STUDY OF THE MATHEMATICAL AND DIDACTIC ORGANIZATIONS OF THE CONICS IN THE CURRICULUM OF SECONDARY SCHOOLS IN THE REPUBLIC OF MALI

Saddo Ag Almouloud
saddoag@pucsp.br

Pontifícia Universidade Católica de São Paulo – PUC/SP

Cyrille Koné
cyrillekone@gmail.com

Mamadou Souleymane Sangaré
mamadoussangare@yahoo.fr

Ecole Normale Supérieure de Bamako – ENSUP/Mali

ABSTRACT

This work analyzes the mathematical and didactic organizations concerning the conics, from the standpoint of the history, epistemology, concept, and the teaching and learning of the conics. The curriculum of the Secondary School (High School) and the most used didactic books for this level were studied from the perspective of the Anthropologic Theory of Didactics by Chevallard (1999). More specifically, we want to understand how conics are configured in the educational system in the Republic of Mali, how the subject is approached in didactic books, and what the conditions of their existence in the educational system are. The historical and epistemological examination led us to identify several approaches used to study the different aspects of conics. Our analysis enabled us to situate the study of the Conics in the scope of both the affine Euclidean Geometry and the Analytical Geometry; to identify the different aspects of conics that are studied in both areas; and highlight the relationship between their differences (epistemological and didactic relationships).

Keywords: Conics; Mathematical and Didactic Organizations; Epistemological and didactic relationships.

RESUMO

Nosso objetivo é analisar as organizações matemáticas e didáticas relacionadas com as cônicas, do ponto de vista histórico, epistemológico, conceitual e dos processos de ensino e de aprendizagem. O estudo foi realizado apoiando-se na Teoria Antropológica do Didático de Chevallard (1999), nos currículos prescritos do Ensino Médio e de livros didáticos mais usados nesse nível de ensino. Mais especificamente, queremos entender como este conteúdo se configura no sistema de ensino da República do Mali, como é tratado em livros didáticos e quais são as condições de sua existência no sistema de ensino. O estudo

histórico e epistemológico levou a identificar várias abordagens que foram usadas para estudar os diferentes aspectos de cônicas. Nossa análise permitiu situar o estudo das Cônicas no âmbito da geometria euclidiana afim e da geometria analítica, identificar os diferentes aspectos de cônicas que são estudados nesses dois quadros e evidenciar as relações entre esses diferentes aspectos (relações epistemológicas e didáticas).

Palavras-chave: Cônicas; Organizações matemática e didática; Relações epistemológicas e didáticas.

1. Introduction

This article was from the work of DEA Cyrille Koné (KONÉ, 2008) of the conics. The study of the conics is admittedly important due to its cultural and social dimension, and to the privileged position that conics occupy in the development of the Mathematics. However, we observed that the teaching and learning of the subject endure many difficulties, such as the continuous trace of the conics with tools of geometric construction, and the study of their properties in the scope of Euclidean Geometry.

In the Republic of Mali, it is in the third grade of the Secondary School (Exact Sciences and Biology) that the teaching and learning of conics occur. . The analysis of the students' copybooks enabled us to observe an important discrepancy between the knowledge to be taught and the knowledge actually taught, and we could observe that the geometric aspects are almost ignored when compared to the analytical aspects. Thus, we pose the following question: *Why do we favor the analytical aspects over the geometric aspects of conics in our classes?*

In this text we analyze the mathematical and didactic organizations related to conics from the perspective of the history, epistemology, conception, and teaching and learning of the conics. The study of the curriculum of the Secondary School and the most used didactic books at this level was based on the Anthropologic Theory of Didactics, by Chevallard (1999). To be more specific, we want to understand the configuration of the conics in the educational system of the Republic of Mali, the approach of the didactic books, and their conditions of existence in the educational system. From the epistemological viewpoint, the following aspects are discussed: the notion of geometric locus, and the mathematical and didactic organizations.

2. Geometric locus: theoretical and didactic aspects

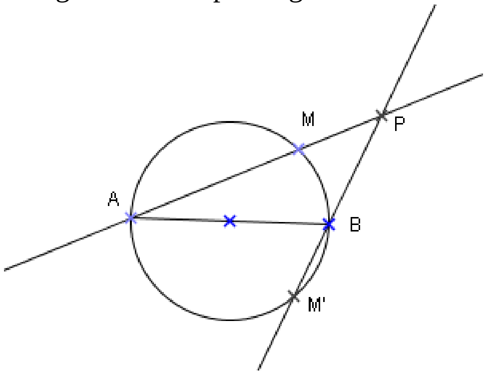
In Mali, the study of the concept of geometric locus occurs in the Secondary School. The students are usually asked to solve less clear problems, which must involve, at this level, an investigation and/or a demonstration. We propose the following situation as an illustration:

Situation 1: Let ($\mathcal{C}$) be a circumference with diameter AB; M a point on ($\mathcal{C}$) and M' the symmetric of M in relation to the line segment AB.

Given the definition of the P point as: {P}= (AM) ∩ (BM'), for M # M'; P= M, for M = M'. Determine and build the locus of P when M moves in $(\mathcal{C})$. (Fig. 1)

Indication for resolution: Given the Cartesian referential $\mathbb{R}(O, \vec{i}, \vec{j})$ such that: $\vec{i} = \overrightarrow{OB}$, with O as the center of the circumference $(\mathcal{C})$. There we have: A(-1; 0); B(1; 0).

Figure 1: Example of geometric locus.



Source: Authors of this text.

Given the point M(x ; y) $\in (\mathcal{C}) \Leftrightarrow \overrightarrow{AM}.\overrightarrow{BM} = 0 \Leftrightarrow x^2 + y^2 = 1$. Therefore M(x ; $\sqrt{1-x^2}$) and M'(x ; - $\sqrt{1-x^2}$) where $x \in [-1;1]$.

$\{P\} = (AM) \cap (BM)$, so det($\overrightarrow{AM};\overrightarrow{AP}$) =0 and det($\overrightarrow{BM};\overrightarrow{BP}$) =0, with P(x_P; y_P). Thus we deduce that P($\frac{1}{x};\frac{\sqrt{1-x^2}}{x}$) with $x \in [-1;0[\cup]0;1]$ and $x_P = \frac{1}{x}$ and $y_P = \frac{\sqrt{1-x^2}}{x}$

$\Rightarrow y_P = \sqrt{x_P^2 - 1}$ $x_P \in]-\infty;-1] \cup [1;+\infty[$.

The locus of P when M moves in curve $(\mathcal{C})$ is, therefore, a hyperbola of equation $x^2 - y^2 = 1$.

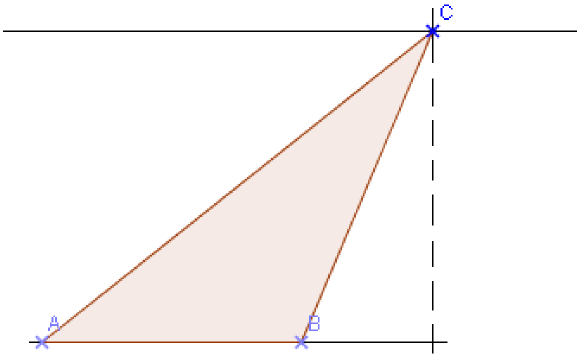
The appropriation of the characteristic properties of the classic loci becomes, therefore, indispensable to address the situation aforementioned. The notions of distance and equidistance are fundamental for the definition of several loci, for example, as the circumference, bisector of a segment, bisector of an angle, conics etc.

In the Republic of Mali, the notion of locus is object of study and its teaching is projected for the curriculum of the Secondary School, more specifically, the second and third years of the Scientific Secondary School. The noosphere indicates that such notion is likely to pervade the study of different concepts, and, consequently, cannot be studied separately. It is presented in the curriculum as originating from the investigation of possible positions for a point submitted to demands and restrictions of a certain figure. As an example, we consider the following situation:

Situation 2: Given AB=5 cm. Construct point C such that the measure of the area of the triangle ABC is 15cm^2. Determine the locus of point C. (Fig. 2)

We know that the measure of the area of a triangle can be calculated from the formula $\frac{base * height}{2}$

Figure 2: An example of locus.



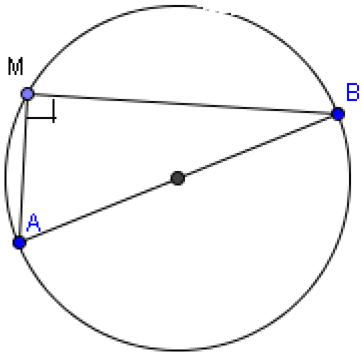
Source: Authors of the text.

Therefore, point C is such that its distance to line AB is equal to 6 cm. The locus of point C is the line that passes through C, and is parallel to line AB.

The notion of locus intervenes particularly in the implementation of significant mathematical activities of geometry learning. It is generally used in activities of reinvestment of the scalar product, of the center of gravity, of the transformations of the plane, in the notion of angle, of distance, in the localization of points in the Cartesian plane, in the study of complex numbers etc. As an example, we present situations 3 and 4.

Situation 3: Let A, B be two distinct points of the plane. Determine the locus of point M such that the triangle ABM is a rectangle in M.

Figure 3: Example of geometric locus.



Source: The authors of this text.

The locus of point M is the circumference of diameter $\overline{AB}$, except for points A and B. We observe that the point M verifies the following relations:

1- The locus of M is the set of points of the plane such that $mes(\widehat{AMP})=\dfrac{\pi}{2}$;

2- The locus of point M is the set of points of the plane such that $\overrightarrow{AM}.\overrightarrow{BM}=0$, because M is a point of the circumference of diameter $\overline{AB}$ (here it is possible to use the analytical method).

Condition and restriction. We must have M # A and M # B, otherwise ABM will not be a triangle.

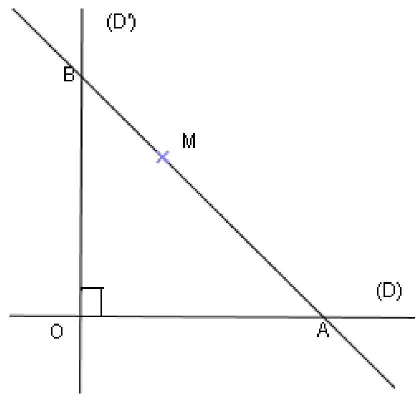
Most problems involving locus and proposed in the Secondary School in Mali are focused on the search for a set of points either:

- Respecting certain conditions and restrictions or having a certain property as, for example, "What is the set of points that are equidistant from two points A and B?". This set of points M is the perpendicular bisector of the segment AB.
- Or associated to variable data.

Situation 4: Segment AB measures λ and the point M verifies the relationship AB = 3AM. Determine the locus of the point M when A moves along line d and B moves along line d'.

We can consider that $\text{mes}(\widehat{OBA}) = \alpha$
Indication of solution: We demonstrate that the locus of M is the ellipses of equation $\frac{x^2}{\frac{\lambda^2}{9}} + \frac{y^2}{\frac{4\lambda^2}{9}} = 1$.

Figura 4: Exemplo de lugar geométrico



Source: CIAM Terminale SM MATHEMATIQUES - EDICEF p. 169, exercise 36.

There are several methods used to determine the loci. We list the most frequent ones as follows:

1st method: analysis and synthesis

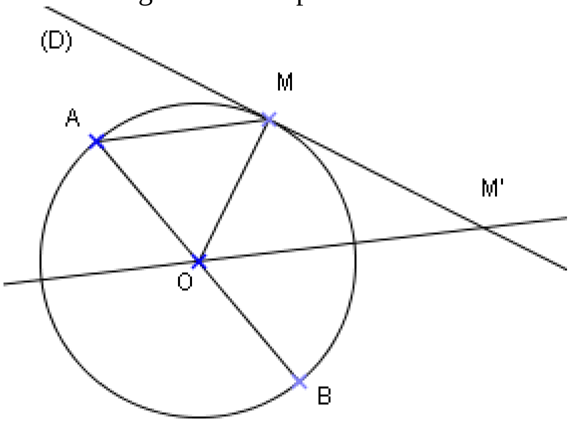
Analysis: After constructing a figure according to the conditions and restrictions of the situation, we seek to prove that every point M of the locus (L) verifies a characteristic property of a given set (E). In this way, we demonstrate the proposition: "For every point M of plane (P), if M belongs to (L), then M belongs to the set (E)", therefore (L) $\subset$ (E).

Synthesis: So, we place the reciprocal problem: "Let M be an element of (E); can we affirm that M belongs to set (L)?". If the answer is yes, then (E)$\subset$(L) and, taking into account the two inclusions established, the sets (E) and (L) are equals. The locus of point M is set (E). If the answer is no, a more « fixed» characterization of a new set (E^1) will be needed through a deeper analysis and we proceed to another synthesis.

In situations 4 to 7 we show examples of those resolution processes forthis kindof problem.

Situation 4: Let ($\mathcal{C}$) be a circumference of diameter $\overline{AB}$ and center O. Let M be a point of ($\mathcal{C}$), (D) the tangent at M to ($\mathcal{C}$), and M' the intersection point of that tangent with the parallel to line $\overleftrightarrow{AM}$, and passing through point O. Find the locus of point M´ when M describes ($\mathcal{C}$) (Fig. 5).

Figure 5: Example of situation.



Source: Authors of the text.

Techniques to solve this situation

1- Hypothesize the wanted locus using some examples.

2- First step: Demonstrate that $\overleftrightarrow{AM}$ is the bisector of line segment AB, and deduce that M´ belongs to the tangent (Δ) at B from circumference ($\mathcal{C}$).

3- Second step: Consider M' a point in tangent (Δ).

a-) Assume M' # B and e demonstrate that M' is a point of the locus.

b-) Examine for M' = B.

4- Note: For M = A, can the procedure to construct point M be carried out? And replacing line MA by the tangent to circumference ($\mathcal{C}$) at point A.

(Collection TERRACHER – MATH 1res S et E HACHETTE Lycées page 23, exercice 26)
The technological-theoretical elements of this method come from the Set Theory. It is the demonstration that two subsets are equal through a double inclusion.

2nd method: utilization of transformation of the plane

Principle: Given the locus (L_1) of a point P of the plane and evidenced that M is the image of P by a usual transformation f of the plane (translation, dilation, reflection etc.). The locus (L) of M is then f (L_1).

Situation 5: Given figure 6 in which the triangle AMP is an isosceles rectangle in A. Determine and construct the locus of P when M covers the semi circumference ($\mathcal{C}_1$).

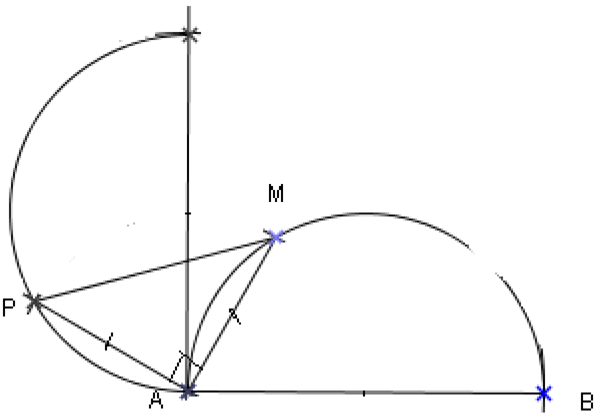
Technique: Assume that point A is fixed and $mes(\overrightarrow{AM}, \overrightarrow{AP}) = \frac{\pi}{2}$, with M # A.

If M is a point of ($\mathcal{C}_1$), we can say that P is that image of M by the counter clock rotation of center A and of angle $\frac{\pi}{2}$.

The locus of P is the image of ($\mathcal{C}_1$) by the counter clock rotation of center A and of angle $\frac{\pi}{2}$.

If M = A, AMP is not a triangle.

Figure 6: Example of situation.



Source: The authors of this text.

3rd method: use of a Cartesian reference frame

Principle: choose a convenient Cartesian reference frame, determine an equation of the wanted locus and recognize it.

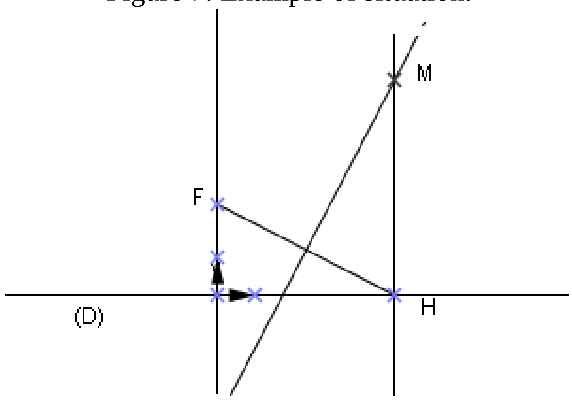
Situation 6: Consider a fixed line (D) and a point F not belonging to the line (D). The distance of F to the line (D) is a positive real number p. Let (E) be the set of equidistant points of F and (D).

1- Demonstrate that (E) is not empty by constructing geometrically a point belonging to it.
2- Choose a conveniente Cartesian reference frame, determine an equation of (E) and identify this set.

Technique

We choose the Cartesian reference frame $(O, \vec{i}, \vec{j})$ defined as follows: O is the intersectional point of (D) and of the perpendicular to line (D) passing through F; $\vec{j} = \frac{1}{p}\overrightarrow{OF}$; $\vec{i}$ is unitary and directly orthogonal to vector $\vec{j}$. A Cartesian equation of (E) is therefore y=$\frac{1}{2p}$ x^2 + $\frac{1}{2}$ p.

Figure 7: Example of situation.



Source: Authors of the text.

4th method: Determining level lines.

Given a function f such that at a point M of the plane associates f (M) in IR (set of the real numbers) and given k a real number. Level line k of the set of points M of the plane such that f(M) = k. From that definition, we deduce that for this category of locus we must determine the reciprocal image of a number k by a numerical application of the plane.

Situation 7: Let A and B be two separate points of the plane, Determine the set of the points M such as $\frac{MA}{MB}$ =2.

Technique: $\frac{MA}{MB}$ =2. $\Leftrightarrow MA^2 - 4MB^2 = 0 \Leftrightarrow (\overrightarrow{MA} - 2\overrightarrow{MB}).(\overrightarrow{MA} + 2\overrightarrow{MB}) = 0$.

Let I be the center of gravity of (A, 1) and (B, -2) and J the center of gravity of (A, 1) and (B, 2). We have that $(\overrightarrow{MA} - 2\overrightarrow{MB}).(\overrightarrow{MA} + 2\overrightarrow{MB}) = 0 \Leftrightarrow \overrightarrow{MI}.\overrightarrow{MJ} = 0$. Therefore, the set of the wanted points M is the circumference with segment IJ as diameter.

As from the discussion and situations presented above, *we can define the locus as a geometric figure characterized by a set of points of the plane that have the same mathematical property*.

In this sense, the bisector of a segment, the bisector of an angle, a circumference, the conics, and the level lines are classic loci within the curriculum of the Secondary School in Mali; their characterization as loci justify their geometric constructions through their characteristic properties. However, the geometric (monofocal and bifocal) definitions of the conics we know just allow their construction point by point, because we do not have a usual tool of geometric construction to enable our tracing the conics in a continuous form in our classrooms, as we can do with the bisector of a segment, a bisector of an angle or of a circumference. That is why the geometric study of continuity (too complex) of those figures is indispensable.

Under the light of this introductory study we have just done on the notion of locus, we realized that the analytical study of the conics occupies a relevant place in our classroom practice. Therefore, we would like now to discuss some of theoretical aspects of the mathematical organizations as far as conics are concerned.

3. The study of the mathematical organizations

The study of the conics will enable us to describe the knowledge of those mathematical organizations that can be constructed or that can construct themselves in the classroom. This work was based on the Anthropologic Theory of the Didactic (CHEVALLARD, 1999), of which we discuss some characteristics in the next paragraph.

According to Chevallard (1999), concepts such as *task, technique, technology and theory* allow the modelling of social practices in general and, in particular, the mathematical activity, based on three assumptions:

a) *Every institutional practice can be analyzed from different points of view and different manners, in a relatively well traced system of tasks*.

b) *The accomplishment of every task results from the development of a technique*.

As from those two assumptions we obtain a "practical-technical" block formed by a type of task and a technique that can be identified in current language as a "know-how-to-do*"*. (Chevallard, 2002, p. 3)

The third assumption refers to the ecology of the tasks:

c) *The ecology of the tasks, that is, the conditions and restrictions that allow their production and utilization in institutions.*

> [...] a ecologia das tarefas e técnicas são as condições e necessidades que permitem a produção e utilização destas nas instituições e supõe-se que, para poder existir em uma instituição, uma técnica deve ser compreensível, legível e justificada [...] essa necessidade ecológica implica na existência de um discurso descritivo e justificativo das tarefas e técnicas que chamamos de tecnologia da técnica. O postulado anunciado implica também que toda tecnologia tem necessidade de uma justificativa que chamamos teoria da técnica e que constitui o fundamento último. (BOSCH; CHEVALLARD, 1999, p. 85-86, tradução nossa).

We assume that the existence of an institution requires a comprehensible, readable and justified technique, which would be the least condition to allow its control and to guarantee the efficacy of the tasks performed, which are usually tasks that assume the collaboration of several actors. Those ecologic conditions and restrictions imply then the existence of a *descriptive and justificatory discourse of the tasks and techniques,* called *technology* of the technique by Bosch and Chevallard (1991). Every technology also needs a justification, which was called the *theory* of the technique.

Inn item 3.1 we examined the mathematical objects "Conics", and then the types of tasks related to them, the techniques used to accomplish those tasks, and the technologies used to justify those techniques.

3.1. Study of the mathematical objects

3.1.1. Conics as locus

Conics can be defined as the loci of points in a plane that verify a particular geometric relationship. Those definitions are usually based on the focal properties of conics. Thus:

1. Monofocal definition of conics

Given line (D), let F be a point in the plane not belonging to line (D), and a real positive number ***e***. The conic of directrix (D), focus F and eccentricity ***e*** is the locus of the points M of the plane, such that $\frac{MF}{d(M,(D))} = e$. The curve that represents that set of points depends on the value of ***e***: If ***e*** = 1, then the conic is a parabola; if o < ***e*** < 1, then the conic is an ellipse; if e > 1, then the conic is a hyperbola.

2. Bifocal definition of conics with center (e $\neq$ 1)

b.1. Given two points F and F’ and a positive real number ***a*** such that FF’ π 2***a***; the set of points M such that MF + MF’=2a is an ellipse of focus F and F’, center O, the midpoint of segment FF´, eccentricity $\frac{OF}{a}$, and directrix (D) defined by: $P \in (D) \Leftrightarrow \overrightarrow{PO}.\overrightarrow{FO} = a^2$.

b.2. Given two distinct points F and F´ such that FF’> 2a, the set of the points M of the plane such that $|MF - MF'| = 2a$, is the hyperbola of focus F, center O, midpoint of the segment FF´, eccentricity $\frac{OF}{a}$ and directrix (D) defined by $P \in (D) \lll=\ggg \overrightarrow{PO}.\overrightarrow{FO} = a^2$.

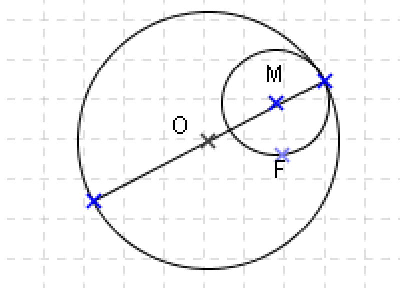
c) Conics as loci of the center of the circumferences passing through a fixed point F and tangent to a circumference or a fixed line

c.1. Circumferences tangent to a given circumference

According to the position of the point F in relation to the circumference of center O and radius r, we have two cases:

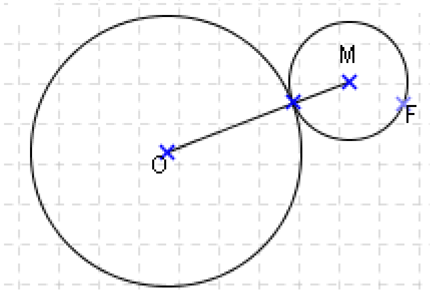
(1) Point F is inside the circle (Fig. 8): if a point M belongs to the wanted loci, then it verifies the relationship MO + MF = r, which characterizes the bifocal property of an ellipse of focus O and F.

Figure 8: Study of an ellipse.



(2) Point F is outside the circle (Fig. 9), a point M belonging to the wanted locus verifies the relationship | MO – MF| = r. This relationship characterizes the bifocal property of a hyperbola of focus O and F.

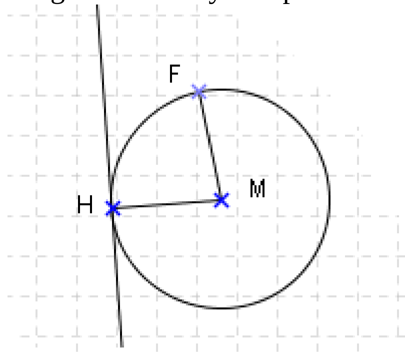
Figure 9: Study of a hyperbole.



c.2. Circumferences tangent to a given line

If a circumference passes through point F and touches the line at point H, then its center is equidistant from points F and H. The wanted locus is a parabola (definition equivalent to the monofocal definition).

Figure 10: Study of a parabola.



3.1.2. Conics with second degree curves

A conic can be characterized by a quadratic equation. The study of the properties of the curves is substituted by the study of algebraic properties of the correspondent equations.

The relevance of the mathematical, didactic and epistemological points of view and of the projective geometry (conic = intersection of the cone and the plane), of the metric geometry (conic = set of points in a plane that verify a given condition of distance), of the algebraic and analytical geometry (conic= second degree curve) underscore the need to study the existing relations between those points of view. Those relations are at the level of the didactic transposition of knowledge, more specifically in the Anthropologic Theory of Didactic by Chevallard (1999).

The study of the conics as the intersection of a cone of revolution and a plane of reference is the object of knowledge (producer institution), with locus as the object to be taught (transpositive institution), and with quadratic curves as the taught object (educational institution).

3.2. Study of the types of tasks

The analysis of the curricula and the know-how-to-dos of mathematics, of didactic books and classroom practices enabled us to identify eleven types of tasks often present during Secondary School activities:

Type of task - 1: construct geometrically a point of a conic given the directrix (D), focus F and eccentricity.

Technique 1: Let H be a point in (D);

- Construct the level line and the numeric application of the plane defined by $M \ \alpha \ \dfrac{MF}{MH}$;
- Construct line (Δ_H) passing through H and perpendicular to line (D);

An intersectional point of that level line with the line (Δ_H) (if it exists) is a point of the conic.

Technology: geometric properties of the figures involved, and monofocal definition of conic.

Type of task - 2: determine the reduced equation of a conic, given its directrix, its focus F and its eccentricity e.

Technique: choose a convenient Cartesian reference frame;

- For the parabola:

Let ($\mathscr{P}$) be the parabola of focus F, directrix (D) and vertex S, at the orthogonal reference frame $(S;\vec{i},\vec{j})$ such that $\vec{i} = \frac{1}{SF}\overrightarrow{SF}$. The parabola ($\mathscr{P}$) is the curve of equation $y^2 = 2px$, where p is the distance from F to line (D).

- For the conics with center:

Let (Γ) be a conic of focus F, directrix (D) and eccentricity ***e*** (*e* # 1); let A and A' be the vertices of (Γ) situated at the focal axis. At a orthogonal reference $(O,\vec{i},\vec{j})$ such that O is the midpoint of segment AA' and $\vec{i} = \frac{1}{OA}\overrightarrow{OA}$, (Γ) is the curve of equation $\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1$, where a = OA and c = OF.

Technology: the coordinates of a point at a Cartesian reference; distance from a point to a line or between two points; and monofocal definition of a conic.

Type of task - 3: determine the nature and the characteristic elements of the set of points of the plane characterized by a second degree equation in $\Re^2$.

Technique: write an equation in the canonic form, and then change the Cartesian reference to describe its reduced equation (E).

If (E): $y^2 = 2px$, then it is the parabola of parameter $|p|$, vertex O, focal axis $(O,\vec{i})$, directrix (D) : $x = \frac{-p}{2}$, and focus F($\frac{p}{2}$; 0).

If (E) : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b) , then we have an ellipse of a focal semi-distance $c = \sqrt{a^2 - b^2}$, of eccentricity $e = \frac{c}{a}$, vertices A(a ; 0), A'(-a ; 0), B(0 ; b), B'(0 ; -b), and major focal axis $\overline{AA'}$ and segment $\overline{BB'}$ as minor axis, of focus F(c ; 0) and F'(-c ; 0), of lines (D) : $x = \frac{a^2}{c}$ and (D') : $x = \frac{-a^2}{c}$ as directrices.

If (E): $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we have an ellipse of focal semi-distance $c = \sqrt{a^2 + b^2}$, eccentricity $e = \frac{c}{a}$, vertices A(a ; 0), A'(-a ; 0), B(0 ; b), B'(0 ; -b), of major axis $\overline{AA'}$, and minor axis $\overline{BB'}$, focus F(c ; 0), F'(-c ; 0), directrices (D) : $x = \frac{a^2}{c}$ and (D') : $x = \frac{-a^2}{c}$, and of asymptotes (Δ) : $y = \frac{b}{a}x$ and (Δ') : $y = \frac{-b}{a}x$.

Technology: Properties of equivalence between two equations (calculation in $\Re$); and reduced equation of a conic section.

Type of task - 4: determine the nature and the characteristic elements of a conic given its parametric representation.

Technique: Use an algebraic treatment to obtain an algebraic expression. Without the parameter, then change the Cartesian reference (if necessary) to determine reduced equation of the conic section.

If (E): y^2 = 2px, then we have a parabola of parameter $|p|$, vertex O, focal axis $(O,\vec{i})$, directrix (D) : $x = \frac{-p}{2}$ and focus F($\frac{p}{2}$; 0).

If (E): $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b) , then we have an ellipse of focal semi-distance $c = \sqrt{a^2 - b^2}$, eccentricity $e = \frac{c}{a}$, vertices A(a ; 0), A'(-a ; 0), B(0 ; b), B'(0 ; -b), major axis $\overline{AA'}$, minor axis $\overline{BB'}$, focus points F(c ; 0) and F'(-c ; 0), directrix (D) : $x = \frac{a^2}{c}$ and (D') : $x = \frac{-a^2}{c}$.

If (E): $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we have then an ellipse of focal semi-distance $c = \sqrt{a^2 + b^2}$, eccentricity $e = \frac{c}{a}$, vertices A(a ; 0), A'(-a ; 0), B(0 ; b), B'(0 ; -b), major axis AA´, minor axis BB´ and focus F(c ; 0), F'(-c ; 0), directrices (D) : $x = \frac{a^2}{c}$ and (D') : $x = \frac{-a^2}{c}$, and asymptotes (Δ) : $y = \frac{b}{a}x$ and (Δ') : $y = \frac{-b}{a}x$;

Technology: properties of the relationship of equivalence in $\Re$; and reduced equation of a conic section.

Type of task 5: Determine an equation of a tangent to a conic section, given its reduced equation.

Technique: utilize a rule of unfolding of variables of the reduced equation, as follows:

Given a point M(x_0 ; y_0), we have the parabola: $yy_0 = p(x + x_0)$, for ellipse: $\frac{xx_0}{a^2} + \frac{yy_0}{b^2} = 1$, and the hyperbola : $\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$.

Technology: definition of the derived of a function and definition of a tangent at a point of a curve.

Task 6: determine equations of asymptotes of a hyperbola defined by its reduced equation.

Technique: lines (Δ): $y = \frac{b}{a}x$ and (Δ'): $y = \frac{-b}{a}x$ are asymptotes of the hyperbola.

Technology: Concept of limit and its properties.

Type of task 7: Determine the equation of hyperbola, given its asymptotes.

Technique: Change the Cartesian reference frame, which has vectors of the associated base which are director vectors of the hyperbola asymptotes. Let $(o, \vec{u}, \vec{v})$ be a Cartesian reference in which $\vec{u}(a,b)$, $\vec{v}(a,-b)$, x = a(X + Y) and y = (X - Y), therefore the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ can be written as $Y = \frac{1}{4X}$.

Technology: Cartesian reference changing formula; and the characteristic properties of a hyperbola.

Type of task - 8: regionalization of a plane by a conic section defined by its reduced equation.

Technique: Let M(x ; y) be a point in the plane. For the parabola we have: if point M is inside the internal region of $\mathscr{P} \Leftrightarrow y^2 < 2px$, and if M is outside $\mathscr{P}$, then $y^2 > 2px$. For the ellipse we have: if M is inside the ellipse £, then $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ <1, and if M is outside the internal region of £, then $\frac{x^2}{a^2} + \frac{y^2}{b^2} \phi 1$. For the hyperbola, if M belongs to the internal region of $\mathscr{H} \Leftrightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} > 1$, and M is external to $\mathscr{H}$, then $\frac{x^2}{a^2} + \frac{y^2}{b^2} \pi 1$.

Technology: Let M be a point on the plane, M belongs to the conic, equivalent to MF < d(M, (D)); M is external do the conic equivalent to MF > d(M, (D)). Besides those characterizations of point M, it is necessary to be based on the monofocal definition of a conic.

Type of task - 9: determine the image of a conic (characterized by an analytical equation) by a transformation of the plane.

Technique: determine an equation of the image, through the transformation considered.

Technology: geometric properties of the transformation considered and the nature of the characteristic elements of a conic.

Type of task- 10: Determine the parametric equations of the conics.

Techniques: Determine the parametric equations of each of the three conics.

a. Parameterization of a parabola: let (P) be a parabola. There is a Cartesian reference in which (P) has the reduced equation: $y^2=2px$.The parabola is, therefore, the set of points of coordinates $\left(\frac{t^2}{2p}, t\right), t \in R$. We chose as parameter, in this case, a parameterization of the parabola of equation: $y^2 = 2px$ and $\begin{cases} x = \frac{t^2}{2p} \\ y = t \end{cases}, t \in R$.

There is another reference in which parabola (P) has equation $x^2= 2py$ and another parameterization is thus: $x^2 = 2py$ and $\begin{cases} y = \frac{t^2}{2p} \\ x = t \end{cases}, t \in R$.

b. Parameterization of an ellipse and a hyperbola:

b.1. Let E be and ellipse. There is a Cartesian reference in which E has as its reduced equation $\frac{x^2}{a^2} + \frac{y2}{b^2} = 1$. A point M (x, y) belonging to the ellipse if there is a real number t such that $\frac{x}{a} = \cos(t)$ and, at the same time, $\frac{y}{b} = sen(t)$. The ellipse is, therefore, the set of the points of coordinates $(a\cos(t), \mathrm{bsen(t)}), \mathrm{t} \in \mathrm{R}$. A parameterization of the ellipse of center O and semiaxes a and b is: $\begin{cases} x = a\cos(t) \\ y = bsen(t) \end{cases}$. If the ellipse has center $\Omega(x_0, y_0)$ and semiaxes a and b, its parameterization is: $\begin{cases} x = x_0 + a\cos(t) \\ y = y_0 + bsen(t) \end{cases}$.

For a hyperbola, as we have for every real number, $ch^2(t) - sh^2(t)=1$ and that, on the other hand the function $t \rightarrow sh(t)$ is bijective of R in R, a branch parameterization of the hyperbola of equation $\frac{x^2}{a^2} - \frac{y2}{b^2} = 1, x \geq 0$ is $\begin{cases} x = ach(t) \\ y = bsh(t) \end{cases}$.

Technology: properties of the parabola, of the ellipse and of the hyperbola, trigonometric relations; coordinates of points on the Cartesian plane.

Type of task - 11: Determine the polar equation of a conic

Technique: Let (Γ) be a conic of focus F, directrix (D) and eccentricity $e \in]0,+\infty[$. We consider the Cartesian plane of origin F, which has the focal axis (Δ) of (Γ) as axis of the abscissas, such that the point K has a positive abscissa. If M is a point of the plane, we have (r, θ) as polar coordinates of M. Given d= distance(F, (D)) and p=ed the parameter of the conic. In the considered Cartesian reference, |r|= FM, x_M= d. We have, then: $M \in (\Gamma) \Leftrightarrow FM = e \times d(M,(D)) \Leftrightarrow |r| = e|r\cos(\theta) - d| \Leftrightarrow r = e(r\cos(\theta) - d)|$

$$or\ r = -e(r\cos(\theta) - d \Leftrightarrow r = \frac{-ed}{1 - e\cos(\theta)} \text{ or } \mathrm{r} = \frac{\mathrm{ed}}{1 + \mathrm{e}\cos(\theta)}$$

Since p= ed, (Γ) is therefore a reunion of two curves (Γ_1) and (Γ_2) of polar equations respectively: $r = \dfrac{p}{1 + e\cos(\theta)} = f_1(\theta)$ and $\mathrm{r} = -\dfrac{\mathrm{p}}{1 - \mathrm{e}\cos(\theta)} = f_2(\theta)$.

We are now going to determine the coordinates of the points M of (Γ): Let $\mathrm{M}_1(\theta)(\text{resp.}\mathrm{M}_2(\theta))$ be the point of polar coordinates $[f_1(\theta), \theta](\text{resp.}[\mathrm{f}_2(\theta), \theta])$. Those two points belong to the curves (Γ_1) and (Γ_2) respectively. We can observe that

$$M_1(\theta + \pi) = \left[\frac{P}{1 + e\cos(\theta + \pi)}, \theta + \pi\right] = \left[\frac{p}{1 - e\cos(\theta)}, \theta + \pi\right] = \left[-\frac{p}{1 - e\cos(\theta)}, \theta\right] = M_2(\theta).$$

Knowing that $\mathrm{M}_1(\theta + \pi) = \mathrm{M}_2(\theta)$, every point of ($\Gamma_2$) is a point of ($\Gamma_1$). Likewise, as $\mathrm{M}_1(\theta) = \mathrm{M}_2(\theta - \pi)$, every point in ($\Gamma_1$) is a point in ($\Gamma_2$). Therefore the curves (Γ_1) and (Γ_2) are coincident and are the same curve (Γ).

Technology: properties of conics; polar coordinates and trigonometric relations.
We will use these types of tasks in the analysis of the didactic books adopted for the Secondary School in the Republic of Mali.

4. Study of the didactic organizations

In this section, we will determine and analyze the choices made in Mali for the teaching and learning of conics from:

- The Mathematics curricula in the classes of Biologic Sciences and Exact Sciences of the third year of the Secondary School, edited by the National Pedagogic Institute of Mali in 1990;
- The know-how-to-dos of the third year of the Secondary School of the Biologic Sciences and Exact Sciences defined in the document "Mathematical Operation", Bamako, June 1996, Edition IPN;
- Didactic books: Colection IRMA Mathématiques CE Algèbre e Géométrie CEDIC/NEA 1987 and Colection CIAM Mathématiques Terminales SM EDICEF 2002. They are the most used didactic books in the Secondary School in the Republic of Mali.

In these documents we will analyze two types of praxeology: the mathematical praxeology (or mathematical organization) and the didactics praxeology (or didactic organization). In the teaching and learning of conics the mathematical knowledge is constructed. Our aim is to describe how that study is proposed, more particularly, we will analyze the introductory activities to the courses, exercises, syntheses, assignments etc.

4.1. Analysis of the curriculum and know-how-to-dos

In the curriculum of Mathematics in the Secondary School from *SBT (Third year of Biologic Sciences) – Edition IPN – Mali 1990*, the noosphere recommends that the conics should be taught in activities (p.10) which focus on: the bifocal definition of conics with a center, geometric properties, reduced equation, analytical study and regionalization of the plane by a conic.

"*Objectifs généraux (page 31 – savoir faire – terminales – opération « mathématique » Bamako juin 1996",* indicate that the geometric introduction of conics should help to prevent the misperception of students by using a set of algebraic equations. According to these authors, it is important that the results from the analytical geometry are learned, or that the students know how to determine them as from geometric verifications. However the discovery of a disclosure of the figure, the construction of a figure through an empirical or a geometric means should be developed at this school level; more specifically, this should be an opportunity to create conditions for students to analyze a figure and to deduce, from that image, a construction program of the representation of the Mathematical organization in discussion.

The analysis of that document highlights the skills that must be learned, that is to say, the students must be able to:

- Present a point-by-point construction program of each of the three conics, and be able to execute it;
- Recognize geometrically the characteristic elements of a conic;
- Construct geometrically the tangent of a conic in a certain point;
- Choose a convenient orthogonal Cartesian reference to obtain the reduced equation of a conic;
- Given a Cartesian reference and a reduced equation of a conic: identify the characteristic elements of that conic, and build it; if the conic is a hyperbola; determine an equation from each of its asymptotes; given a Cartesian reference, recognize the set of points M(x ; y) of the plane such that: $ux^2 + vy^2 + w = 0$ (u, v, w given real numbers);
- The reduced equation of a conic given in a Cartesian referent, a point M of the plane given by its coordinates, situate the point M in relation to that conic;

The analysis of the curriculum of Mathematics prescribed by *SET* (third year of the Secondary School Exact Sciences) – *Edition IPN – Mali – 1990*, highlights the most important aspects (p. 4 e 5) that should be taught/learned, namely: geometric definitions (bifocal, focus and directrix); reduced Cartesian equations; equation of hyperbola from its asymptotes, tangent of a conic at a certain point; activities of regionalization of the plane by a conic.

Concerning the general objectives *(*p. 20, know-how-to-do), the document "*Opération « mathématiques » Bamako juin 1996*" emphasizes the following aspects: the study of curves which are admittedly important for Mathematics and Physics. The chapter on conic will be introduced geometrically. Therefore, the consideration of the plane sections of a cylinder allows conjecturing naturally a particular property that will be the starting point of a study where the focus will be the geometric aspect of the properties of the conics and their construction. According to this document, this study will be the opportunity for students to use of tools built during the first year of the Secondary School, such as the center of gravity, scalar product, *arco capaz*..., and to search for new sets of points. Finally, based on those properties, they can try to show the similarities that justify the generic term "conics".

This document also appoints to the skills that should be met in the processes of teaching and learning of conics. According to this document, the students should be able to:

- While investigating a set of points, recognize a conic (through a geometric, analytic or parametric property), and explicit the mathematical object and its characteristic elements;
- Construct a point of a parabola, given its focus and directrix; a point of an ellipse (or of a hyperbola) , given its focus and eccentricity; a point of an ellipse, given focus and major axis; a point of a hyperbola, given focus and the distance between its vertices;
- Construct, in a practical way, a parabola using a set-square and a string; an ellipse, by the method called "gardener"; a hyperbola, using a ruler and a string;
- Given geometrically a conic geometrically, choose an appropriate Cartesian reference to obtain a reduced analytical expression that represents that conic;
- Given a certain Cartesian reference, (Γ) the set of points of the plane, with coordinates that verify the equation $ax^2 + by^2 +cxy + dx + ey + f = 0$ (in which a, b, c, d, e, f are given real numbers), utilize a change of Cartesian reference to determine the nature of (Γ), if c # 0, the change of reference must be indicated in the enunciation;
- Given the reduced equation of a conic, use the method of unfolding of variables to determine the equation of the tangent to that conic at one of its points; find, in case of a hyperbola, the equations of the asymptotes.

We have realized that the syllabuses focus on the mathematical organizations chosen by the noosphere, and suggest the observation of some types of tasks we mentioned before. The analysis of the prescribed syllabuses and of the know-how-to-do show that one of the goals of the teaching and/or learning of conics is to make an geometric introduction which would allow the study of the geometric knowledge of conics, thus preventing it to become reduced to the management of simple algebraic equations. However, we observe that this goal is not generally reached, as we will argue as follows in the investigation of the mathematical and didactic organizations proposed by the didactic books for the teaching and learning of conics.

5. Analysis of didactic books

We based this analysis on the notion of "didactic moments" introduced by Chevallard (1999).

The concept of moment was introduced by Chevallard (1999) to describe a didactic organization, and it refers, - just apparently - to the temporal structure of the study process. The meaning given to the word “moment”, is, initially, multidimensional, a factor in the multifactorial process. The didactic moments are, primarily, before being a chronologic reality, a functional reality of the study. When we want to describe a didactic organization around a mathematical object, any form of that study; certain types of situations; moments of the study; or didactic moments are necessarily present. The author defines six didactic moments, warning for the fact that they can occur simultaneously, because, as there is not a predefined sequence for its occurrence, they may be repeated during the study.

The first moment refers to the encounter with the praxeological organization through tasks; this encounter will guide the development of the institutional and personal relations with the object. The relations will be constructed along all the process of study and play an important role in the apprenticeship[1]. This moment consists of finding the Mathematical organization (OM) through at least one of the types of its constituent tasks and which, however, does not determine totally the relation with the object, because it is constructed and modified during the process of study.

In a second moment, the tasks must be explored and there must be a technical elaboration to solve that type of tasks. At this moment the teacher must guide the students to constitute, at least partially, a technique that can basically solve the problem that represents the kind of tasks studied. Later this action must show the emergence of other more elaborated, general and complete techniques. Thus, to study certain kind of problems is a permanent way of creating and improving one or more techniques that will become a means to solve almost routinely this type of problems.

The third moment concerns the construction of the technologic/theoretical environment which begins to be constituted since the first encounter and becomes increasingly accurate during the study. In general, this moment begins through a relationship between the formerly constructed technologic/theoretical environment, and the beginning of a new environment which will become more exact with the emergence of the technique. In general, this moment is in close interaction with each of the other moments, establishing a dynamic and dateless process in the evolution of the process triggered by the mathematical organization, and which is the focus of the didactic organization study. Therefore, since the first encounter with a certain type of task, we have inter-relations and/or connections with a formerly elaborated technologic-theoretical environment. It is worth mentioning that this moment, in the traditional education, constitutes the first step of the study and the tasks appear as the application of the technologic/theoretical block.

In a forth moment, there is the work with the technique in different tasks, which can, eventually, be improved by its relative mobilization to a set of qualitatively and quantitative representative tasks of the mathematical organization under study.

[1] To Chevallard, apprenticeship exists when the personal relation of the subject with the object is modified or created by the interaction with the institutional contract, that is, the interaction with the institutional relation R_I (O).

In a fifth moment, the institutionalization, the mathematical organization is defined. Elements which were part of the study in former stages can be discarded, and others can be definitively integrated after the official elicitation of those elements by the teacher or by the student, becoming an integral part of the culture of the institution or the class. That is, new elements can be introduced by the modification of the valid institutional relation or by the creation of a new institutional relation with those elements.

The sixth moment is considered from these perspectives: the evaluation of the personal relationships and the evaluation of the institutional relationship, both concerning the constructed object, the constructed technique, aiming at verifying an intellectual capacity.

The moment of the evaluation is an important step of the TAD because it is assumed that it is the moment when the teachers take as the object of study the solutions produced by their students. On their turn, the students observe in the execution of their solution (in class or in the book) certain "ways of doing", analyzing them and evaluation them to "develop" their own solution.

As stated by Chevallard (1999), the scheme proposed by the anthropologic approach is universal, and considers the evaluation stage fundamental, which must not be considered just from the perspective of the school evaluation. Rather, the evaluation act will always be necessarily relative, because the value recognized for an object is neither intrinsic nor absolute, since the attribution of a value always refer, implicitly of not, to a determined social use of the evaluation object. We always evaluate from a particular perspective.

In moments 1, 2, 3, and 4 the knowledge is contextualized and used as tool. In moment 5, the knowledge is decontextualized, it is considered object of study, and, in moment 6, the knowledge is re-contextualized to solve new tasks.

This model of moments of study can, on one hand, be used as a scheme for the analysis of didactic process; and, on the other hand, allow to reflect about the problem of the effective execution of a didactic organization for the appropriation of a mathematical organization by students and/or teachers.

We analyzed, essentially, the two most employed didactic books in the Secondary School in the Republic of Mali. They are from Collections IRMA – Mathématiques CE. Algèbre et géométries – CEDIC/NEA-1987, and CIAM – mathématiques terminales SM-EDICEF – 2002. Our investigation was based on the didactic moments and the kinds of tasks according to the Anthropologic Theory of the Didactics by Chevallard (1999).

The didactic book of Collection *IRMA* (p. 445 a 505) shows in the summary of chapter 9 that this chapter is about conics: an introduction, definitions, analytical characterization of a conic, parametric representation of a conic.

For the book of Collection CIAM (p. 147 a 170), the summary shows that conics are approached in chapter 7: a general study on conics, study of parabola, ellipse and hyperbola, respectively.

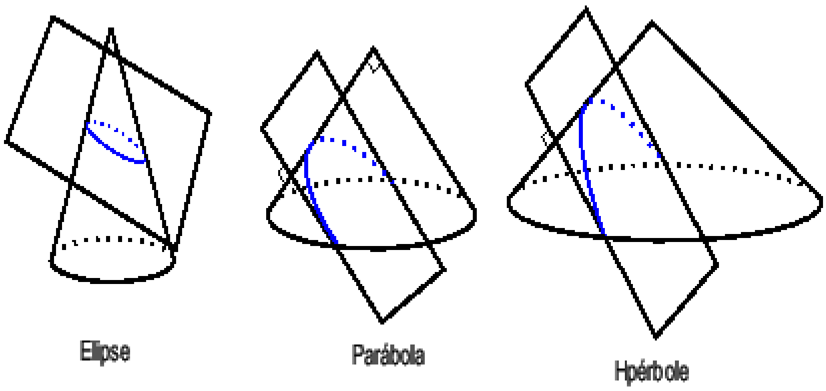
Moment 1: 1st encounter with the notion of conic

In chapter 9 of the book of collection IRMA, the first encounter with the conception of conic is through the disclosure of these curves in several areas of knowledge such as mathematics, physics, astronomy etc. There the book explains that the conics appeared in the Ancient Greece around 350 B.C., and their discovery is attributed to Menaecmus, disciple of Eudoxus. There were, at that time, two ways of defining the new curves: through combinations of uniform movements and as intersections of geometric surfaces.

The first works on conics date back to Euclid of Alexandria (-320?; -260?) and to Menaecmus (middle of the fourth century B.C.) and were thoroughly studied by Apollonius of Perga, around 200 B.C., in its *"Les coniques"*. Apollonius studied and named three types of conics: the *ellipse* (from the Greek *elleipein*: to fall short, or leave out); the *parabola* (from the Greek *parabolê*: *para* = beside; *ballein* = to throw); the *hyperbola* (from the Greek *huperbolê*: *huper* = over; *ballein* = to throw).

He describes his constructions based on a cone of revolution intersected by a plane. (Fig. 11)

Figure 11: Conics generated by the section of a cone by a plane.



Source: SATO (2014).

In this way the study of the intersection of a cone and a plane enables the discovery of a characteristic geometric property of the conics.

In the book of CIAM collection, the first encounter with the mathematical organization is the conic defined as the intersection of a cone and a plane. Then, the book presents the scheme of the point-by-point construction of set (Γ) of points M of the plane such that $\frac{MF}{MH} = e$ (the values chosen for e are : $1, \frac{3}{4}, 2$), where H is the orthogonal projection of M on a line (D), and F a point of the plane not belonging to line (D). This allows the disclosure of different conics.

Moment 2: the exploration of types of tasks and the elaboration of techniques

We present in chart 1 the types of tasks proposed by both books

Chart 1: Types of tasks presented by both books.

Types of task	IRMA	CIAM
1	x	X
2	x	X
3	x	X
4	x	
5	x	X
6	x	X
7	x	X
8	-	-
9	-	X
10	-	X
11	-	-

Source: Our production.

This chart shows that both didactic books use the same kinds of tasks 1, 2, 3, 5, 6, and 7, and that none of them emphasizes the types of tasks 8 and 11 (T8: regionalization of a plane by a conic defined by its reduced equation, and T11: determine the polar equation of a conic).

Moment 3: constitution of the technologic – theoretical environment

Every similar finite-dimensional space n is isomorphic to $\Re^n$.

Moment 4: Working the technique

Generally speaking, both books focus on the kinds of tasks that privilege the following skills:

- To sketch a conic using the graphic representation of functions associated to the reduced equation of a conic;
- Determine the reduced equation of the conic using a conveniently chosen Cartesian reference frame;
- Determine an equation of the tangent to a conic at a point that belongs to it, using a rule of unfolding of variables of the reduced equation;
- Determine the equations of the asymptotes of a hyperbola using the method of investigation of asymptotes of functions associated to the reduced equation;
- Determine the equation of the hyperbola from its asymptotes using the change of Cartesian reference which has as directing vectors the vectors of the asymptotes;
- Determine the type of conic and the characteristic elements of the set of points of the plane defined by an second degree equation in $\Re^2$.

Both books do not present kinds of tasks that enable to mobilize the parameterization of conics and the determination of the polar equations of those mathematical objects, as we have showed before.

Moment 5: institutionalization

From the perspective of the mathematical organization, we present on chart 2 the definitions and properties that the authors of both books institutionalized:

Chart 2: Institutionalized knowledges.

IRMA	*CIAM*
1- In the introduction the work underscores that the conics are in several different areas of knowledge as the Mathematics, Physics, Astronomy, and present the origin of the conics. 2- Definition: 2.1- study of the set (Γ) and points that are equidistant of a line and a given point: a- evidence an symmetry axis (Δ) b- search for points of (Γ) that belong to (Δ) c- search of regions of the plane containing the points of (Γ) d- Specific study of case $e \neq 1$ and identification of the symmetry axis, bifocal definition 2.2-construction and definition a- Study of case e = 1, point-by-point construction and monofocal definition b- Study of case $0 \pi e \pi 1$: point-by-point construction using the monofocal and bifocal definitions. c- Study of case $e \phi 1$: point-by-point construction using the monofocal and bifocal definitions. d- Circumference case 3- Analytical characterization of a conic 3.1- Reduced Cartesian equation of a conic a- Parabola case: Tangent at a point of the parabola/interior and exterior to the parabola b- Ellipse case: Tangent at a point of the ellipse/interior and exterior to an ellipse c- Hyperbola: Tangent at a point of the hyperbola/interior and exterior of a hyperbola 3.2- Set of the points of the plane defined by a second degree equation in $\Re^2$: $(x,y) \in \Re^2, ax^2 + by^2 + cxy + dx + ey + f = 0$ a- 1st case: a ≠ 0 et b≠ 0 b- 2nd case: a = 0 et b = 0 c- Equation of the hyperbola from its asymptotes	1- General study of the conics 1.1- conics defined by the focus and directrix Introduction: point-by-point construction of the conic from a monofocal definition Definition and properties Regionalization of the plane by a conic 1.2- Reduced equation of a conic: reduced equation of a parabola, reduced equation of a conic with center. (e≠1) 1.3 curves of equations $Ax^2 + By^2 + 2Cx + 2Dy + E = 0$ 2-Study of the parabola**:** 2.1- Analytical study: construction of the parabola, characteristic elements of a parabola, equation of the tangent at a point of the parabola, regionalization of the plane by a parabola. 2.2- Geometric study: method of construction of a point of a parabola and its tangent. 3- Study of an ellipse 3.1- Analytical study: construction of an ellipse, characteristic elements of an ellipse, equation of the tangent at a point in an ellipse. 3.2- Ellipse and circumference: parametric representation of an ellipse, main circumference and affinity, method of construction of an ellipse using its parametric representation 3.3- bifocal definition of ellipse: construction of a point of an ellipse and its tangent given its bifocal definition, construction of an ellipse by the gardener's method. 4- Study of the hyperbola 4.1- Analytical study: construction of a hyperbola, characteristic elements of a hyperbola, equation of the tangent at a point of the hyperbola, equation of a hyperbola given its asymptotes

4-Parametric representation of a conic: parabola, ellipse and hyperbola.	4.2- bifocal definition of a hyperbola: construction of a point of a hyperbola and its tangent given its bifocal definition.

Source: The authors of this text.

Moment 6: evaluation

The techniques studied in the book from IRMA collection did not allow to solve the following kinds of tasks: to determine the type of conic and characteristic elements of a conic; to search for a locus; to determine the image of a conic by a usual transformation (translation, symmetries, rotation etc.); geometric construction of the characteristic elements of a conic; to determine the image of a circumference by an orthogonal transformation; to determine the bifocal definition of conics with center; and to determine the parametric representation of a conic. Our analysis shows that the study of the conics is focused on the reduced equations of the conics.

The techniques studied in the didactic book of the CIAM collection also did not allow to solve the following tasks: the geometric construction of a conic; the geometric construction of the asymptotes of a hyperbola; to calculate the measure of the area of the ellipse; to determine the type of conic and of the characteristic elements of a conic; to search for a locus; to determine the image of a conic by transformation; to pass from a parametric representation of a conic to its reduced equation. This didactic book focuses more on the construction of the reduced equations of the conics.

6. Some considerations on the analysis of didactic books

The book from CIAM collection introduces the concept of conic as from an activity of construction of points of a curve, while the book from IRMA collection approaches the conics from their essential geometric properties (monofocal definition, bifocal definition, symmetry axis, symmetry center, and vertex).

The authors of IRMA book study the geometric properties of conics before discovering the form of the curves that represent them, whereas CIAM use the forms of the curves that represent the conics to disclose their geometric properties.

CIAM adopts the monofocal definition as follows: a conic is a set of points M on a plane such that $\frac{MF}{MH} = e$ (e a positive real number), where H is the orthogonal projection of M on a line (D), while IRMA defines a conic as the set of points M of the plane such that $\frac{MF}{d(M,(D))} = e$.

Moment 3 “constitutions of the technologic-theoretical environment” is almost inexistent in CIAM, while in IRMA collection we find some evidence of that environment.

The study of the geometric aspect of the conics is more present in IRMA than in CIAM. The types of tasks and techniques studied are almost identical in both collections, and the work of the techniques is almost nonexistent in IRMA collection.

Unlike the mathematical and didactic organizations proposed by the authors of the analyzed didactic books, we observe that, in classroom practice and our experience as teachers, in the first year of the Secondary School a parabola is the graphic representation of the function defined by $x \ \alpha \ x^2$, and hyperbola the graphic representation of a function defined by $x \ \alpha \ \frac{1}{x}$. However, in the second year of the Secondary School the parabola appears as the graphic representation of a quadratic function, and the hyperbola as a homographic function defined by $f(x) = \frac{ax+b}{cx+d}$ where $c \neq 0$. In the third year of the Secondary School a parabola, an ellipse and a hyperbola are studied as conics.

Conics are generally introduced in a geometric form and their study begins from the notion of locus. However, the geometric study of conics is rapidly substituted by an algebraic study. This fact can cause epistemological difficulties concerning the “meaning” that the students may attribute of the concept of conics and their characteristic elements (focus, directrices, eccentricities, elements of symmetry). The construction of knowledge on conics can be problematic. In fact, the study of conics from their geometric definitions (monofocal or bifocal definitions) is replaced by the study of the conics defined as second degree curves in $\Re^2$ characterized by an algebraic equation. The meaning of the characteristic elements (focus, directrices, eccentricities, elements of symmetry) of a conic is reduced to the only appellation that is attributed to them in the picture of the analytical geometry.

7. Considerations and perspectives

The historic and epistemological study promoted the development of several approaches that were used to investigate the different aspects of the conics. It helped to situate the study of the conics in the realm of the affine Euclidean geometry and analytical geometry, to identify the different aspects of the conics that are studied in these two scenarios, and to put in evidence the relations between those different aspects (epistemological and didactic relations).

The study of the didactic transposition concerning the teaching and learning of conics in the Republic of Mali conducted us to observe the following:

- *Concerning the prescribed curricula and the know-how-to-dos,* the teaching and learning of the conics, as presented in the programs and how-how-to-dos, occur in the affine Euclidean geometry and analytical geometry scenarios with practically the same types of tasks to which different techniques are associated, according to the register in the place.
- *Concerning the analyzed didactic books,* the study of the conics in didactic books focuses analytical aspects of the conics, which favors the picture of the analytical geometry rather than the Euclidean geometry, because the techniques associated with the types of tasks in those manuals refer mainly to the analytical geometry.

- *Concerning classroom practices,* beyond the monofocal definition of conics that is presented to the students without their participation in their construction, the proposed didactic organizations refer to the analytical geometry context.

The teaching and learning of the conics in the Republic of Mali face many problems. We list here the continuing disappearance of geometric tools which enable a deep study of the geometric properties of the conics (radical axis, power of a point with respect to a circumference, theory of the poles and polars etc...), insufficient teachers' formation to overcome the difficulties in relation to the learning and teaching of conics, and the (continuous) tracing of conics given their geometric definitions..

Nevertheless, one of the goals of the prescribed curricula and know-how-to-dos is to make a meaningful introduction of conics, preventing students to reduce the conics to simple algebraic equations, as it happens in our classroom practices.

We propose (forthcoming work) to make a very significant study of the notion of conic in the scenario of the affine Euclidean geometry before passing to the picture of the Euclidean geometry, in which we have efficient and effective tools to resolve the difficulties faced by the students in the context of the affine Euclidean geometry. This approach has a triple advantage:

- To make the concept of conics come through for the students;
- To make the concepts of the students on the notion of conics evolve;
- To offer the students alternatives based mainly on problem solving geometric tools with respect to conics.

It becomes clear that this goal is far from being reached, if we consider our practice in the classroom: the monofocal definition that is presented - without the participation of the student to introduce the conics geometrically - impairs the construction of geometric knowledge of conics, and has a meaning only for the establishment of reduced equations of conic, favoring the picture of the analytical geometry. A correspondence is immediately done between those scenarios, the analytical geometry and the Euclidean geometry, to substitute the study of the geometric properties by the study of the algebraic properties of the corresponding equations. The conceptual study of the object is substituted by the technical study: effect of cognitive slippage.

This early passage from the picture of the affine Euclidean geometry to the picture of the analytical geometry is the source of the difficulty in the construction of the geometric knowledge of the concept of conics: the meaning of the characteristic elements of the conic section (eccentricity, directrix, elements of symmetry), the apprehension of the monofocal definition of conics, and the comprehension of the geometric construction of the conics (given its monofocal definition).

8. References

Chevallard. Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique, *Recherche en Didactique des Mathématiques*, vol.19, n°2, p.221-266.

CIAM. (1999). Terminale SM, Mathématiques, *EDICEF*.

Collection Fractale Mathématiques 1re S Géométrie. (1991). *BORDAS*.

Collection IRMA. (1987). Mathématique Terminale CE, Algèbre et Géométrie, CEDIC/NEA.

Collection Terracher. (1991). *Mathématiques*, Géométrie 1res S et E Hachette.

KONÉ. C. (2008). *Etude des coniques à partir de la notion de lieu géométrique*, diplôme d'étude approfondie (D.E.A) en didactique des mathématiques, École Normale Supérieure de Bamako, Mali.

MALI-Programmes de Mathématiques (Terminales). (1990). Edition IPN Mali.

MALI, Savoir-faire de Mathématiques (Terminales). (1996). *Opération Mathématique*, Bamako, Edition IPN, juin.

Sato, Jocelino. (2004). *As Cônicas e suas Aplicações*. Retrieved October 18, 2014, from http://www.sato.prof.ufu.br/Conicas/node2.html.