UNPACKING INTERACTIONS USING BROUSSEAU'S DIDACTICAL MILIEU

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ABSTRACT

In this work we used Brousseau's Theory of Didactical Situation to examine ways in which interactions with a researcher/teacher influenced mathematical practices of a child. The findings suggest an extension of teaching actions associated with TDS to include the process of stabilizing mathematical understanding of children as they interact with and engage in task-milieu.

Keywords: Didactic, Interaction and Cognition.

RESUMO

Neste trabalho, utilizou-se a Teoria da Situação Didática de Brousseau para analisar as maneiras pelas quais as interações com o pesquisador /professor influenciaram as práticas matemáticas de uma criança. Os resultados sugerem uma extensão das ações de ensino associadas com TDS para a inclusão do processo de estabilização do entendimento matemático de crianças e como se interagem e se envolvem em tarefas.

Palavras-Chave: Didática, Interação e Cognição.

1. Introduction

Over two decades ago Arsac, Balacheff, and Mate (1992) asked whether it was possible to identify the effect of specific teaching actions and to describe their impact on students' learning. In a recent synthesis of literature on classroom-based interactions and learning, Kahveci and Imamoglu (2007) drew attention to the typology of factors that researchers had considered in exploring the question referenced above. They identified that when conducting genre of research scholars have considered how lessons are structured, the sort of norms/context that is established and maintained for mathematical arguments, student motivation to learn, teacher goals and teacher knowledge when monitoring group exchanges, ways that the teacher interacts with students and leads discussion, physical and social environment of the classroom along with tools used in advancing children's mathematical work. The authors identified a paucity of scholarly reports that provided insights on aspects of learners' mathematical thinking influenced by classroom interactions and the structural situations that enhance their development. The synthesis documented that understanding the relationship between teaching and learning of mathematics remains an issue in need of inquiry.

Of particular interest to the research reported here was to identify ways in which *interaction* shaped children's practices during mathematical problem solving sessions. Relying on *Brousseau's* Theory of Didactical Situations (TDS) (1997) as a theoretical tool for grounding the methodology, and an analytical tool for interpreting interactions; we inspected the relationship between interaction and cognition focusing on structural elements that shaped mathematical practices of children including the teachers' particular modes of interacting with the child and the type of learning that seemingly emerged from it. Our study was guided by one question: *What conditions does interaction place on mathematical work of children?*

2. Context and Theoretical Framework

The current report is a part of a larger research project in which we study and trace development of mathematical thinking among 60 middle school and high school children over the course of three years and as they progress from 7^{th} to 10^{th} grade in their respective schools. In the larger project, we document ways in which children draw from school mathematics when they solve mathematical problems in an informal setting. There we also trace how the mathematical practices and identities of children transition overtime manifested in how they solve problems and interact with mathematical contexts. Fundamental to our research capacity to elicit and interpret children's thinking has been our reliance on semi-structured interviews during which the interviewers engage in Exploratory talk (Mercer & Littleton, 2007) with learners as they express their thinking on how they perceive, process and solve mathematical tasks. An *exploratory talk* is the kind of talk in which *partners engage critically but constructively with each other's ideas. Statements and suggestions are offered for joint consideration. These may be challenged and counter-challenged, but all actively participate, and opinions are sought and considered before decisions are jointly made* (Mercer & Littleton, 2007, p.59). These interviews, in line with Goldin's guidelines (1985, 1992, 2000) serve the dual purpose of (a) observing mathematical practices of children in problem solving contexts and, (b) drawing inferences from the observations about the children's cognition. This approach to learning about children's thinking is informed by two key considerations on our part. First, the aim of our research is not to build a model of children's instantaneous knowledge but that of their sense making process when encountering mathematical tasks. Second, our epistemological grounding lies in the domain of social psychology. Of interest to us is not the study of cognition in its static or final form but that of how knowledge is transformed through interactions with the environment. Brousseau's TDS (1997, 2003) provides an *enabling* framework for researchers of learning with such an epistemological stance, as described below.

2.1.Theory of Didactics

The Theory of Didactical situations (TDS) (Brousseau, 1997) is grounded in four foundational epistemological principles: (1) knowledge construction is the result of an attempt at finding an optimal solution to a problem, (2) learning is a form of cognitive adaptation, (3) genuine learning occurs in the course of inquiry into fundamental situations that motivate sense-making, (4) autonomy is the chief ingredient to learning (Radford, 2008). The essential component of learning of mathematics, from the TDS perspective, is that learners be viewed as rational individuals whose adoption of scientific knowledge must be guided through the teacher's actions and as they explore mathematical contexts free of didactical conditioning (Brousseau, 2003).

The *situations* within the TDS construct consist of the interactions between the teachers and students around task milieu aimed at practice and construction of mathematical knowledge. Learning is expected to derive from children's interactions with fundamental situations; the types of contexts that can foster sense-making and promote a knowledge construction process, organized around a type of game, then gives rise to the development of an *didactical situation* (1997, p. 30), the kind of mathematical activity that is freed from the teacher's direct intervention and instruction. According to Brousseau, it is within these adidactical situations that knowledge adaption and true learning is inspired to occur. As such, TDS positions the teaching-learning phenomenon within the teacher-knowledge-pupil triangle (see Figure 1), suggesting mathematics learning to be a function of milieu in which children are enabled to engage as they transform their own knowledge.

Brousseau uses the metaphor of *play* to highlight the dynamic and yet complex nature of teacher/learner interactions as children are placed in a context of free interactions, given autonomy to decide what information they choose to share, what questions to pursue, what representations to use when sharing their ideas. The teacher then participates with the students in an interaction-game format and deals with the problems they may experience, authenticating their work against accepted conventional disciplinary knowledge.

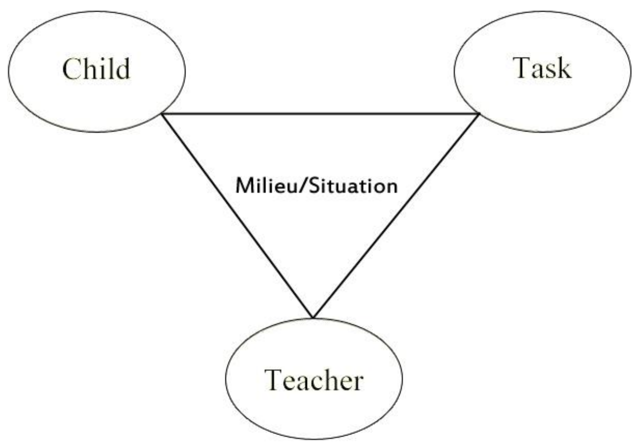

Figure 1. Triangular Milieu

Brousseau posits that learners' transformation of knowledge relies on *devolution*, the process a teacher uses to make learners accept (in an implicit way) the responsibility for solving new problems while remaining conscious of the consequences of this transfer. A teacher's role then is to be fully aware of the learners' state of achievement and to offer them all the necessary means to excel knowledge. Achieving productive interaction within an adidactical situation resides also on the condition that the students accept the challenge of dealing with new problems. The teacher's particular charge then is to make *the students to act, speak, think, and evolve by their own motivation* (Brousseau, 1997, p. 30). In doing so, Brousseau offers four specific criteria that need to be met in order for the environment to generate new learning:

1) Action situation: The environment provides opportunities for the learners to enter tasks and develop theoretical knowledge.

2) Formulation situations: The environment makes possible the potential for developing language and representational tools through communicational situation.

3) Validation situation: Learners are expected to explain their thinking and justify their methods.

4) Institutionalization situation: The institutionalization of knowledge is the process that allows students to navigate, reconsider, and alter their previous knowledge in light of new experiences under the teacher's endorsement.

Although action, validation and formulation situations are taken as shared within both didactical (direct instruction) and adidactical (free play) situations, the institutionalization situation is one that demands the teacher to meet the obligation of allowing genuine learning to arise from the individual's own actions, giving them cultural (mathematical) meaning and authority, helping learners to build a relationship with the object of knowledge (Mano, 2012; Radford, 2008 a,b).

In this work, we adopted Brousseau's model as a lens to examine adult-child interactions set within a semi-structured interviewing design (Goldin, 1992, 2000). As such, our approach to the use of TDS is novel on at least two levels. First, to date, the potential of TDS in the study of cognition within a clinical interviewing research setting has not been widely explored. Didactical situation, as Brousseau conceptualized it, concerns classroom interactions within the institutional setting of formal schooling[1].This distinction, from our point of view, should not raise concern regarding the applicability of the model for our design experiment as we view any kind of interaction around mathematics to be instructional, regardless of whether this element is deliberately considered in the research efforts focused on the study of cognition (MacBeth, 2000). Second, by utilizing the model we also consider the researchers themselves as interventional apparatus that by their sheer presence shape mathematical practices of those whose cognitive actions are elicited and studied. By considering the interviewers as an interactional constituent, and including analysis of their actions as a part of the research design, we ***endorse*** the view that what children choose to show or withhold during cognition- revealing interviews is directly linked to, and deeply intertwined with ways in which the researchers' own epistemologies and knowledge may condition children's thinking (Steffe, 2013). Furthermore, we posit that the type of artifacts children may use to represent and communicate their own internal mathematics can be influenced by what they consider of value to the interviewers and/or their perceptions of what the researchers' epistemological expectations might be[2]. Our interpretation of researcher as a teaching agent is certainly not novel. Indeed, Steffe and Cobb (1983) advocated the need for researchers to take on such a role when conducting teaching experiments (Steffe & Thompson, 2000).

3. Methodology

In analyzing the quality, content and nature of interactions organized around a task and the influence of those interactions on mathematical situations that arise, we rely on data from one specific interview (approximately 40 minutes in length) with one child (Dana-a pseudo-name) as she was prompted to solve a pattern-generalizing task in Algebra. At the time of the interview Dana was enrolled in 8th grade and taking Algebra I. This particular interview session was chosen deliberately since it provided persuasive illustrations of the impact of interactions on practices of the child whilst highlighting the particular epistemological responsibilities and analytical demands that they placed

[1]Indeed, the myriad of reported research studies that have used TDS as a theoretical lens for analysis of data have been concerned with whole or small group interactions (CERME, 2010, 2012).

[2]We do acknowledge the controversial nature of such a claim and consider a debate regarding its merit beyond the scope of this paper.

on the interviewer when eliciting and responding to the child's mathematics, unpacking specifically the institutionalization process as articulated in TDS. The problem on which Dana worked asked her to predict the growth pattern of a sequence of coins arranged in two intersecting arrays of equal length (see Figure 2).

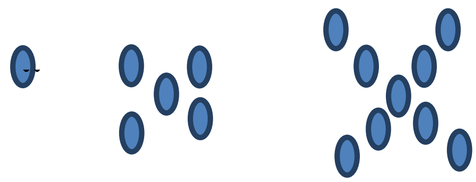

Figure 2. The Coin Problem

3.1. Data Coding and Analysis

The entire interview was transcribed fully and verbatim, recording time, gestures, and particular representations used or introduced during each message unit by both the interviewer and the child (consisting of a total of 260 turns). Data coding followed four stages. First, to break down the conversational exchanges and the moment-by-moment construction in the ongoing interactions, we conducted an analysis of language functions of verbal exchanges during the interview, according to turns and segments of utterances offered. Analysis of language functions was used to identify the thematic nature of interactions globally, relying on Kovalainen and Kumpulainen (2009) codebook that distinguishes eight types of talk visible in classroom interactions. A second round of coding data identified specific mathematical actions that the interactional participants exhibited during the interactional episode. These codes are presented in Table 1 below.

Table 1: Discourse Coding

Language functions	Description	Mathematical actions	Description
Informative	Providing information	VER	Seeking for evidence or counter examples
Reasoning	Reasoning to support judgment	PVG VAL	Proving Validating
Evaluative	Evaluating work or action	CNJR	Conjecturing new relationships
Interrogative	Posing questions requiring information	ABST	Abstracting new ideas or generalizing
Organizational	Organizing and/or controlling behavior	CRD	Coordinating, using a different representation
Reproductional	Repeating what had been said	STR	Structuring
External thinking	While working at a task, student is thinking aloud	FOR	Formalizing
Heuristic	Expressing having found out something	THK	Reflecting and reviewing

At the third stage, we coded each of the interviewer's turns and utterances according to Brousseau'ss*situational* forms (Action, Formulation, Justification, Institutionalization). This coding stage allowed us, for the purpose of our final analysis, to trace the presence or absence of certain elements within the interactional episode that enhanced or inhibited the institution of a productive task-child milieu (An example of coded data is presented in Appendix A). The coded transcript was reviewed again to isolate places where Dana's mathematics shaped the interviewer's modes of participation in the interaction and vice-versa. This last stage of coding was accomplished in order to document how each of the interactional agents may have been influenced by ideas shared during the interview so to describe the structural climate of the didactical milieu.

Two independent researchers coded the interview episode. In places where interpretive disagreements of data existed, the segments were reviewed and coded once more. The process was repeated until both coders reached consensus. Once coding was completed, we first conducted a quantitative analysis of the data in order to identify the most prominent actions, behaviors, and practices of the interactional participants. We then juxtaposed our quantitative analysis with qualitative description of how interaction had influenced the participants during the interview. Findings are outlined below.

4. Findings and Discussion

4.1. Dana's Terrain

Mason (1996) offered two main strategies used by learners when finding the general term of a sequence. The first strategy seeks to establish a relationship between some terms of the sequence in natural language relying on perception. The general term of the sequence in this type of approach is represented using implicit and iterative relations. The second strategy focuses on representing the relationship in the general form, using an explicit model. Dana began her work focused on an implicit relationship though cognizant of the need for defining a general explicit rule and yet unable to do so. The interaction was focused on her attempts at constructing an explicit formula. Dana's formalizing process took approximately 40 minutes.

An overview of the major mathematical events of the episode is presented in Table 2. The table is organized around the important shifts that occurred in Dana's approach to solving the task and her responses to the interviewer's comments or questions. The numerical values noted in parenthesis in each cell correspond with the specific turns during the exchange from the original transcript. The time intervals listed in each cell present the length of time that was spent on each phase of the activity.

0:00 -6:00 D tried to make sense of the problem by looking for a general pattern. By drawing out the picture of the next row, D announced the rule was adding four each time. Dana began the session by immediately stating that she noticed a coin was added to each of the four corners of the intersecting arrays. Referencing, using and extending the visual model she announced that the rule was adding four each time (#9).
06:00 - 06:51 Interviewer asked D for extension to row 20th . D expressed that "adding four" rule could not help her find the twentieth row without drawing the diagram. Dana announced that she was unable to find the number without drawing. Encouraged to pursue the task again, she tried to express the relationship multiplicatively (#16) but in finalizing a response she reverted to the pattern of adding coins each time hinting higher maturity as she now referenced adding groups of four to each consecutive row iteratively (#17).
07:01 – 08:50 Interviewer asked D to go back and determine the number of coins in row five and to form a convincing argument for why her answer would be correct. Dana claims that she thought the number of coins could be determined by multiplying the row number by itself (#38, #50). Typing numbers on her calculator she concluded that the fifth row would have 25 coins in it. It is not clear whether she is using the additive process for

finding the number of coins or her multiplicative relationship (times row by itself) as the calculator screen is not visible.	
09:27 – 11:40	Interviewer asked D if she could draw out a table to organize the data. Looking at the two-columns, D began to formulate the relationship between the row number and the number of balls. She sets up a table of value, noting number of coins in rows 1 through 7. She expressed that she was trying to find how numbers in one column could be generated from the other (#66)
11:40-12:56	Interviewer challenged D to figure out the 20th row without completing the table. D announced that the rule can be the row number times three and then add something but that the rule did not work for 2 (meaning the second row) (#79)
12:57- 13:13	D starts reading the number coins in each row, stating how she viewed the relationship: row 4=3(4)+1, row 5=3(5)+2, row 6=3(6)+3. (#81, #87)
13:13-17:00	Per interviewer's request, D computed the number of coins in several different rows. Dana thenrefined her rule to become "times three and add the row number minus three." (# 119)
19:05 - 21:40	Interviewer challenged D to apply her nth rule to work backwards to find the row number given the number of coins. (#146 through #169)
27:11 - 30:10	Interviewer challenged D to use pictorial representations for her nth term rule. The interviewer explained her thinking regarding what she meant by pictorial representation as she offered suggestions for how the coins could be rearranged to form particular geometric shapes (#178, #180, #183). In the process of the interviewer's demonstration, D made a conjecture about perfect squares- identifying specific rows in the pattern that generated squared number of coins#196)
30:12 - 33:25	Interviewer questioned D about the row number of next perfect square. D mentally applied the "adding four" to the pictures and figured out the row number (i.e. row thirteen).Interviewer asked D to reason the next perfect square after 49, which promoted S to realize the feature of "odd number" of the number of rows. (#200 through #226)
33:26-34:42	Interviewer challenged D to find out the row number of 81 coins. D applied her backward counting nth term rule and the "adding four" rule swiftly to figure out the accurate row number.(#227)
36:05 –36:45	Interviewer challenged D to look for connections between her nth term rule and the "adding four" rule. D connected the middle coinand the "three" in her nth term rule,which justifiedthe "adding four". She generalized her rule to account for rows 1, 2 and 3.

Figure 3 depicts a roadmap of Dana's mathematical production in the process of generating an explicit rule for describing the pattern under study, highlighting the specific relationships she identified and approaches she developed and used. The turn numbers listed in the model correspond with particular segments in the interactional episode.

Dana's initial framing of a solution to the task, as Mason (1996) had outlined, was based on the observation that four coins were added to each term of the sequence each time. This framing remained strong throughout her work though how she referenced and used this construct varied. Her auxiliary approach, imposing a multiplicative structure on the pattern, also appeared to remain prominent in her thinking. Her desire to combine these two frames and to reconcile their differences served as Dana's primary force of free play, and one, which granted her autonomy over the task.

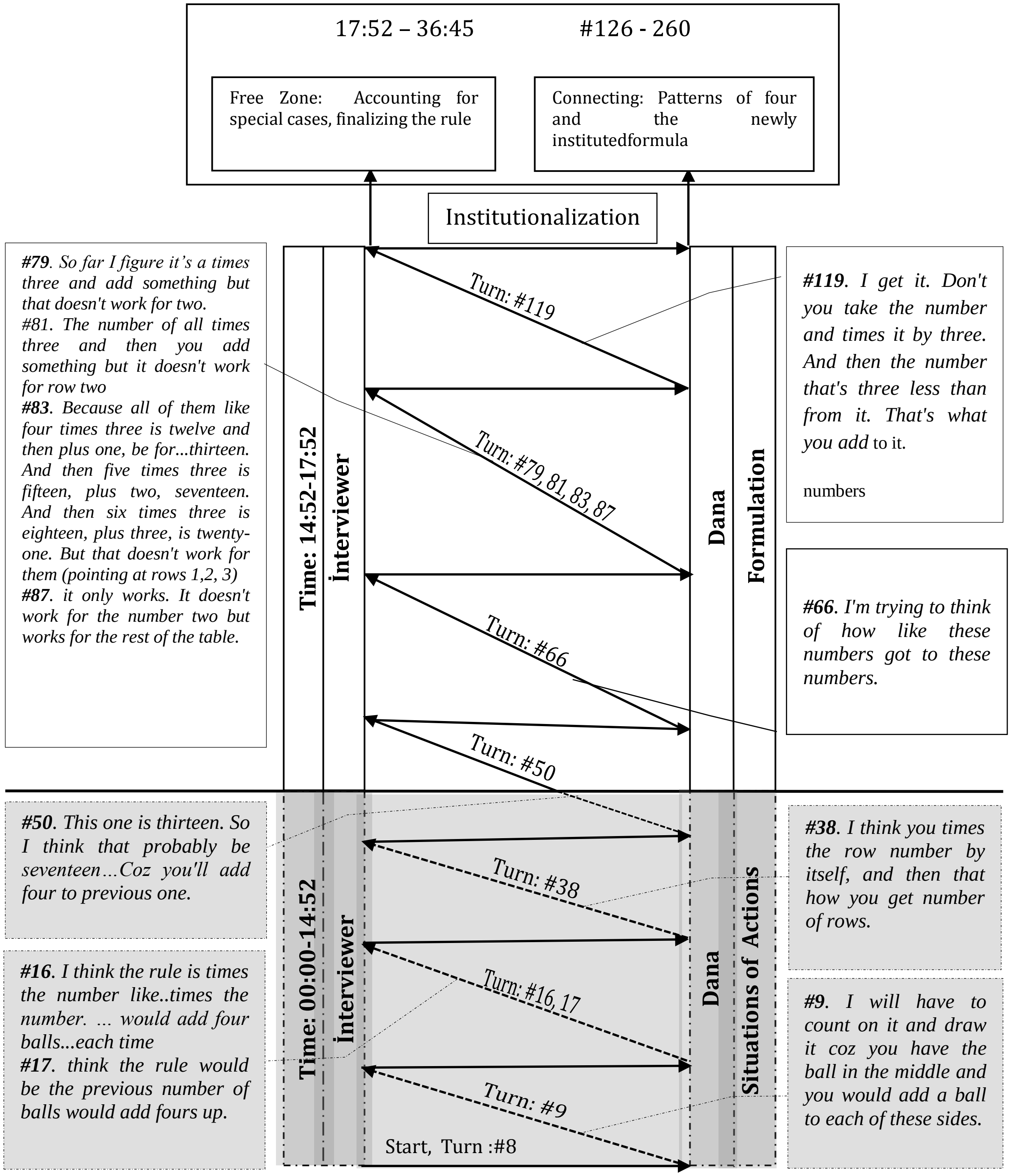

Figure 3. Dana's Mathematical Roadmap

4.2. Interactional Pattern: A Quick Quantitative Look

Table 3 summarizes the percentage of action types initiated or provoked in the course of interactions with the task situation. In computing the percentages of each comment type we compared the number of times a particular action was either resulted from the interaction or was elicited by it for each of the interactional agents. In augmenting data on coded verbal utterances and mathematical behaviors, we realized the need to document the number of prolonged silences (longer than 15 seconds) in our analysis of mathematical exchanges. We considered these prolonged silences as actions, equaling in value as verbal and non-verbal semiotic tools (language, representations, graphs, tables, drawings) used by the participants. Double counts in certain categories were inevitable. In places where an action or utterance fell under more than two categories of coding they were included in only two categorical counts. This happened on approximately 10 occasions.

As the data indicate, both the interviewer and Dana engaged in mathematical exchanges that forced epistemic actions of reasoning, justifying, formulating, structuring and validating. While the discourse of the interviewer consisted primarily of comment types that elicited or evoked understanding, explaining or aimed to challenge (with over 60% of the utterances shared) these exchanges prompted substantial episodes of reasoning, structuring, formalizing and evaluating actions from Dana (76%).

The number of prolonged pauses by both the interviewer and Dana during the session is noteworthy since these pauses resulted from a "pressure" to respond to a comment shared, a question asked, or an explanation offered. These pauses manifested reflection on a comment, or a reconsideration of an action performed, in an attempt to strategize what to do or say next. In almost all cases, the opening comment following a prolonged silence showed an extension of what had been done or said before. Lastly, the large number of shifts in the use of representations by Dana throughout the interview episode, augmented with the large percentage of reflective talk in the interviewer's discourse, is indicative of the presence of responsiveness within the milieu.

When reviewing the distribution of the interviewer's comments according to time intervals it was revealed that nearly 75% of all occasions of challenging, confirming, evoking occurred during the first 17 minutes of the interview promoting situations of action and formulation for Dana. However, 90% of all her comments during the remaining interview time aimed to institutionalize Dana's mathematical work. We will elaborate on the significance of this statistics in the later part of this analysis.

Table 3: Interactional Patterns

Participant	Dana	Interviewer
% of evoking/eliciting	1%	31%
% of Reasoning	20%	5%
% of Explaining	26%	12%
% of Proving	2%	-
% of Structuring	22%	7%
% of Formalizing	12%	-
% of Conjecturing	6%	2%
% of Challenging	3%	25%
% of Confirming	-	5%
% of Evaluating	15%	2%
% of Reflective talk (let me see if I understand)	3%	10%
# of shifts in the type of representations used	13	5
# of Reflective pauses	25	35

Although the quantitative summary of data provides a global account of interactions, offering a general view of epistemic actions that Dana produced and the type of interventions the interviewer used to provoke them, it fails to capture how *interaction itself was shaped during the interaction.*

That is, *how the didactical milieu itself evolved*. In the following section we will elaborate on specific elements of interaction and ways in which they shaped the participants' particular practices.

4.3. Interactional Patterns: Unpacking the Didactical Milieu

In TDS, the function of the teacher is not to offer suggestions that ensure a certain knowledge is produced but to assist the learner in fruitful interaction with the milieu in such a way that through interactions the learner engages in scientific activities using processes to *produce, formulate, prove, and construct models, languages, concepts and theories* (Brousseau, 1997, pg. 22). In this capacity the teacher is expected to ground and extend the children's work and their ability for production of more sophisticated knowledge, including the use of conventional semiotic tools.

A close inspection of the participants' interactions revealed two distinct ways in which the interviewer impacted Dana's interaction with the task under consideration: evoking reflection by eliciting reasoning and explaining, and providing a structure for Dana to form and solidify theoretical knowledge (abstracting generalization) through questioning. In turn, Dana's mathematical actions and observations forced a shift in the interviewer's own position towards the task, creating a climate for sense making beyond what was originally anticipated. The interactional episode then was divided sharply into two parts. During the first 17 minutes of the episode a predictable pattern of interviewer asking/eliciting and Dana responding and acting was evident. Struggle on the part of the interviewer toward untangling Dana ideas and to help her to articulate her thinking was prominent. This interactional configuration was interrupted by an unanticipated contribution by Dana, suggesting a novel mathematical model for describing the pattern under study. Dana's new insight did not seem to connect to her previously articulated expressions of thinking, which left the interviewer with no choice but to first attempt to determine the accuracy of Dana's idea while mapping out how she may have arrived at the formula, and then to try and provide a structure for Dana to connect her recent observation to those she had previously expressed so to set her up for establishing and using more abstract reasoning (Cobb, 1988). This was not an undemanding undertaking.

4.4. Dana's Actions: The interviewer's influence

According to Brousseau a didactical situation might be described as a game between a person serving in the capacity of a teacher and the student-milieu system. The rules and strategies between the teacher and the student-milieu system, which are specific to the task under study or knowledge under construction, are called the *didactic contract* (1997, pg. 41). He offered that within a didactical game the teacher plays two major parts: *devolution* and *institutionalization*. In devolution phase the teacher puts the learner in an adidactical or pseudo-adidactical situation as a starting point into the didactical game. In *instutionalization* situation the teacher defines the relationships allowed between the student's free behavior and production of scientific knowledge by providing a way of *reading these activities and giving them a status* (p. 56). Brousseau considered these two types of negotiations central to productive child-task milieu. The interviewer played these roles and enacted them both implicitly and explicitly throughout the episode.

The primary function of interventions, particularly during the first 17 minutes of the interaction episode, was that of establishing *devolution*, providing Dana space for action, formulation and justification by asking her to explain her ideas. The interviewer's comments and her questions focused on provoking Dana to reflect and to reason. This was done either implicitly by asking, "Are you sure?" "What do you mean?" "How do you know?" or by presenting the expectations more explicitly, demanding her to produce particular mathematical actions that the interviewer privileged (e.g."Convince me that you are right?" "Convince me that this method works!" "Can you prove your conjecture?" "Can you prove why this rule is right?") In nearly all occasions of this sort Dana fulfilled the interviewer's requests, as she showed attempts at either constructing a different explanation/representation or a more abstract/generalized argument that satisfied the parameters of the task. Throughout such exchanges Dana experienced some tension when first trying to merge the mathematical relationships she had observed and then in communicating them to the interviewer. She tried rephrasing her comments so they could be understood and upon failure resorted to restating what she had expressed before. Despite this, she remained in tune with the interviewer's demands and engaged in the task. Repeatedly, she articulated her own understanding of what the task expected. She routinely monitored efficiency of the rules she proposed and coordinated her responses by considering when and how her conjectures and theories were fragile. This *dialogic game* (Brousseau, 1997, p, 69) indeed helped Dana to continuously refine her mathematical statements. As such, the interviewer organized an environment for Dana to *overcome obstacles* (Brousseau, 1997, pg. 88). Dana, in turn, through her actions, conveyed an understanding of both the objectives of the didactical game; formulating, testing and validating statements, and that of permissible practices within the interactional milieu; noting, writing, pausing, reflecting and reviewing and routinely appraising what the interviewer considered as satisfactory and complete.

Excerpt I from the transcription depicts one of such situations. Prior to this interactional chain, Dana had computed the fourth term of the sequence using primarily concrete approach of drawing and counting the number of coins in each term. The interviewer then asked if Dana could determine the number of coins in the 20^{th} row, seemingly an attempt to help her transition from the specific to general. This led to two important events. First, Dana tried to offer an explicit rule involving multiplication by 4 (she wrote: 4*20=80) but quickly abandoned this technique since she noticed the formula did not work for rows 1, 2 and 3 (later she acknowledged that the number of coins must be odd due to the addition of 1 coin in the center). She then tried multiplying the row number by itself. In both cases she discarded her multiplicative approach and reverted to the "adding four each time" explanation. When asked again to proceed with finding the number of coins in row 20^{th} she argued that she was "not good at drawing," and after several attempts she lost interest and focus, observable in her gestures as she put down the pencil, pushed her chair away from the table, deliberately creating a distance between herself and the interviewer. The exchange detailed below accounts for what occurred in reaction to Dana's weakened interest.

At the point of entry into the dialogue the interviewer asked Dana to regroup (#22) and to explain what she believed the problem asked her to do. The interviewer's second turn (#24) was an additional attempt at understanding what Dana had assumed she was expected to do so to remove ambiguities regarding the direction she may have tried to pursue. Her fourth question (#28) asked explicitly that Dana to express the pattern she saw. Dana's response (#29) indicated that while she observed the pattern of adding four coins to each row each time she was unable to express a rule without relying on a physical models. The interviewer's request to Dana to generate the number of coins for row number 5 (#31) and then for her to offer an argument supporting her claim (#33 and #35) were aimed at grounding her back into the task. Dana's expression (#36)evidenced a shift in her approach, a stab at defining a multiplicative rule. This exchange suggests that during her prolonged silences she had indeed been actively trying to construct a rule that fitted the pattern. It is plausible that her new conjecture [(row #)^2] was conceived as the result of the interviewer's request for her to find the number of coins in the fifth row. Dana extracted from these two cases (row 3 and row 5) that the general rule would be the row number multiplied by itself. The interviewer's follow up question (#45: *so, how many would you have in row 4*) alerted Dana that while squaring the row number generated correct responses for the number of coins in third and fifth rows it failed to accommodate other cases. Hence, she opted to resort to describing the pattern recursively (adding four to previous one).

Excerpt I from the transcript:

22. *I2: You wanna tell me what the problem is?(regrouping)*
23. *S: Ok. How many balls would be in the twentieth row, hundredth row and any row?(restating the task)*
24. *I2: So when you see a problem like that what do you think you have to do?(regrouping, assessing)*
25. *S: Find the rule.*
26. *I2: Find the rule. So the question is essentially what? What's the pattern?*
27. *S: Yeah?*
28. *I2: Ok. So do you see a pattern?*
29. *S: Yeah but I can't find like the definite row. I am now in-between them. You'll add four but I don't know, like, how you would *** without drawing it.*
30. *S: prolonged silence*
31. *I2: Okay. So row five would have how many? Do you think?*
32. *S: Would have…fourteen…seventeen?*
33. *I2: How do you know?*
34. *S: Because you will add four to the previous one you got?*
35. *I2: Okay, so convince me that row number five has seventeen circles in it.*
36. *S: I think, I think you times the row number by itself, and then that how you get number of rows.*
37. *S: pause*
38. *S: So the fifth row would have...*
39. *S: pause*
40. *S: I think it would have twenty-five.*
41. *I2: You think so?*
42. *Prolonged silence*
43. *I2: I am not saying it's wrong. I am just saying...And you don't have to do it in your head. You can write things down.*
44. *S: I don't know. I will..I will write down.*
45. *I2: What did you get for row 4?*
46. *S: Er...so...and now I think that one's seventeen.*
47. *S: Er, huh...This one is thirteen. So I think that probably be seventeen…Coz you'll add four to previous one. [going back to the pattern of adding four]*
48. *I2: Okay. Do you guys? Er, do your teachers teach you things like different problem solving strategies, like how to organize the data, set up a table? Do you do that sort of things?*
49. *S: Data tables but we don't do like nothing like that.*
50. *I2: Okay. Do you think you can use a table if you set up a table and organize the numbers, you might be able to...to look for a pattern? Do you think?*

In the above exchange the interviewer's attempts at creating occasions of *Action* and *Formulation* situations are prominent. Evidence of reflecting, reviewing, revising and justifying and validating is present in Dana's utterances. Note also that in places where organizational heuristics were not visibly used by Dana which seemingly decreased her level of involvement in task (not working, refraining from commenting on her thinking) the interviewer activated her task milieu by suggesting problem- solving techniques that she may have known. Additionally, by posing questions that required Dana to engage in reproductions (What did you get for row number 4?) she structured progress when Dana's attempt at generalizing seemed too rushed to be productive or the expectation that she would do so seemed immature due to insufficient time devoted to specializing and testing of cases. Although the interviewer did, in places, make suggestions to Dana to consider certain approaches, these suggestions did not appear to channel Dana's thinking toward a privileged mathematical path. Indeed, they maintained an invitational flavor (i.e. Can you think of a way of presenting this information in a table? Is there a different way to look at this pattern?). Most specifically, Dana did not seem to have perceived these suggestions as mandates she felt obligated to fulfill. She chose to ignore the suggestions as she desired and continued to focus on refining her own idea. Further, the interviewer did not persist on persuading Dana to pursue a venue beyond her own first reference. Dana's own claim to the didactical space is visible through her frequent requests for time to think (*Hold on; oh, okay*). Indeed, her growing enthusiasm in formalizing her own thoughts brought to the fore the need for the interviewer to gauge her own interactions with the task.

4.5. Institutionalization

A careful examination of specific epistemic actions (i.e. conjecturing, validating, justifying, reflecting, abstracting, generalizing) that learners exhibit and the ability to respond to them appropriately sits at the heart of effective interaction within Brousseau's didactical triangle and most instrumental to the institutionalization process. We interpret this to mean that the teacher's effort within the child-task milieu to be geared not only toward provoking actions or episodes of formulation but also assisting the learners to formalize their intuitive and informal ideas.

A significant aspect of the interviewer's interventions concerned ***stabilizing*** Dana's own observations; seemingly a strategy to help Dana to either resolve conceptual ambiguities she experienced or to consolidate her seemingly disparate observations. The dialogue presented in Excerpt II provides a powerful illustration of this teaching function. Note that prior to the sequence of exchanges presented in the excerpt, Dana had, again, expressed that she was unable, even after having set up a table of values and considering quantities in corresponding columns, to find the number of coins in row 20^{th}without computing the values each time using recursive addition of fours. Interviewer asked Dana to articulate what she knew and whether she could predict values without relying on repeated addition (#78). In response, Dana offered a new observation, a relationship she had noticed (#79: *So far I figure it's a times three and add something*[3]*.)* The insight manifested her careful consideration of the givens of the task and constraints of her own technique for producing a generalized method (#79: *but that doesn't work for two*). It signaled her "internal" processing of problem without having shown evidence of such using any externally detectable tools up to that point. Throughout the remaining exchanges, the interviewer's effort was focused on unpacking Dana's new observation, which ultimately enabled her to suggest a closed form formula approximately 37 minutes into the interview.

Excerpt II from the transcription:

77. *I2: Now before you do that just think you could look up how to get down there? How to get that number without actually filling out the whole table . Is there a way of doing that?*
78. *S: So far I figure it's a times three and add something but that doesn't work for two.*

[3] Although Dana abandoned her newly constructed formula $(row\#)^2$ at this point, this formula was revisited later during the interview when she abstracted and argued that the formula did indeed account for odd numbered rows.

79. *I2: Why times three?*
80. *S: The number of all times three and then you add something but it doesn't work for row two*
81. *I2: Why times three?*

Note that in the preceding dialogue the interviewer asked twice why Dana had considered "multiplication by 3." Though Dana seemed unable to explain directly how she had deduced the relationship, she immediately acknowledged that she was aware that her technique did not accommodate the number of coins in all rows. Her specific reference to rows numbers where the rule failed gestured her reflective stance and the personal relationship she had constructed with the task. Her next explanation reveals her thinking far more vividly.

Excerpt III from the transcription:

82. *S: Because all of them like four times three is twelve and then plus one, be for...thirteen. And then five times three is fifteen, plus two, seventeen. And then six times three is eighteen, plus three, is twenty-one. But that doesn't work for them (pointing at rows 1,2, and 3)*
83. *I2: Interesting!*
84. *I2: interesting, yeah. (Prolonged pause)*
85. *I2: Let me let me test it. So you say two times three is six.*
86. *S: Yea, it only works. It doesn't work for the number two but works for the rest of the table.*

The rule that Dana began to describe (#82: starting from row 4: 4*3=12+1 be 13 and, 5*3=15+2 be 17) suggests a shift toward structuring and formalizing her technique, a far more sophisticated mathematical processing than the one she had shown earlier. Most remarkable about this particular exchange is that although Dana had not previously offered any indication of how she might have been processing the relationship among the numerical value, her statements disclosed that even when her mathematical work appeared flat she had continued her own free exploration of the task. The follow up conversations provided her with an opportunity to do so as the interviewer asked her to use her own formula to generate the number of coins in various rows; starting with row 4 and extending to row 12 (#88 to #118).

Excerpt IV from the transcription:

87. *I2: Three times three is nine. So you get that. Then four times three.*
88. *S: Is twelve plus one is thirteen.*
89. *I2: Twelve plus one. Three times five*
90. *: Is fifteen and plus two is seventeen.*
91. *I2: And then six times three*
92. *S: Is eighteen and plus three is twenty-one.*
93. *I2: So next one would be what? (Together with S) Seven times three*
94. *S: Plus four*
95. *I2: Is?*
96. *S: Er..it's ***.*
97. *I2: You can use your calculator.*
98. *(S uses calculator)*
99. *S: Twenty-five*
100. *I2: Is it correct?*
101. *S: Yea. Hold on. It's four. Twenty-one plus four is twenty five.*
102. *Long Pause*
103. *I2: Kay, so may I interject?*
104. *Pause*
105. *I2: So what was it? Seven times three*
106. *S: It's twenty-one plus four is twenty-five.*
107. *I2: So, fif...row fifteen would be. Oh, row ten, what do you think it would be? Ten times three is thirty.*

108. *S: Plus seven?*
109. *I2: You think so?*
110. *S: Yea, plus seven.*
111. *I2: You wanna test that?*
112. *S looks at the table*
113. *S: Okay.*
114. *(S is writing)*
115. *[00:15:30.02] S: Fifty-five....eight...is...twenty-nine...is...thirty-four*
116. *I2: thirty-three*
117. *S: Oh, thirty-three. (drawing) thirty-seven...*** So would be ten times three plus seven. Yea.*
118. *S: Oh, Okay. It's like *** I get it. Don't you take the number and times it by three. (I2: Okay) And then the number that's three less than from it. That's what you add to it.*
119. *I2: Hahahahaha! Are you sure? Okay, so let's get serious. Row fifteen?*
120. *S: Fifteen.*
121. *I2: Use your theory. Use your own theory ...to test it.*
122. *S: Fifteen times*
123. *I2: Got an extra pen? (talk to the other adult present!)*
124. *I2: Okay. Go ahead and test it.*
125. *S: Fifteen times three is forty-five plus twelve is fifty-seven.*
126. *I2: So, teach me how to do this. So when I...when I say row fifteen. You want me to multiply fifteen by three and then what?*
127. *S: Add twelve, coz it's three numbers less than that.*
128. *I2: So that would be fifteen minus three?*
129. *S: Yea.*
130. *I2: Are you sure?*
131. *S nods.*
132. *I2: Okay, so if I say row fifty. What would you do?*
133. *Fifty times three plus fifty minutes three.*

As Brousseu argued the institutionalization of knowledge is the process that allows students to navigate, reconsider, and alter their previous knowledge in light of new experiences under the teacher's guidance. The institutionalization of knowledge is not forced but organic (Radford, 2008). The above dialogue captures features of *institutionalization* process, focused primarily on creating a bridging media for Dana so that she could stabilize her mathematical observation. By asking Dana to use her own theory to predict the number of coins in various rows (#121)the interviewer created an environment for Dana to build stronger trust in her own technique and to confirm its validity. In order to facilitate Dana's transition, the interviewer encouraged her to use a calculator (#98) so to reduce the residue of computational errors that could interfere with her focus. She officially endorsed the authority of work to Dana by asking her to teach her how to apply her formula to find the desired values (#127: *So, teach me how to do this. So when I.. when I say row fifteen. You want me to multiply fifteen by three and then what?*). She also asked Dana to test and verify her answers several times (#110, #112, #125, #131). These actions collectively, enabled Dana to formalize her own method.

4.6. The Interviewer's Actions: Dana's Influence

One of the main claims of Brousseu's PDS is teacher sensitivity to the child's state of knowledge in ways that *free* action is activated and enabled. Hence, the notion of feedback in TDS model has less to do with controlling mathematical practices of the child but with the teacher's ability to be present in her zone of thinking. As such, the child's intentions are interpreted and assessed within the context in which those intentions and actions were formed. Therefore, the teacher's feedback within the child-milieu system is not gauged towards achievement of some absolute truth but constructed in response to the child's milieu system. That is, the teacher's feedback within the didactical system is an attempt to understand the child's way of thinking and expanding it in ways that may have little to do with the knowledge that was intended by the teacher. This description is consistent with Wagner's notion of effective interaction. Wagner (1994) argued that an interaction is effective when the environment is responsive to change. Within the didactical situation, the environment consists of the two main interactional agents, teacher and learners. Central to the concept of effective interaction is the idea that participating agents in the interactional space respond to each other (Bloome et al., 2009), making the relationship reciprocal in nature.

According to Cobb and colleagues (1997), the role of the teacher within a learning environment is to initiate reflective shifts in discourse in ways that what is said or done in action becomes an explicit topic of discussion. To do this appropriately, the teacher must possess a deep understanding of what happens in action. In the previous sections, we illustrated ways in which Dana exhibited responsiveness to the ideas and suggestions of the interviewer. Drawing from the same examples we will comment on the manifestations of mutuality of interactions pertaining to the influence of Dana on the activities of the interviewer. This analysis is central to unpacking the *institutionalization* situation proposed in TDS framework, from our standpoint.

4.7. Dana's mathematical insights and how they were treated and used

Previously we elaborated on the nature and content of interactions between Dana and the interviewer during the first 17 minutes of the interview episode. During that particular time frame although the interviewer and Dana seemed to gauge their comments and questions against the new information they obtained from one another, the autonomy and power within the interactional space was not always equally shared, the interviewer continued to instigate, providing space for Dana to structure and to articulate her thinking en route for defining an explicit rule. This pattern was changed when Dana, who had shown no observable progress in moving from the specific to general through her numerical analysis, announced a statement unpredicted by the interviewer (see #119). Dana's new observation forced the interviewer to engage, simultaneously, in sense-making and problem solving in the course of interactions. This tilted the balance of control and ownership of the didactical milieu. The synergy of interaction then focused on solidifying the mathematical observation of the child and created an environment in which the interviewer and the child shared equal claims to the action space. According to Brousseau when the learner disregards the didactical contract and acts with reference to the characteristics of the a didactical situation the ideal devolution situation is achieved. Such was the case.

In the previous section we examined the interviewer's reaction to Dana's discovery of the rule, 3(row number) + (row number -3), and ways in which her questioning stabilized Dana's conviction of its merit. Excerpt V from the transcript captures utterances 145 through 185, which portray the interviewer's efforts towards evoking Dana's free play, extending her thinking drawing from her own ideas. The target goal during this exchange was not about fulfilling the interviewer's desire to help the child to express the iterative representation of the sequence in an explicit form (4*(row number)-3) but to the institution of venues for testing the reliability and efficiency of the child's idea (Balacheff, 1991).The dialogue reveals the interviewer's attempt at grounding Dana's observation by asking her to consider an inverse relation (given the number of coins in the pattern, can you identify the row number), a mathematical action central to understanding functional relationships (Mason, 1996).

The excerpt begins with Dana stating again that her rule did not accommodate the pattern she had observed (#145). Since she had previously repeatedly expressed that she was aware that the formula did not work for rows 1 through 4 (#83, #87) her responses were perceived as signals that she may potentially abandon her line of reasoning. The interviewer extended Dana's mathematics by asking if given the number of coins she could determine the row number associated with it. The interviewer's first choice, 36 coins (#149) appeared deliberate since the table of values Dana had previously produced could potentially help with her connection making process. Drawing from the table, Dana offered an estimate for the row number (#155) however she did not acknowledge the condition that the row number had to be an integral value. The interviewer then asked Dana to find the row number associated with a larger number of coins (#159). We interpret this move to have been made to serve one of two functions: to evaluate how Dana's processing might conform or contradict her previous strategy or to find another point of entry for anchoring her mathematical construction[4].

Excerpt V from the transcription:

145. *S: Because nine times three is...is twenty seven plus six is thirty three. And it does that for eight, seven, six, five, four and three, and one... but don't that for two*
146. *I2: So here's what I'm going to do. I am going to give you a number and I would like for you to tell the row number is associated with it.*
147. *S: Okay.*
148. *I2: You're ready?*
149. *I2: Thirty-six.*
150. *(S looks down the paper) Prolonged pause*
151. *S: You talk about thirty-six rows or thirty-six balls?*
152. *I2: Thirty-six balls.*
153. *(S is using the calculator.)*
154. *I2: What are you doing? You wanna tell me what you are doing?*
155. *S: Urm...It has to be nine point something. So, I was gonna try nine point five to see if it gives back..coz that will be closer than ten.*
156. *I2: How did you decide that it has to be nine point something?*
157. *S: Because it has to be between nine and ten. Because it has to be between thirty-three and thirty seven.*
158. *I2: Got it. So, here is my other number. One hundred...two hundred and thirteen.*
159. *S: Urm (silence) It has to be...ok. So...(prolonged silence) has to be around the seventies. Or late sixties.(not writing and not using a calculator)*
160. *I2: How do you know? How do you know? Convince me that is around seventies.*
161. *S: Because two hundred thirteen divided by three is seventy-one. (Okay.) So has to be around sixty..to seventies.*
162. *I2: Why do you say sixty, seventy?*
163. *S: Because it's either the late ones or the early ones because the rule you had to add. You had to subtract three...from it. All right. Hold on. I don't know. I just know it has...coz when you divided by three. It's seventy*

[4] We don't intend to characterize the interviewer's move here (or at any point during the interactional episode) as right or wrong. We merely reflect the events as they unfolded; what seemed to be the motivation behind questions the interviewer asked and how she used the knowledge gained to organize the subsequent interaction. Indeed, we question the appropriateness of having made such a quick leap from 36 to 213 in such a short time interval and without having unpacked Dana's response to the previous question. We do speculate that this quick assessment provided the interviewer with a better view of conceptual links that the child may have made at the time of construction and informative in how she may have needed to proceed next.

one. So it has to be (?) seventy one. Sixty-seven is above (?) seventy one. So you have to be sixty-seven or the seventies..the early seventies. (she means low seventies)

164. *I2: Makes perfect sense to me.*

Dana's response (# 159)and her follow up explanations (#161 and #163)alerted the interviewer that she had not yet made an explicit connection between the dependent and independent variables. In response, the interviewer first acknowledged that she understood what Dana was doing (#164:*Makes perfect sense to me*) and then initiated a new line of questioning which seemed to have been chosen to help Dana to gain greater control over her formula, seemingly another attempt at ***stablizing*** Dana's *bridging* process. At this point the task was reformulated, at least implicitly, by the participants.

In her new approach, the interviewer asked Dana whether building a geometric model of the pattern could help in connecting her new formula to her previous observation (addition of four coins each time) (# 165). This struggle became far more visible as her comments and questions seemed to go unnoticed by Dana (#173, #177, #179, #182) although Dana seemed to have built on these exchanges to refine her own thinking (#168, #172).

Excerpt VI from the transcription:

165. *I2: You said here that each time we add four (Urm um) Correct? (Yea) Here you said get the row number multiply by three and then add three less than the row number. So what is the connection? Is there a connection between this formula that you generated here and that adding four pattern that you recognized there?*
166. *S: So I'm tryin' to think what the relation would be..coz..I don't know.* (Prolonged silence)..Urm...
167. *I2: You wanna tell what's going on in your head?*
168. *S: I'm trying to think what would it would look like because I'm trying to see the..if this would be together with that. (Look at the table) It would be. You have four because of would like...the one number difference we put in the table. So..er! One and four would be between as three. And one n three is what? I don't know. Er..(Silent)*
169. *I2: What are you? What are you seeing in your head right now? Some numbers? Are you seeing pictures? What are you seeing like now?*
170. *S: Numbers?*
171. *I2: What? What numbers do you see?*
172. *S: I'll trying...like the difference between… I'm trying to think of the difference between this and like we do have four.*
173. *I2: Ah...Can you look at this? Pictorially for seeing why the difference would be?*
174. *S: Because the number you..add... wait. Coz, coz each time you add a ball to the end should be four ends. So you add one ball each. That would be four balls.*
175. *I2: Right. It's like you put four...add to the outside.*
176. *S: (nods) Um um.*

Dana's comment indicated her lack of progress in attaching a visual imagery to numerical results she had obtained. The interviewer tried to help her overcome this obstacle by asking her if the general form could be represented geometrically (#177 and #179). The subsequent exchange however, demonstrated a particular disconnect between the interviewer's mission (helping Dana reflect on connections between the implicit and explicit formulas she had generated) and what Dana actually extracted from the exchange. Throughout this mathematical exchange the struggle for sense making on the part of the interviewer is evident[5].

[5]We are not convinced that the interviewer may have had a particular destination in mind as she prompted Dana to seek a geometric interpretation. Indeed, her particular emphasis depicted in long utterances of her explaining signals at attempt at co-constructing potential relationships in the course of interaction.

Excerpt VII from the transcription:

177. *I2: And I am thinking why is, what would be like the visually, pictorially, what could that be? How could that translate...because this one we can visually see the additional four each time, right? (um..um) It makes perfect sense, right? Because if I start from here. I'm telling you this because I wanna know if you can pictorially build that one too. I see this. (I2 starts writing on the paper). I see one, right?*
178. *S: Um um.*
179. *I2: And then...one plus four, right? One two three, throw it in, right? And then I see the previous one plus those four. So it's one plus four plus four, right? (um) So again, I see this one. Go again, and for the next one, it would be four more added here, right? One plus four plus four plus four, which is pretty nothing that you solved that so quickly. Right? I wonder...if we can have like a pictorial representation of what this might look like, because, here, for instance, you say, it's two times three plus*
180. *S: Negative one*
181. *I2: Er... two minus three, right? Which is negative one. You're absolutely right.*
182. *I2: So I wonder...if there's a way of pictorially show this relationship. So this would be three times one plus...one minus three? which is negative two? Or let's do this, three times two.*
183. *(Silent)*
184. *S: Coz like, when you, if you keep doing that for all the different numbers, it will be like one, or one, one different, one different, and then one different. (Point at the paper)*

Brousseau (1997) argued that *the more the teacher gives in to her demands and reveals whatever the student wants, and the more she tells her precisely what she must do, the more she risks losing her chance of obtaining the learning which she is in fact aiming for* (p. 41).Although it is not quite clear, based on what Dana chose to share publicly what she may have extracted from the explanations that the interviewer offered or the type of analytical tool she tried to introduce to Dana, the above exchange provides a compelling instance of independent discovery on her part. Although Dana's comment (#180) seem unrelated to the interviewer's descriptions of a geometric representation of formula, it is reflective of the progress she made in explaining how her explicit formula accounted for anomalous cases she had identified earlier. As such it does suggest that she did use the space of interaction to stabilize her explicit formula further by accounting for the number of coins in the second row [3(2) -1]. The unsolicited explanation she offered next (#184) can be taken as evidence of her autonomous knowledge construction process.

5. Discussion

The primary purpose of the study was to examine the impact of interaction on learning and particular situations that were generated in the process using Theory of Didactical Situations as a lens for analysis and interpretations of events. Brousseu's TDS served as a powerful tool for isolating micro-genetic aspects of interaction that enhanced the potential of didactical milieu for learning. Analysis of the in-the-moment actions of Dana and the interviewer around the task, the mathematical relationship they formed in the course of the interaction and the quality of feedback they gave to or received from one another revealed two key issues central to increasing the potential of didactical situations for learning, both of which sharpen focus on the teacher and her capacity for activating and sustaining children's presence and participation in the learning space. These include, (1) the relationship of the teacher with the mathematics under study- and her ability to manipulate her disciplinary knowledge in responding to the unexpected events in the session and, (2) the teacher's relationship with the child- and her ability to attend to both affective and cognitive needs that arise in the course of didactical interaction. As such we offer an elaboration to Brousseau's model that utilizes these elements, as described below.

5.1. Child-Task-Teacher

Figure 4 conveys our findings regarding influences shaping the didactic situation analyzed in this work. Note that the gray regions in the model account for spaces of interactions present in the milieu, which correspond with three specific ordered pairs: teacher-task, teacher-child, and child-task relationship.

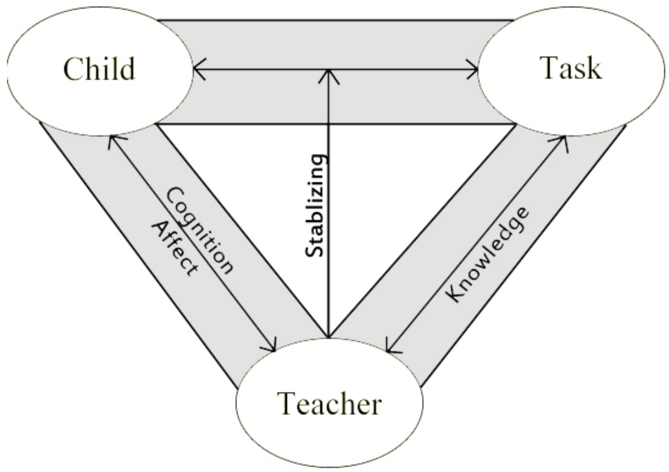

Figure 4: Child-Teacher-Task

In our analysis, the teacher-task relationship served as a chief ingredient in what knowledge was elicited from the learner and ways in which the relationship between the child and the task was facilitated. That is, the presence and absence of mathematical insights, the ability to recognize the unfamiliar and/or unpredicted mathematical ideas of the learner, the aptitude to make sense of the child-instantaneous ideas and to simultaneously make connections between her implicitly stated ideas and cognitive structures from which the child may have drawn when making those statements. Most importantly, the interviewer's willingness to re-evaluate or even abandon epistemological stance with which she may have entered the interaction determined whether the feedback she provided did indeed increase autonomous mathematical thinking so to allow generation of more sophisticated ideas without imposing a desired form of thinking on the child. As such, our findings suggest that a teacher's own knowledge and her relationship with the mathematics of the task can influence the interactions of the child with didactical milieu, an element seemingly taken as given in the TDS. Specific conditions that the teacher's own relationship with the didactical task might impose on the child-task milieu and most importantly on the devolution process and institutionalization situation of the environment merit careful scholarly investigation. This point is particularly important since our findings suggest that Brousseau's notion of institutionalization might need to be extended beyond actions aimed at legitimizing knowledge, from a disciplinary standpoint, and to include also the deliberate activity of stabilization of knowledge. During Dana's exploration of the task she made and expressed different relationships she either observed or tried to construct. If not prompted by the interviewer or given the opportunity to strengthen conceptual links or make analytical explanations, it is not obvious whether Dana would have managed to move forward in formalizing her thinking. As such, the interviewer helped stabilize Dana's understanding.

An equally important constituent in the teacher's capacity to navigate child-task relationship was the degree of attentiveness to the manifestations of the child's particular affective state communicated though gestures and pauses, as she was involved in the knowledge constructing process. That is, realization of, and sensitivity to, when the child might have felt defeated by the task, understanding the impact of emotional constraints placed on the child due to inability to produce results as immediately as she had perhaps assumed she was expected to do, an absence of mathematical language she felt needed to make her ideas understood, along with reluctance to make public her internal thoughts in fear of public scrutiny was fundamental to the interviewer's success in grounding the child in a sustained and fruitful manner.

6. Epilogue

Mercer (2000) coined the construct of inter-mental development zone (IZD) to describe a communicative space co-created between a student and teacher by staying tuned in to each other's knowledge creation. The IDZ is the ideal space, where the student, via continual negotiation, can work on the edge of his/her knowledge with the help of the instructor. Although we find this construct useful in understanding the complex nature of knowledge construction process through interaction, as evidenced in our analysis, we argue that elements of IZD apply equally as much to the teacher. That is, not only the child but also the teacher (the more knowledgeable agent) must resume the position of "sitting on the edge of his/her knowledge" so to provide appropriate feedback during the interaction. Advancing mathematical thinking of children, and transitioning them towards forming and articulating more sophisticated representation of knowing, or articulation of such, relies heavily on the interactional agent's knowledge of the child's particular state of knowledge-- its intricacies and its potential for enrichment. This implies that the agent acknowledges existence of learning opportunities within the didactical space for improving her own capacity to increase children's mathematical capacities. This point has immediate implications for research of any kind that concerns mathematical thinking of children.

First, children's mathematics as articulated is not always in polished or final form. Children may not propose or use representations immediately recognizable to the interviewer. Indeed, as we saw in Dana's case the child may even withhold ideas, we saw only *what she allowed visible during the interaction*. What she chose to articulate and make public did not follow an idealized linear progression or refinement.

Second, when eliciting thinking, naturally, a researcher of cognition targets and seeks to localize evidence of children's learning and knowing using their own vision and horizon of mathematics as a framework for interpretation. This tendency might lead to an assembly of flawed inferences regarding the efficiency of methods produced by those under study-- their state of knowing and thinking, and their instantaneous or cumulative potential for development. This issue has rarely been addressed in mathematics education or carefully contemplated. Indeed, while scholarly efforts aimed at unpacking classroom-based interactions and learning have forced a sharp focus on inspecting the quality of teachers' knowledge and its impact on what is elicited or promoted in classrooms, researchers themselves have been safeguarded against compatible examination and analysis. Here, we acknowledge our vulnerability in our interpretations of Dana's mathematical work based on what she chose to exhibit (Steffe, 2013). Nonetheless, we do recognize that the development of any sort of theory that concerns human mind and its functioning should be assumed as work in progress. Grounding analysis of mathematical practices of children, potential and viability of didactical situations, unpacking interactional episodes in multiple lenses that take into account affect, culture, personalities, and personal ways of expressions is essential in order to increase the community's capacity to offer a grand theory of knowing and learning to know mathematics.

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Appendix I

Segments		Transcription	*Language functions*	Mathematical	Situational
Sgt 1	1	S: How many balls in the twentieth row, and the hundredth row? And any row?	*Reproductional*	Entering	Action
0:00 - 01:46	2	S: Other...don't it's like…er.Well, I am trying to figure out how it was different. And it's different because the outside ball each group of ball. So, guessing...like...that...that would be**	*Reasoning*	Structuring	Action, Formulation
	3	S:Think that you'll keep the ball in the middle and then just keep adding the balls to the…the ball. These balls. Just keep add on it.	*Heuristic*	Explaining	Formulation
	4	S: But since it's in. The five's an odd number, it. You can… hold on, I'll think about it.	*Organizational*	Reasoning	Action: Thinking
	5	S: Ok, since five is an odd number, then you would do. You can add balls, so like every sides you have an extra one. Coz it's like.	*Reasoning*	Reasoning	Justification
Sgt 2	6	I1: So what is the question?	*Informative*	Reflecting	Formulation
1:50-5:00	7	S: How many balls in the twentieth row, hundredth row and any row? ***second row?	*Reproductional*	Responding	Formulation
	8	I1: how many balls would be in the fourth row?	*Interrogative*	Confirming	Validation
	9	S: The fourth row has? Like…ok…one…So I will have to count on it and draw it coz you have the ball in the middle and you would add a ball to each of these sides. Then after that I will figure out though.	*Eternal thinking, heuristic, reasoning*	Formalizing	
	10	I1: Ok let's do it.	*Compositional*	Confirming/Challenging	Institutional