education was not enough to carry out an educational practice able to transform themselves and their students. Such situation ended up revealing that this knowledge, which mostly derived from their experiences as students or from education processes that did not value the reflection about and the awareness of the actions, is unable to provide teachers with the possibility of efficiently organizing their teaching practice.

Thus, we agree with Lopes (2004), who explains that this teaching-learning process requires these confrontation moments to emerge so that future teachers may show a critical attitude towards their knowledge. "[...] when [the future teachers, the pre-service teachers] face the teaching practice, they find out that many other actions, and other ways of developing them, can and should be carried out, aiming at elaborating [re-elaborating] and developing such practice" (Lopes, 2004, p. 136).

According to such perspective, in order to monitor the relations future teachers established with the mathematical knowledge, we selected two episodes in which it is possible to see that a new perspective, which is linked to this type of knowledge, appears in the pre-service teachers' actions.

Episode: The Math class beyond mechanical procedures

The first part of the herein presented episode happened in a Collective Meeting. The purpose of the meeting was to provide information to the pre-service teachers so that they could analyze their teaching practice. In order to achieve such goal, we analyzed the video recordings of the classes taught by each of the future teachers in the campus school. In this particular meeting, we analyzed sections pertaining to the classes taught by Donizete.

The next part took place after we had watched the first two classes taught by him. The aim of these classes was to introduce *origami* to the students and to propose them to solve the problem situation planned by Donizete, i.e., the Delian problem (which is the cube volume doubling problem).

Part .1

n	Author	Speech
1	Professor	Based on what you have seen, do you believe he could achieve his goals?
2	Laurinda	I don't think so.

3	Tereza	I think that, up to the last class we have watched, he did not achieve his goals.
4	Donizete	You say it because of the students' apathy.
5	Laurinda	Do you know why, Donizete? The same thing happens to my students in the adult and youth education project I work in. They are there to learn and study and stuff, but they want to see something that clearly gives them results, what they look for. Something that allows them to say: I learned it, I'll use it for something. When you use origami and folding activities, they think: what does it have to do with Mathematics?
6	Professor	Our big challenge is to convince the students that this is Mathematics, and mostly, to convince ourselves that we really can teach Mathematics in this way. If we do not believe that it can be Mathematics, we do not do anything different from what is said to be Mathematics. I mean, what is the idea of Mathematics these students have?
7	Tereza	Calculations, numbers.
8	Professor	And when do we calculate?
9	Tereza	In the Math class.
10	Professor	But, what is the Math class? How can we understand the mathematical knowledge?
11	Donizete	As a language.
12	Professor	We must perceive Mathematics as a language. Such perception leads us to search for several ways to take ownership of such language. This is why we use origami, computer, and problem situations. The function of these theoretical elements we are now discussing is to show that Mathematics is a language that must be significantly developed and appropriated by the subjects; but, how? In a way that the subject facing problem situations can be challenged and motivated to solve some things, and that it occurs within a context prepared by the teacher.

This part of the episode begins with the teacher's inquiry (1). His intention is to make the pre-service teachers think about the following question: *how the teaching organization in Donizete's classes reflects the mathematical and scientific knowledge perspective?* The proposition of his problem situation led to a new order in the classroom. The students, who were used to listen, copy and memorize, suddenly found themselves faced with a challenge. They had to work together with their colleagues in order to think about and solve a problem created by the teacher. Without a doubt, this situation caused a feeling of strangeness in the students, which was expressed by speeches such as those reproduced in Donizete's CCP (extract 1).

Let us analyze the following sentence by John (not his real name), "… new role ...". We were questioned through this phrase until the fifth week of class. This question is explained by another one: "… when are we going to do math?".

CCP extract 1 – The students' feeling of strangeness

Based on such perspective, Tereza (3) and, mainly, Laurinda (2 and 5) explain the reasons for Donizete's lack of success. Laurinda's speech (5) may seem to be impregnated with common sense and influenced by a mathematical knowledge conception marked by tradition. However, an example extracted from their lecturing period shows how their Math class conceptions significantly changed.

Extract 2 shows that Tereza, who is represented by the letter P, is leading the completion of a task in the classroom. According to the task she has proposed, the group of students had to search for problems that could be mathematically represented in a number of textbooks provided by the pre-service teachers. After the search, they should represent the problems using the symbolic language of Mathematics and, then, find a solution for them. In the highlighted part, Tereza asks one of the groups to describe the selected problem to the others. Although the issue chosen by the children cannot be classified as a problem, the teacher values the students' mental resolution form and asks the other students to validate or not the solution presented by the group. The form of intervention chosen by Tereza shows her intention to establish a confirmatory interaction with the class and to create a space in which the students go beyond memorizing procedures through the development of actions that value their creativity and intuition. Thus, Mathematics is understood as an important element in the development of the individuals' higher mental functions, since it is essentially a symbolic tool (Bishop, 1997).

P: The girls will read the question they have chosen and then explain how they solved the equation.

A7: The sum between the half of a number and twenty-three is sixty. What is the number?

A4: This is what we thought. We had already had ... and a number added to twenty-three would equal sixty. So, we wanted to know the number that would be added again. So, we subtracted twenty-three from sixty to find the ...

A3: The half

A4: So, we found thirty-seven. Then, we found out that the x they talked about would be …, that the thirty-seven would be the half. So, we multiplied thirty-seven by two and found sixty-four, which equals x. Now, A3 will represent the equation.

A3: We took half of x plus twenty-three, which equals sixty.

A4: And x will equal ...

A3: x will equal sixty-four.

P: So, did everyone get that?

(Some students said no and others said yes. Thus, A4 decided to explain it again). A4: It's like this. They said that the half of a number plus twenty-three equals sixty. So, we subtracted twenty-three from sixty ...

CCP extract 2 – The development of a task in the Math class taught by Laurinda and Tereza

By analyzing the previous extract, which shows the way Tereza and Laurinda developed their teaching practice, it is possible to see the professor's speech echoes (6 and 12) in the way they conducted their class. The professor emphasizes the need of a new design for the Math class in (6) (*Our big challenge is to convince the students that this is Mathematics*) for learning to occur in (12) (*Mathematics is a language that must be significantly developed and appropriated by the subjects*). It indicates to the pre-service teachers that they need to *revisit their conceptions about the teaching and learning process*.

This new need can be seen in the next part of the episode in which the pre-service teachers demonstrated why they used a pedagogical mode of action that went beyond the traditional perspective of Math class. This part also took place in a Collective Meeting in which the first two video image-based classes taught by Laurinda and Tereza were analyzed. The selected episode occurred after we had watched a few moments of the videos showing the classes conducted by the pre-service teachers. It is worth clarifying that, before we watched the videos, the professor had made a statement, which summarized and highlighted the main fundamentals that featured the work proposal developed by the group. By making that statement, the professor wanted to promote a qualitative leap in the pre-service teachers' perception about what they had done until that moment. In a way, the professor (1, 3 and 9) suggested that the future teachers should exceed the simple description of their teaching practice and think about the reasons and the main factors that led them to act the way they did in the classroom.

Part 2

n	Author	Speech
1	Professor	You have seen that Teresa took over ten minutes trying to systematize what had happened in the previous class before she started the next activity. What do you think about it?
2	Tereza	At that point, we wanted to make them link …, to make a link between the classes, for example, the next activity, we needed them to get into the activity, to find out what they would do. Because we proposed the last activity and they did it.
3	Professor	What do you mean by get into the activity?
4	Laurinda	It is realizing that knowledge is taking shape. Because they did it, but they had to understand what they did.

5	Professor	This is important. They did it, didn't they? In the previous lesson, which comprised the task with the strings, you got there. You told the students to do it, there was a problem situation and they solved it. If you, teachers, leave the situation by itself, if in this class here, you started saying: hey guys, today we will do this and that activity. This with the strings [...]
6	Laurinda	It would be confusing.
7	Professor	Why would it be confusing? Because, yesterday, we worked with the strings, and today we will work with what? A little box!
8	Tereza	In fact, the aim of both the string and the box activities was the same.
9	Professor	Tell me, what was the aim?
10	Tereza	Making them realize, begin to realize that there is an x, y form, an unknown factor. We did not want to tell them, it was an unknown factor. We wanted them to realize that there may be an unknown factor that can replace the number.

The analysis of the interventions made by the professor allowed Tereza to understand in (2) the importance - for the learning process - of setting a meaning to the actions the individual performs during the course of the activity (*Find out what they would do*). In addition, she was able to see (10) the importance of establishing moments to enable the students to develop their understanding about the meaning of the equations and, more specifically, of the unknown factors (*We did not want to tell them, it was an unknown factor*). Tereza's action reflects a mathematical knowledge approach that escapes the overestimation of the algorithmic memorization and learning, which is based on the excessive and automatic repetition of the correct mathematical procedures. This very same qualitative leap may be seen in Laurinda's speech (4), when she emphasizes the relation between acting and reflecting on what has been accomplished.

Figure 2 summarizes the key points of such episode:

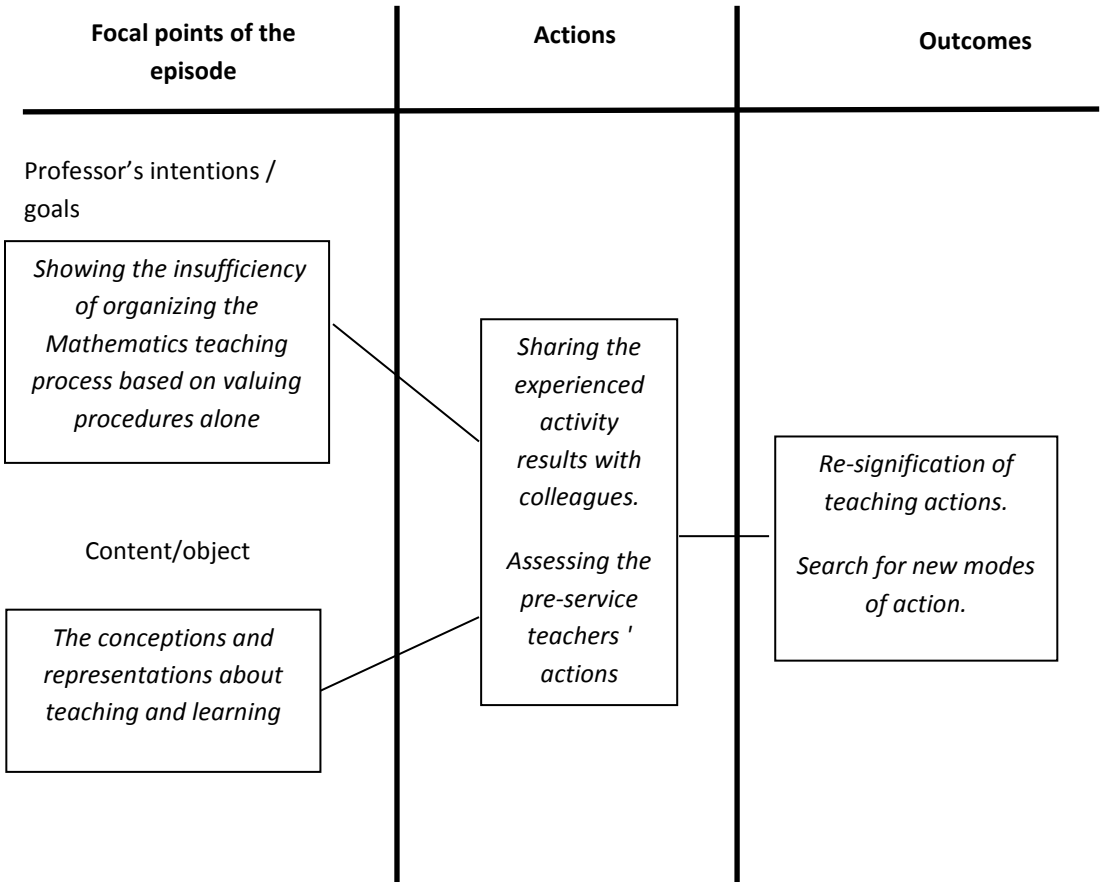

Figure 2 – The main aspects of the episode

Source: the author

7. Final considerations

It is worth highlighting that an important aspect found in the transformation of reasons - Cedro (2008) addressed different aspects - is the mathematical knowledge essentiality. It is common knowledge that the initial basic requirement to become a teacher of any discipline is that the individual must master the contents of it. However, it is also known that knowing these contents alone is not enough to trigger the students' learning process. Such statement gains even more strength when it is brought to the Mathematics field, which is customarily marked by formalism and by the prevalence of formal logic. It is possible to find the teaching-learning process framework within such relation. By showing to the individuals the limitations of their own knowledge and by offering them the possibility of collectively reconfiguring their conceptions, perceptions and representations, we help them to take ownership of their pedagogical actions.

According to this viewpoint, with regard to the teaching and learning of Mathematics, we were able to see that many obstacles have emerged in the educational planning period, during the course of the study. The obstacles faced by the future teachers ranged

from difficulties to deal with the selected mathematical concepts to those related to teaching organization, namely: the difficulty to overcome the old reproductivist practices, to introduce new instruments, and to plan in a significant manner, etc.

However, we observed that the introduction of elements such as valuing playful activities, understanding the logical-historical movement of the concepts, and developing problem situations allowed the future teachers to experience moments that surpassed the Mathematics-related teaching practices already rooted in the prevailing school culture. Thus, we were able to show that learning may be significant, not only when we use the most modern teaching materials and the most innovative practices, but also when we join a conceptual deepening movement that allows us, teachers, to develop our praxis.

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