

# Contextualized tasks of Brazilian textbooks from a Critical Mathematics Education perspective

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## ABSTRACT

**Background:** considering the dimension of the National Textbook Program (PNLD) in Brazil, the textbook plays a relevant role in student formation as it, as in many countries, defines the real curriculum. **Objective:** to propose reflections on whether, and how, contextualized tasks can contribute to changing students' attitudes and beliefs regarding Mathematics and society. **Design:** In view of this goal, based on the assumptions of qualitative research. **Settings and participants:** we analyzed a total of 41 Mathematics textbooks, 20 of which were Middle School and 21 of High School, all approved by the PNLD. **Data collection and analysis:** We sought for contextualized tasks that promoted reflections and critical discussions about everyday themes - economic, social, technological, sustainable, ethical, among others - present in society. To analyze the selected tasks, we adopted the Critical Mathematical Education as a theoretical lens, revealing social, political, and cultural concerns, proposing the reading of the world with Mathematics, in addition to promoting a Pedagogy for Liberation, based on social justice. **Results:** The results indicated that the selected tasks could promote reflections and contribute to the change of students' attitude towards Mathematics and society, however we emphasize that the teacher's role is fundamental in this process. **Conclusion:** In view of the high number of tasks present in Brazilian textbooks, we consider that they could present a greater number of contextualized situations, increasing the number of options for the teacher.

**Keywords:** Textbook analysis; Reading the world with Mathematics; Contextualized tasks; Relevant teacher's role; Critical Mathematics Education.

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## Tarefas contextualizadas em livros didáticos brasileiros em uma perspectiva da Educação Matemática Crítica

### RESUMO

**Contexto:** considerando a dimensão do Programa Nacional do Livro Didático (PNLD) no Brasil, o livro didático desempenha um papel relevante na formação do aluno, pois, como em muitos países, define o verdadeiro currículo. **Objetivo:** propor reflexões sobre se, e como, tarefas contextualizadas podem contribuir para mudar as atitudes e crenças dos estudantes em relação à Matemática e à sociedade. **Design:** Tendo em vista esse objetivo, com base nos pressupostos da pesquisa qualitativa. **Ambiente e participantes:** analisamos um total de 41 livros didáticos de Matemática, sendo 20 dos Anos Finais do Ensino Fundamental e 21 do Ensino Médio, todos aprovados pelo PNLD. **Coleta e análise dos dados:** Buscamos tarefas contextualizadas que promovessem reflexões e discussões críticas sobre temas do cotidiano - econômicos, sociais, tecnológicos, sustentáveis, éticos, entre outros - presentes na sociedade. Para analisar as tarefas selecionadas, adotamos a Educação Matemática Crítica como lente teórica, revelando preocupações sociais, políticas e culturais, propondo a leitura do mundo com a Matemática, além de promover uma Pedagogia Libertadora, com base na justiça social. **Resultados:** Os resultados indicaram que as tarefas selecionadas podem promover reflexões e contribuir para a mudança de atitude dos alunos em relação à Matemática e à sociedade, porém ressaltamos que o papel do professor é fundamental neste processo. **Conclusão:** Tendo em vista o elevado número de tarefas presentes nos livros didáticos brasileiros, consideramos que elas poderiam apresentar um maior número de situações contextualizadas, aumentando o número de opções para o professor.

**Palavras-chave:** Análise de livros didáticos; Leitura do mundo com a Matemática; Tarefas contextualizadas; Papel relevante do professor; Educação Matemática Crítica.

### BRAZILIAN SCHOOLS AND TEXTBOOKS

Brazilian Basic Education comprises three cycles (0 to 5-year-old), Fundamental School, which is divided into Elementary School (6 to 10-year-old) and Middle School (11 to 14-year-old), and High School (15 to 17-year-old). A total of 48.6 million students and 184.1 thousand schools<sup>1</sup>, distributed as follows: 72.2% of preschool students, 81.7% of

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<sup>1</sup> Data from the National Institute for Educational Studies and Research Anísio Teixeira (INEP): [https://drive.google.com/file/d/1ul8OptGdTzory5J0m-TvvSzILCrXmWeE/view\\_.2017](https://drive.google.com/file/d/1ul8OptGdTzory5J0m-TvvSzILCrXmWeE/view_.2017)

Fundamental School students and 87.8% of High School students from public schools<sup>2</sup>. The number of students enrolled in public schools reflect the large dimension of the country. Brazilian public-school students receive textbooks through a free distribution program from the Ministry of Education.

Since the use of textbooks has been a common practice by Brazilian teachers and students, governmental programs<sup>3</sup> have been created to ensure the quality of the material. According to Carvalho (2018), a group from the Ministry of Education (MEC) with more than 250 professionals has been responsible for evaluating the textbooks submitted by private publishing companies since 1997 and the teachers can choose the most appropriate ones based on their context and the school's political-pedagogical project.

The approved books are bought by the federal government from privately owned publishing houses and distributed to public school in the whole country. The financial resources are provided by FNDE, *Fundo Nacional de Desenvolvimento da Educação* (National Fund for Educational Development), which funds most national educational programs, with resources from many sources. This textbook distribution program, called *Programa Nacional do Livro Didático*<sup>4</sup> (PNLD, National Textbook Program) is, by far, the biggest, in financial investment, of all MEC programs (Carvalho, 2018, p.773).

It is important to emphasize that authors and publishing companies do not have complete autonomy in the elaboration of the materials to be submitted, since a public edit with norms and regulations was issued. The textbooks are evaluated every four years for each cycle – childhood education, elementary school, middle school and high school; therefore, the material is distributed to all the students of the same segment every year (Brasil, 2020). Except for consumable books (from

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<sup>2</sup> Public schools and universities in Brazil offer free education; therefore, the students do not pay any fee or tuition.

<sup>3</sup> For further information, see Mazzi and Amaral-Schio (2021), Mazzi (2018) and Carvalho (2008).

<sup>4</sup> In 2017, the program was reformulated and its name was changed to National Textbook Program, The acronym, however, was maintained (PNLD).

the first to the third year of elementary school), the textbooks are expected to be returned in good condition to be used in the subsequent years. However, replacement copies are bought and distributed in all grades<sup>5</sup>. Table 1 shows the last three governmental acquisitions.

**Table 1**

*Books acquired over the last 3 years (FNDE Portal<sup>6</sup>, 2020)*

	<b>Total of benefitted schools</b>	<b>Total of benefitted students</b>	<b>Total of copies</b>	<b>Values (US\$)</b>
<b>PNLD 2020 (middle school assessment)</b>	123,342	32,010,093	172,571,931	347,550,258
<b>PNLD 2019 (preschool and elementary school assessment)</b>	147,857	35,177,899	126,099,033	275,506,413
<b>PNLD 2018 (high school assessment)</b>	117,566	31,137,679	153,899,147	366,808,028

According to Carvalho (2018), the PNLD is a successful initiative, since it has provided the public-school students with access to quality material, although some of this material has become stifled<sup>7</sup>. The program is constantly reviewed, and the last alterations took place in 2019.

In this scenario, and considering the dimension of the PNLD in Brazil, the textbook plays a relevant role in student formation. Unavoidable fact is that “textbooks in Brazil, as in many countries, define the real curriculum [...]. Therefore, any improvement in textbooks, in their contents or methodologies, opens the way for improvements in

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5 This replacement occurs due to loss or the relation between number of students and available books, which annually change.

6 <https://www.fnde.gov.br/index.php/programas/programas-do-livro/pnld/dados-estatisticos>

7 Significant items were altered, among them the evaluation team and the periodicity of each cycle.

teaching, and possibly in learning” (Carvalho, 2018, p.776). And textbook can be seen as a curriculum not only in Brazil (Dias, 2011). Not surprisingly, the textbooks have been object of interest in research, as stated by Fan (2013) and Valverde et al. (2002), especially over the last 15-20 years, with the global expansion in the production, distribution and improvement of textbooks.

The present paper was conducted by two researchers of Theorem - Reflections on Geometry Mathematical Education research group, which has addressed different themes associated with mathematics education under different theoretical lens. The group has lately focused on the collections distributed by the PNLD from 2014<sup>8</sup> 2020, having analyzed almost a hundred didactic books and the results of this investigation are described herein. Thus, this analysis is aimed at providing information on the contributions of textbooks in transforming the students’ view on mathematics, changing their attitudes and beliefs and increasing their knowledge on mathematics, focusing on the last approved textbook editions to be used in the final years of Fundamental School and High School.

For this, we adopted the Critical Mathematics Education (CME) as theoretical lens, revealing social, political, cultural and economic concerns and proposing reading and writing with mathematics (Gutstein, 2003, 2006), aiming at a pedagogy for liberation (Freire, 1987, 2013, 2019), based on liberty, equity and social justice (Skovsmose, 2008). In this direction, we looked for evidence that would help us to understand the following question: *how is CME fostered by contextualized tasks of textbooks in middle school and high school?*

Regarding contextualized tasks, we understand that contextualization exceeds a mere task presented by a text, it is intended to promote thoughts about broader aspects - economic, technological, sustainable, social, ethical, among others - and cannot be applied in a *naïve* way, once it is aimed at allowing students to approach situations where their own knowledge will exceed the limits of the classroom (Alrø & Skovsmose, 2004), and can become “a vehicle for assessing insight, understanding, and concepts” (De Lange, 1999, p. 27).

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<sup>8</sup> It is important to note that the reference year is the one in which the textbook will reach the school; therefore, the evaluation and distribution of the books actually start two or three years earlier.

Considering this scenario, the present study elucidates our view on CME, used as theoretical reference for data analysis, as well as the methodological assumptions and the textbooks analyzed. It also presents the analysis of ten tasks, highlighting the convergences and divergences regarding CME and proposing possible improvement strategies that the teachers could use to conduct the tasks. Finally, some conclusions about the topic are presented.

## **THEORETICAL FRAMEWORKS**

Critical Mathematics Education (CME) first appeared in the 1970s in Europe, and a new version of the concept emerged in the 1980s in the United States. From the first accounts concerning the foundations of CME, three different theoretical perspectives can be identified in the literature: the Critical Theory developed by Frankfurt; the Pedagogy for Liberation by Paulo Freire and the Ethnomathematics by Ubiratan D'Ambrósio (Valero et al., 2015).

According to Valero et al. (2015), from its very roots, CME focuses on the ways mathematics is used, on its consequences and implications in educational practices, seeking to understand how mathematical knowledge and the school mathematical knowledge are intertwined in and with society and its political, historical, economic and social processes (Valero et al., 2015, p. 9).

Considered the father of CME (Ernest, 2010), Skovsmose (2007, 2011) claims that, rather than a branch of mathematics education, it is “an expression of some preoccupations or concerns with respect to mathematics education” (Skovsmose, 2011, p. 2); therefore, it cannot be defined as a classroom methodology or be constituted of specific curricula.

One of these preoccupations regards the formatting power of Mathematics. Skovsmose (1988, 1994) draws attention to the fact that, in a highly technological society, structured on mathematical models, a reflection upon what is behind decision making is needed. In this sense, the society is expected “to challenge the neutral and innocent view of Mathematics through reflection and critical thinking” (Valero et al., 2015, p. 10). According to Skovsmose (1994, p. 38),

If education, as both a practice and a research, should be critical it must discuss basic conditions for obtaining

knowledge, it must be aware of social problems, inequalities, suppression etc., and it must try to make education an active progressive social force.

In this direction, CME can develop critical understanding and challenge the students to question the dominant powers (Frankenstein, 1983), challenging the ideology of certainty and the view that Mathematics is a perfect and irrefutable system (Borba & Skovsmose, 1997). However, to develop such knowledge, some competences are required: mathematical, technological and reflexive – aspects that compose mathematical literacy (mathemacy).

Mathemacy is defined as the ability to calculate algorithms and mathematical procedures; apply these tools in different situations and be capable to reflect upon them (Skovsmose, 2011). These ideas convert to what Gutstein (2003, 2006) calls *reading the world with Mathematics*. According to this author,

means to use mathematics to understand relations of power, resource inequities, and disparate opportunities between different social groups and to understand explicit discrimination based on race, class, gender, language, and other differences. Further, it means to dissect and deconstruct media and other forms of representation and to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them (Gutstein, 2003, p. 45)

Despite the importance of reading the world with Mathematics, situations that foster the development of these competences are not frequent in the classroom, the exercise paradigm still prevails (Skovsmose, 2001). This paradigm is associated with a traditional teaching approach, in which the teacher follows a plastered sequence of actions: *concept presentation* → *examples* → *sequence of exercises*. In the activities proposed under this approach,

Any information provided in the text of an exercise can be considered exact and sufficient. And furthermore, the information provided in the exercise is both sufficient and necessary for solving the problem [...] where all measures are exact, and where the information given is

both necessary and sufficient in order to calculate the one and only correct answer. (Skovsmose, 2011, p. 8).

Assuming the exercise paradigm as the only basis is a way to corroborate and perpetuate a mercantilist view of education, which aims at preparing students for the labor market rather than promoting critical thinking and reflection, viewing them as a controlled and submissive workforce. In this sense, Skovsmose (2011, p.7) addresses some issues that foster our reflection as mathematics educators:

Exercises play a crucial role within the school mathematics tradition. Thus, during their time in school, most children will be solving more than 10.000 exercises. However, not much mathematical creativity is cultivated through working on such exercises. Could it be that some deep socio-economic irrationality is maintained as part of mathematics education? Could it be that this part of the educational system the world over sustains a dysfunction? Or could it be that this is not dysfunction, but rather a kind of functionality which is actually much appreciated in today's labor market, but which we as mathematics educators are not really prepared to acknowledge?

In terms of dynamics, traditional teaching resembles the Industrial Revolution: in the latter, workers are gathered and confined in a factory, providing them with the necessary tools to perform their work, while in the former the students are given some information on how to do the exercises and, sitting quietly on their chairs, try to find the only expected solution for the problems proposed. In other words, it is a (often) veiled "*prescription readiness*" (Skovsmose, 2011, p. 9, emphasis added), including verbs like *solve*, *determine*, *calculate*, etc. This approach is associated with the banking concept of education (Freire, 2013, p. 83), which "minimizes or annuls the students' creative power, stimulating their credulity and serves the interests of the oppressors".

In contrast, Skovsmose (2001; 2011) introduces the *landscapes of investigation* as an alternative for the exercise paradigm. A landscape of investigation consists in an environment that supports investigation, where the students are invited to elaborate questions and look for explanations. In this scenario, the students are protagonists and responsible for the whole investigation process and the role of the teacher



is the one of a learner. The dialogue teacher-students and students-teacher allows a dialectic movement where the subjects concomitantly teach and learn, once, according to Freire, there is no teaching without learning. This scenario reflects the principles of a pedagogy for liberation (Freire, 2013, 2019, Freire & Shor, 1987).

According to Skovsmose (2001, 2011), investigation practices heavily contrast those based on exercise paradigm, i.e., these distinct perspectives establish “different milieus of learning” (Skovsmose, 2011, p. 39). The milieus can refer to pure mathematics, to a semireality or to real life. Combining these three references with the two perspectives – exercise paradigm and investigation landscapes – six types of learning milieus are obtained, as shown in Table 2. Each environment has its own characteristics, which are not thoroughly described herein<sup>9</sup>.

**Table 2**

*Milieus of learning* (Skovsmose, 2001)

	<b>Tradition of exercises</b>	<b>Landscape of investigation</b>
References to pure mathematics	(1)	(2)
References to a semireality	(3)	(4)
Real-life references	(5)	(6)

It is important to note that, as Skovsmose (2001), we do not consider to abandon exercise practice/solving. We are aware that there are no *good* or *bad* learning environments, it all depends on how these environments are explored (Skovsmose, 2011). Each milieu of learning presents advantages and disadvantages; thus, teacher and students have to identify the best choices for a particular situation, taking into consideration the pedagogical and mathematical objectives involved. “It is important that students and teacher together find their route among the different milieus of learning. The ‘optimal’ route cannot be determined

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<sup>9</sup> For a better understanding of each milieu see Skovsmose (2001; 2011). For examples of the different environments see Biotto Filho et al. (2017); Faustino (2018) and Moura (2020).

in advance but has to be decided upon by students and teacher.” (Skovsmose, 2001, p. 128-129).

Moving from an environment to another is like moving from a comfort zone to a risk zone. The exercise paradigm is associated with the comfort zone, where the teachers know the answers for the exercises proposed and are prepared to answer any questions the students might ask, they have control of the lesson and the procedures elaborated beforehand. On the other hand, the landscapes of investigation bring the teachers into a risk zone (Penteado, 2001), where they become vulnerable to unpredicted situations that can emerge from the problems proposed. In this new dimension, the inquiries that can emerge and the possible answers for them are unpredictable, and teachers cannot be sure they will master all the contents approached by the students. Even bringing some uncertainty, a risk zone can be considered a “zone of possibilities” (Skovsmose, 2011, p. 48). It is important to note, however, that the textbook can contribute with activities associated with both paradigms, playing a fundamental role in providing the teacher with opportunities to fall into a zone of possibilities.

## METHODS

The present study is a qualitative research developed under an interpretative approach. According to Bogdan and Biklen (1994, p.209), studies of this nature “should be concerned with the process and meaning rather than with causes and effects”. In this sense, the focus of the present study is on the comprehension of how CME can be fostered by contextualized tasks contained in textbooks, and not on results or products, which contemplate one of the central characteristics of a qualitative research Denzin and Lincoln (2000) and Vidich and Lyman (2000).

It is important to note that, according to Alves-Mazzotti (2001, p.131), “the main characteristic of the qualitative research is to follow a comprehensive or interpretative tradition”. Moreover, Bogdan and Biklen (1994) recommend the objective study of subjective matters that are inherent to research. For this, the authors emphasize the importance of selecting methods that substantially *limit* the researcher involvement bias, although it cannot be completely *eliminated*. Still according to these authors, the solution to avoid bias would be to work in groups, sharing data for peer analysis and criticism. Through our experience in Theorem

over the last years, we have noticed the importance of collective work in enriching moments of “peer analysis” (Lincoln & Guba, 1985), in what regards not only a deeper theoretical study, but also the research objectives.

**Table 3**

*Textbook collections included in the research*

<b>Middle school</b>	<b>High school</b>
Matemática – realidade & tecnologia Joamir Roberto de Souza FTD. 2018 A conquista da Matemática José Ruy Giovanni Junior & Benedicto Castrucci FTD. 4.Ed. 2018 Araribá mais – Matemática Mara Regina Garcia Gay & Willian Raphael Silva (Editores) Moderna. 2018. Projeto Apoema Matemática Linos Galdonne Editora do Brasil. 2.Ed. 2015.	Quadrante – Matemática Diego Prestes & Eduardo Chavante SM. 2016 #Contato Matemática Jacqueline Garcia & Joamir Souza FTD. 2016 Conexões com a Matemática Fábio Martins de Leonardo Moderna. 3.Ed. 2016 Matemática: interação e tecnologia Rodrigo Balestri Leya. 2.Ed. 2016
Matemática – Compreensão e prática Enio Silveira Moderna. 3.Ed. 2015	Matemática: ciência e aplicações David Degenszajn, Gelson Iezzi, Nilze de Almeida, Osvaldo Dolce & Roberto Pêrigo Saraiva Educação. 9.Ed. 2016 Matemática – Contexto & aplicações Luiz Roberto Dante Editora Ática. 3.Ed. 2016
	Matemática – Paiva Manoel Paiva Moderna. 3.Ed. 2015

The present research analyzed five textbook collections of the middle school<sup>10</sup> (20 books) and seven collections of high school<sup>11</sup> (21 books)<sup>12</sup>, as shown in Table 3.

The textbooks offer a wide variety of tasks involving the milieus of learning. Some textbooks present fewer than 10 type (6) tasks, and those which present a larger number do not exceed 30-40 task, which is, in our opinion, far behind the ideal. On the other hand, many textbooks present 300-400 type (1) and (2) tasks, the latter followed by some level of contextualization; however, most tasks are categorized as tradition of exercises.

Most of the analyzed textbooks contain questions aimed at problematization, i.e., ones that allow discussion and reflection, and this type of task present a higher potential to contribute with the students' critical formation. Thus, such activities deserved more attention in the present study. These tasks are capable of changing the students' beliefs and, consequently, their attitudes and behavior regarding Mathematics.

The number of tasks analyzed is large and it would not be possible to discuss all of them in this study. Therefore, we chose to *select some contextualized tasks as example*. The main criteria for choosing were to contemplate different editions and contents. The thematic of the chosen tasks are relevant to the formation of a critical citizen, and involve the environment, gender/ethnic wage gap, consumerism, driving awareness, among others. These issues emerge from real contexts (semireality) and involve broad information about the topic to be discussed; therefore, they transcend the traditional mathematics exercises: *determine; calculate; solve*.

## DATA ANALYSIS

This section describes, the contributions of the present research in changing the students' beliefs regarding the relationship Mathematics

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<sup>10</sup> Textbooks from middle school were analyzed at PNLD of 2016 and 2020.

<sup>11</sup> Textbooks from middle school were analyzed at PNLD of 2018.

<sup>12</sup> It is important to note that we were not able to analyze all the textbook collections approved by the PNLD because we did not have access to them. As the books are not commercialized, we depended on school and publishing company donations. As there was no access to the Teacher's Manual of all textbooks, only the copies that are produced for the students were analyzed.

and society, highlighting points for improvement and suggestions on how to enrich the tasks contained in the textbook.

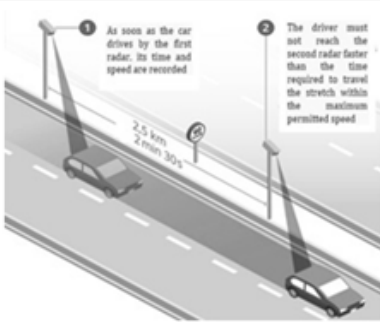
The first task (Figure 1), extracted from a middle school textbook, presents a real and common situation in the Brazilian traffic: the presence of speed measuring radars.

**Figure 1**

*Task 1* (Adapted from Souza, 2018d, p. 113)

Join a partner and solve the following problem.

On roads where there are speed radars installed, the drivers usually drive slowly within the monitored area to avoid getting a ticket, increasing the speed soon after. To avoid this situation, radars that measure the average speed of the vehicles were installed in the city of Sao Paulo in October 2017. Note how these radars work Fig. 1.



- Enumerate the risks of excess speed on public roads.
- Research and write the speed limit of some street of avenue close to your school. Are there any radars installed in this area?

**For items (c), (d) and (e), consider a road where two of these radars delimit a distance of 6km, where the speed limit is 60 km/h.**

- A vehicle passed by the 1st radar at 10h40 and by the 2nd at 10h49.
  - In how many minutes the vehicle completed the distance? How many hours does this distance correspond to?
  - Considering this distance, what is the average speed of the vehicle in km/h?
  - This speed is over the speed limit?
- The times at which some vehicles passed by the radars are indicated below. Calculate the average speed of each vehicle and indicate which ones exceeded the speed limit.
 

I. 1st radar: 10h40	III. 1st radar: 22h48
2nd radar: 10h52	2nd radar: 22h52
II. 1st radar: 13h55	IV. 1st radar: 15h35
2nd radar: 14h	2nd radar: 15h43
- Calculate the minimum time in which a vehicle can complete this distance without exceeding the speed limit. Justify your answer.

Items (a) and (b) can contribute for a rich debate, from the reference to reality, inviting the students to bring information from their everyday life into the classroom. On the other hand, items (c), (d) and (e) involve calculations, the necessary tools to determine the average speed using the given information.

This type of problematization can contribute to raise road safety awareness and should be addressed at school, since Law 9,503 establishes that Road safety Education must be inserted in Basic Education “through planned and coordinated actions by the National Transit System and the federal, state and municipal education departments within their respective scope (Brazil, 1997).

However, none of these questions proposed mathematical discussion on the topics addressed. The first two items do not seem to have a direct connection with Mathematics, while the last three have exact and absolute answers, not allowing any critical reflection.

Although the task is interesting, the author could have explored the opportunities which lie beyond the calculation. Therefore, the teacher will have to be critical and reflexive to apply such an activity and provide the students with elements to be investigated and discussed. According to Skovsmose (2001), knowing about the construction and application of mathematical models and the reflection upon these models is as relevant as the mathematical knowledge itself. From this perspective, a problematization could be proposed on how the data are produced by the radar or how the sensor is programmed to measure the speed of the vehicles. Other question could emerge: how reliable is this information? Is it accurate? What is the importance of this instrument in the relationship traffic-individual-society? These questions challenge the irrefutable character of Mathematics (Borba & Skovsmose, 1997) and provide a fertile ground for discussion.



According to Boaler,

Mathematical activities need to be open enough, not only for students to formulate strategies but to formulate their own meaning. Tasks should require that students develop an understanding of the underlying processes and the way that these link with content; in this way students will appreciate and develop an understanding of the interrelationship of the two (Boaler, 1993, p. 16).

Task 2 (Figure 2) is from a High School textbook and was selected for analysis for having the potential to explore different discussion topic, integrating Mathematics with other areas and enabling interdisciplinary work, provided the teacher is willing to plan the classes with a colleague who teaches another discipline.

## Figure 2

### Task 2 (Adapted from Souza & Garcia, 2016a, p.100-101)

Being conscious. Why do we pay taxes?	
<p>You might not know it, but when you buy a videogame, 70% of its value correspond to taxes. This means that, if a product costs R\$ 1000, around R\$ 700 are collected by the government. In general, all goods purchased are subject to sale taxes. We also pay taxes on utility services, such as water and electricity. What are taxes actually for? The taxes collected by the state, municipal and federal governments are intended to fund public services, such as health, education, security and transportation. When we pay taxes, we are fulfilling our duties as citizens, contributing for the improvement of public assets and services, and, consequently, to improve life quality for the society as a whole. Some examples of taxes are:</p> <p><b>IPJU:</b> municipal real estate tax - or urban property tax, annually collected by the municipalities</p> <p><b>ICMS:</b> tax on sales and services associated with the movement of goods. This tax is usually included in the final price of the products.</p> <p><b>IPVA:</b> tax on the property of motor vehicles, such as cars, motorcycles and buses.</p> <p><b>IRPF:</b> Federal tax on personal income, which includes wages, pensions, rents, etc. In some cases, after the analysis of the annual income tax return, the taxpayer is refunded by the government.</p> <p><b>ISS:</b> tax applied to services provided to a third party by a company or professional and is paid by the service provider. The tax is usually added in the final price of the product or service.</p> <p><b>Tax collection and allocation</b> The appropriate collection and allocation of taxes favor the social development and the economic growth of a country.</p> <p><b>Collection</b> When we buy a product and ask for an invoice or a bill of sale, we are ensuring that the corresponding taxes will be converted into benefits to the citizens. Therefore, we should always ask for an invoice or a bill of sale when we buy something. The invoice and the bill of sale describe the approximate value of the taxes added in the purchase.</p> <p></p> <p>We play an important role in inspecting the application of the money collected through taxes, which is intended to allow the improvement of public services.</p>	<p><b>Analyzing with citizenship criteria</b></p> <p>a) Why is it important to ask for an invoice or a bill of sale when we buy something? b) Search for other types of taxes and describe the situations in which they are charged. c) In your opinion, are the taxes collected in Brazil properly used for the improvement of public services? Talk about this with your teachers and colleagues.</p> <p><b>Analyzing with Mathematics</b></p> <p>d) Bring an invoice or a bill of sales to the classroom and verify the taxes added to the product or service you have acquired. What is the tax percentage? e) Considering an ICMS of 17% on the final price of a particular product that costs R\$ 3,00, calculate the taxes for:</p> <ul style="list-style-type: none"> <li>• 1 unit</li> <li>• 5 units</li> <li>• 35 units</li> </ul> <p>f) Write the function <math>f</math> that associates the number <math>x</math> of products mentioned in item e with the ICMS value <math>y=f(x)</math> paid.</p> <p>Education is one of the public services funded by taxes, which are intended to provide public educational institutions with conditions to offer quality education to all Brazilian students.</p> <p></p> <p>Public health is also funded by taxes. Vaccination is one of the most important services provided by the Brazilian public healthcare system, since it prevents serious diseases as poliomyelitis. The population must get vaccinated according to the official calendars released by the government.</p>

This task starts with an explanation on how federal, state and municipal taxes are collected and exemplifies tax percentage on goods, also elucidating how the collected money is returned to welfare. The proposed exercises explore citizenship, emphasizing the importance of asking for an invoice or a bill of sale when buying any product or service, and suggest that students research about situations in which taxes are charged, raising a reflective discussion on the use of tax resources. In the subsequent questions, mathematics appears more clearly, and the students are presented some content about functions. The task presents two different sections: *analyzing with citizenship criteria* and *analyzing with Mathematics*. This form of presentation can create the misconception that it is not possible to perform an analysis integrating citizenship and Mathematics.

The teacher can explore this type of context by coming up with relevant questions to develop critical citizenship, promoting mathemacy, or mathematical literacy (Skovsmose, 1994, 2001, 2011). In other words, this perspective “assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens”

(OECD, 2013, p.17). This task can be regarded to have present critical perspective because, according to Skovsmose

we can say that critique has to do with: (1) An investigation of conditions for obtaining knowledge, (2) an identification of social problems and evaluation making, and (3) a reaction to problematic social situations. In other words, the concept of critique points out demands about self-reflections, reflections and reactions (Skovsmose, 1994, p. 37).

Still according to Skovsmose (2001), this type of task provides a learning environment that makes reference to reality, and such environment can become an investigation setting (Skovsmose, 2001, 2011), very relevant in the formation of critical and reflective students.

However, the author clearly implies that paying taxes is a positive action: “When we pay taxes, we are fulfilling our duties as citizens, contributing for the improvement of public assets and services, and, consequently, to improve life quality for the society as a whole” (Souza & Garcia, 2016a, p.101). This statement makes one believe that the Brazilian system is efficient. Unfortunately, this is not true. To understand the relationship between social welfare and taxes in Brazil, we must consider that the Brazilian tax burden is one of the highest among developing countries<sup>13</sup> and the financial resources intended to be applied in sanitation, health, education, public mobility and other social welfare are subtracted by corruption [...], the money collected through taxes is diverted or misused”.

Despite the naive introduction regarding the application of taxes, the author offers an opportunity for critical discussion in the questions proposed: “In your opinion, are the taxes collected in Brazil properly used for the improvement of public services? Talk about this with your teachers and colleagues.” (Souza & Garcia, 2016a, p.101). In this task, the author opened space for the students to bring their own experiences, to discuss about what they have lived in their homes, with their friends and about what they have heard on TV or the internet. The teacher can promote a discussion on the accuracy of the information and the importance of critical thinking and of elaborating a personal opinion (the student’s own opinion as a citizen, as a member of society). Thus, the

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<sup>13</sup> <https://financaspeessoais.organizze.com.br/afinal-para-onde-va-o-os-impostos-que-pagamos/>



teacher plays a crucial role in contributing to a critical reading of the initial statement and in fostering a valuable discussion on the question proposed by the textbook, highlighting the importance of taxes for the maintenance of the public system, but, at the same time, providing the basis for a critical and political view of the system.

This type of task works as a reminder that education is a political act, (Freire, 2013, Freire & Shor, 1987) and, therefore, not neutral. The teacher must have the students reflect and talk about the governmental actions. According to Freire,

This is a great discovery, education is politics! After that, when a teacher discovers that he or she is a politician, too, the teacher has to ask. What kind of politics am I doing in the classroom? That is, in favor of whom am I being a teacher? By asking in favor of whom am I educating, the teacher must also ask against whom am I educating. Of course, the teacher who asks in favor of whom I am educating and against whom, must also be teaching in favor of something and against something. This “something” is just the political project, the political profile of society, the political “dream” (Freire & Shor, 1987, p. 60).

This discovery by the teachers is fundamental for them to be aware and carry out their practices without reinforcing dominant discourses and defending elitist policies.

The following tasks present questions related to Environmental Education, which, according to the Brazilian legislation, entails

the processes through which the individual and the collective build social values, knowledge, abilities, attitudes and competences aimed at environmental conservation, seen as people’s commons, essential to a healthy life and sustainability (Brazil, 1999).

According to Law 9,975, Environmental Education has to be present in all learning levels and modalities, in an integrated way, both in formal and nonformal situations, and is considered a fundamental and permanent component of Brazilian education (Brazil, 1999). Thus, the textbooks are fulfilling their purpose when presenting this thematic.

### Figure 3

Task 3 (Adapted from Chavante & Prestes, 2016a, p. 51).

#### Reduce your waste!

The destination of solid waste needs to be rethought by all of us. This includes the federal, state and municipal governments, private institutions and citizens in general. When dumped in inappropriate places, such as landfills, it can cause serious environmental impacts, once its degradation is highly toxic and contaminates the soil, the air and water streams, not to mention the risk imposed to people's health.

In 2010, the National Policy for Solid Waste was instituted in Brazil. The project is aimed at reducing the generation of solid waste, stimulating reuse and recycling and regulating the disposal of solid and organic waste. Still in Brazil, in 2014, an estimate showed that a person produced an average of 1.062 kg of waste on a daily basis. The image below shows the average of waste generated per region.



#### How can we reduce waste?

##### Smart shopping

- Prepare a shopping list to avoid waste.
- Use returnable shopping bags.
- Choose concentrated products, ones with simple packaging or refills.
- Choose products with recyclable or returnable packaging.
- Do not buy products exclusively because they are on sale.

##### At home

- Plan your meals to avoid waste.
- Avoid excessive use of toilet paper, paper towels and napkins.
- Avoid preparing fried food to reduce cooking oil disposal.
- Use stove lighters instead of matches.

- In addition to the attitudes presented, what else can we do to reduce the amount of waste we produce on a daily basis?
- What was, in 2014, the average amount of solid waste generated per day, per person, in the region you live in?
- Write the standard form of a function  $r: N \rightarrow R$  that relates the number of people  $p$  with the average amount  $r(p)$  of urban waste generated in 2014 in Brazil. Then, calculate how many tons of waste were generated on a daily basis, on average, by a population of 400 thousand inhabitants.



The first part of Task 3 (Figure 3) suggests, through an infographic, a discussion on the daily generation of solid waste in each Brazilian region per inhabitant, in kilos. The second part of the task presents some alternatives for waste reduction. Environmental education should be regarded as a general interest, since the whole population suffers with the impacts. Bringing this type of discussion into the classrooms is a way to provide the students – and, consequently, their families - with important information to change daily habits and contribute with waste reduction.

Through this task, the teacher can promote a debate on the factors that influence this environmental issue, stimulating a reflection about sustainable development, which, according to Jara (1998, p. 35)

has environmental, economic, social, political and cultural dimensions, which necessarily implies several concerns: with the present and the future of individuals; with the production and consumption of goods and services; with the basic subsistence needs; with the natural resources and ecosystemic balance; with decision making practices, power distribution, personal values and culture.

The teacher must emphasize that this is a global issue, involving economic interests and the search for collective solutions – rather than individual ones – is of the utmost importance.

The context of task 4, intended to fundamental school students, is the Amazon deforestation. This task was included herein for two reasons.

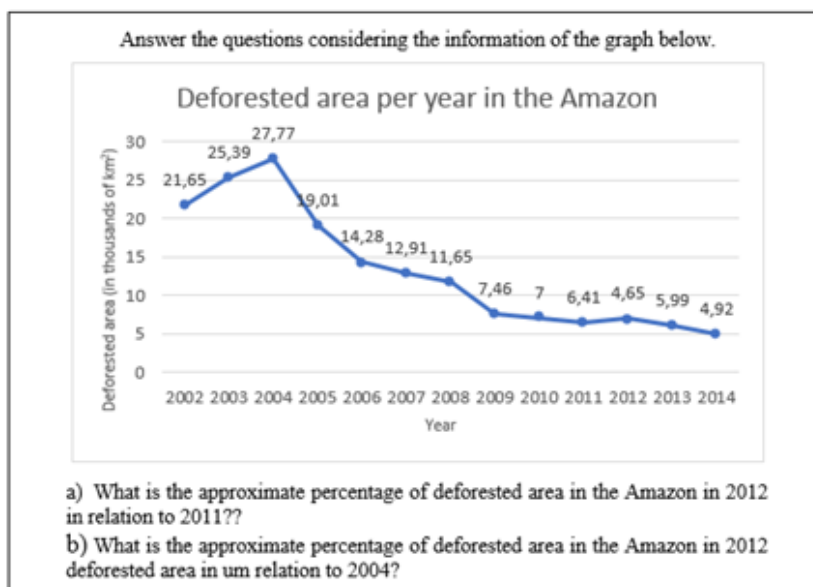
The first reason is that, without a critical view on the subject, the author focuses exclusively on mathematical knowledge, i.e., the information provided by the graph and their importance for the environment, mainly the serious impacts of deforestation in Brazil, are not explored. The non-problematization of such data may lead the reader to believe the figures regarding deforestation are not significant, especially because the graph shows a decrease over the years.

The second reason is associated with the ideology of certainty (Skovsmose & Borba, 1997). The graph presents incorrect information.

Assuming that the numerical information is right<sup>14</sup>, note that, in the year of 2012, the graph shows a deforested area of 4.65 thousand km<sup>2</sup>; however, the dot presenting this data is located in a higher position in comparison with the ones representing the years 2011, 2013 and 2014, which correspond, respectively, to 6.41 thousand km<sup>2</sup>, 5.99 thousand km<sup>2</sup> and 4.92 thousand km<sup>2</sup> of deforested area.

**Figure 4**

*Task 4* (Adapted from Silveira, 2015c, p.193)



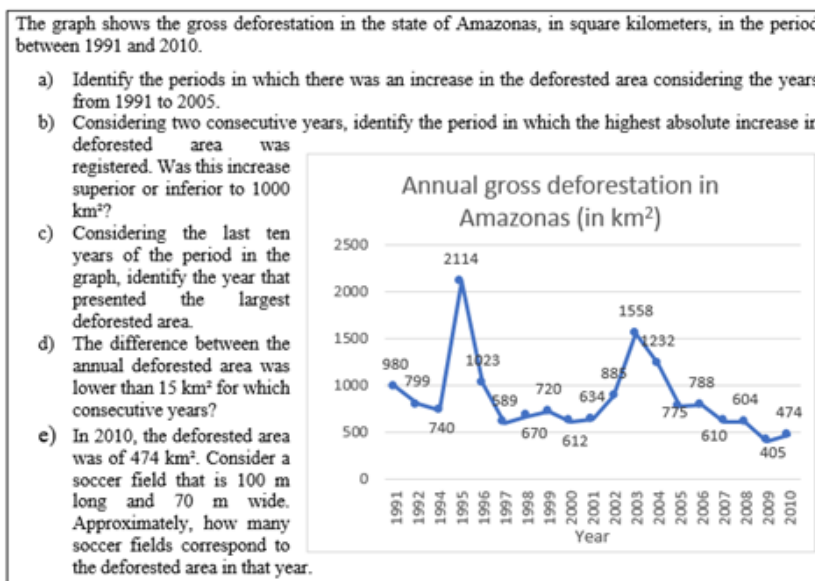
Another high school textbook collection addresses the same topic (Figure 5), showing data from the same source - the National Institute for Space Research (NISR) – and the tasks do not open space

<sup>14</sup> The website in the reference was not available; however, we assume that the values are correct, since the NISR website shows similar data (<http://www.obt.inpe.br/OBT/assuntos/programas/amazonia/prodes>. Accessed on Jun 8, 2020).

for debate, there is only a simplistic concern about reading graphs and dealing with numerical information.

**Figure 5**

*Task 5* (Adapted from Iezzi et al., 2017c, p. 53)



**Figure 6**

*Task 6* (Adapted from Galdonne, 2015d, 224)

10) A gas station charges R\$ 2,90 for the liter of gasoline and R\$ 1,80 for the liter of ethanol.

- Write the standard form of a function providing the amount Q to be paid for x liters of gasoline.
- In this function, what happens to the value of Q when the value of x doubles?
- Write the standard form of a function providing the amount P to be paid for x liters of ethanol.
- In this function, what happens to the value of Q when the value of x doubles?

Despite proposing a thematic of global concern, the deforestation of Amazônia, the textbooks (Silveira, 2015c, Iezzi, 2017c) did not explore the problematization of the topic. Again, it is the teacher's responsibility to promote investigation and create an environment of discussions and reflections. The teacher needs to incorporate the idea of "environmental education for democracy" (Novicki, 2007, p. 164), which leads the individual to understand economic and political influences – and the capitalist ones, in general – on attitudes that attack and harm the environment (Novicki, 2007).

Some contextualized tasks are not presented in specific sections, they are part of a set of extra activities included for each chapter – mostly in the end. Most of these extra tasks are typical examples of what Skovsmose (2001) calls *reference to semireality*. The situations and data presented are fictitious, trying to simulate possible realities. Task 6, extracted from a middle school textbook, illustrates a situation that approaches the price of fuels. The task, however, does not suggest the students to perform any research on the fuel price in the city they live in. Instead, the values are given by the task, disregarding the actual differences in fuel price by region. Additionally, the prices given for ethanol and gasoline were effective in 2010 and 2013, respectively<sup>15</sup>.

Considering that the textbook reached the school in 2017, these values are far behind the current reality, and few students notice this, and the teachers who reinforce the formatting power of mathematics (Skovsmose, 1988) will make the students accept the information without questioning it. In this situation, the students could be led to compare the prices of gasoline and ethanol, to discuss which one would be more convenient to use considering the car's fuel economy or how the fuel production process works, for example.

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<sup>15</sup> For example: <https://www.tabelasdefrete.com.br/planilha/historico-da-variacao-de-precos/25>

**Figure 7**

Task 7 (Adapted from Balestri, 2016a, p. 30)

Effects of alcohol at various Blood Alcohol Concentration levels	
Blood Alcohol Concentration (BAC)	Effects
<p>Small amounts of alcohol can affect the behavior of mental and motor functions, causing serious accidents, mainly when combined with other factors, such as motor vehicle driving. The table shows the effects of alcohol at various blood alcohol concentration levels.</p> <p>a) What is the concentration range in which the person presents some effects but no visual impairment?</p> <p>b) In Brazil, drinking and driving is a crime, and drunk drivers can be arrested or have their driving privilege suspended or revoked. If the BAC is higher than 6.0g/L, the individual is arrested. Represent in a graph the concentration interval that can lead a driver to be arrested.</p> <p>c) According to the table, write the effects that might be presented by a person who can be fined for drinking and driving despite not being subject to arrest.</p>	<p>0.1 – 0.5 Increased heart and respiration rates; decreased attention; inconsistency in behavioral task performance; decrease in judgment and inhibition; mild sense of elation, relaxation and pleasure.</p>
<p>0.6 – 1.0 Decreased attention and alertness; slowed reactions; impaired coordination, and reduced muscle strength; reduced ability to make rational decisions or exercise good judgment; increase in anxiety and depression</p>	<p>1.0 – 1.5 Dramatic slowing of reactions; Impairment of balance and movement; Impairment of some visual functions; slurred speech; vomiting, especially if this BAC is reached rapidly.</p>
<p>1.6 –2.9 Severe motor impairment, e.g. frequently staggering or falling; exacerbated emotional responses (fear, stress, anxiety); Severe sensory impairment (colors, shapes, movements, dimensions); impairment of balance and movement</p>	<p>3.0 –3.9 Non-responsive stupor, loss of consciousness, anesthesia comparable to that for surgery.</p>
<p>&gt; 4.0 Unconsciousness, cessation of breathing, death (usually due to respiratory failure)</p>	<p>In addition to causing a large number of traffic fatalities, indiscriminate consumption of alcoholic drinks can trigger chemical dependency and contribute to the development of diseases like cirrhosis and some types of cancers. Join two colleagues and talk about the harms and consequences of excessive drinking.</p>



**Drinking X Driving**

Law 11,705/2008, popularly known as Dry Law, has the objective of making society aware of the dangers of drinking and driving. Drunk driving can cause serious accidents, since alcohol can potentially affect vital organs, including the brain, which is responsible for coordinating the human body. A small amount of this substance in the blood can lead to slowed reactions and decreased risk awareness and perception, which directly affects the ability to drive a motor vehicle.

A person taking a breath alcohol test in Parana, Brazil, 2009. A breathalyzer is used to detect and measure BAC in milligrams of alcohol per liter of exhaled air.

Another important topic in the process of building citizenship is related with good health, in this case, more specifically, problems associated with alcohol abuse. According to Felipe and Gomes (2010, p. 2), “alcohol consumption is considered a global public health problem that has affected principally adolescents”, and basic education schools represent an important locus to discuss this topic.

The school as a social field is permeable to transdisciplinary approaches and to a profusion of approaches from different theoretical sources in all knowledge areas. Therefore, the school is a privileged territory for the incorporation of health-related knowledge and for the transformation of the current scenario of social vulnerability in which many young Brazilians live (Lopes et al., 2007, p. 713).

Task 7 (Figure 7) addresses the harms caused by alcohol abuse and describes the Brazilian laws regarding “drinking and driving”.

This thematic offers an opportunity for investigation (Skovsmose 2001), in which the students could research about the number of road accidents before and after the Dry Law<sup>16</sup>. The results of their research could be used to reflect about the need of such a governmental action and its impact on society.

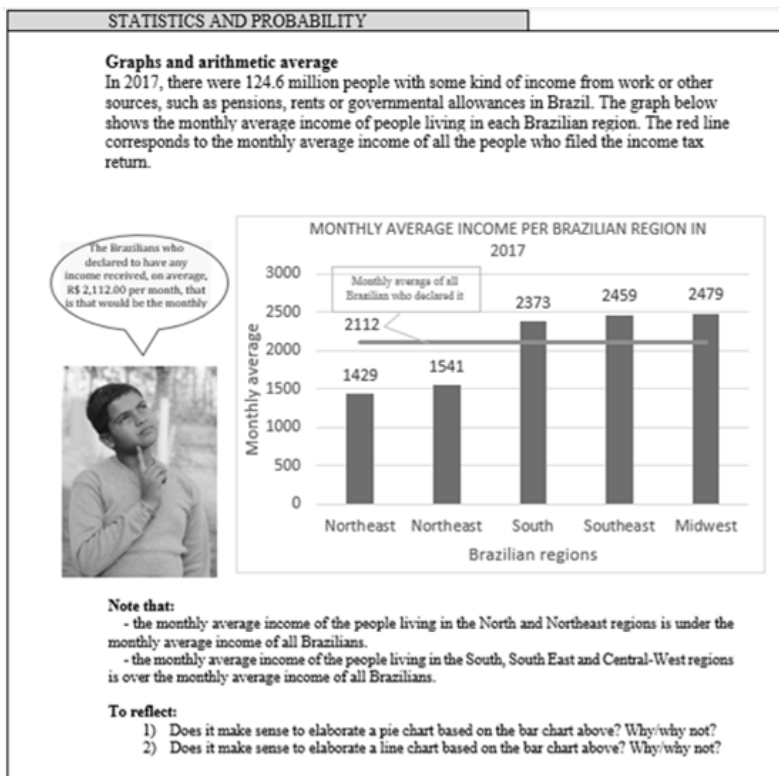
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<sup>16</sup> Dry law is the popular name of the law 11,705/2008, which reduced the maximum level of alcohol in the driver’s blood. The former legislation allowed the ingestion of up to 6 decigrams of alcohol per liter of blood (equivalent to two glasses of beer). The blood alcohol concentration allowed when the law was enacted was of 0,1 mg/l; nowadays, the limit is 0.05 mg/l (Brasil escola, 2020).



## Figure 8

Task 8 (Adapted from Gay & Silva, 2018d, p.166)



The debate for the formation of a critical citizen is suggested at the bottom of the page, while the main activities, rather than fostering reflection, focus solely on numerical calculations.

This task shows that the average monthly wages earned by individuals living in the North and Northeast are lower than the country's average. Several issues could be discussed through the analysis of this graph, such as the fact that these regions concentrate a larger number of people with lower income and that many of these individuals migrate, especially to the Southeast, to find job opportunities and better life conditions. However, the exercises proposed in task 8 are limited to mathematical and technical knowledge, associated with the elaboration of a graph. Although these exercises are fundamental to develop the

ability to read and write information expressed by graphic representation, helping the students to read the world, the authors could have explored the data presented by the charts and tables, and also take the opportunity to debate mathematical concepts. Is the *arithmetic average* the best tool to be used in this case? Does this average represent the reality of the country? Could we use the *median* instead? Such reflections can contribute for the criticism of the ideology of certainty (Skovsmose & Borba, 1997), leading the students to reflect not only about what the data are saying, but also about how these data were collected/created. Thus, the intentionality underlying widely spread information can be explained to the students.

Task 9 has a high potential for debate, since it addresses one of the most important social issues – gender and ethnic wage inequality – and such a debate is fundamental to challenge the dominant elitist discourse (Freire, 1987, 2013).

The first point to be highlighted in this task is that the graph shows the *average wage of men and women over 14 years old*. The teacher must emphasize that, according to the Brazilian law 10,097, art. 403, states that it is forbidden to employ any individual under 16 years of age, except for the position of apprentice, which is allowed for individuals aged 14 or older (Brazil, 2000).

The actual data presented in this task describe a situation that should not be part of our reality – gender and ethnic discrimination. Although the activity focus on gender pay gap, some other relevant points could be approached in the debate over gender and ethnic inequality. According to Teixeira (2008, p.39),

Understanding the causes and consequences of professional segregation is critical for the elaboration of policies regarding the labor market. Policies of affirmative action, for example, can be interpreted as an attempt to minimize professional segregation and policies to equalize salaries can be seen, in part, as an effort to minimize income disparities caused by segregation.

## Figure 9

Task 9 (Adapted from Gay & Silva, 2018d, p.167)

**Activities**

1) Analyze the graph that shows the average wages for men and women over 14 years of age in 2017.

Gender	Average Wage
men	2410
women	1820

Data from: <https://educabge.gov.br/ciencias/bras11/posso-povo/19626-trabalho-e-rendimento.html>

• The answers to activities 1d, 2a e 2b can be found in the *General Guidelines* section of this book.  
 • Have the students reflect on the data presented by the chart and the table and talk about wage inequality in Brazil. Such moments stimulate the dialogue and raise awareness regarding respect, equality and citizenship. The teacher can bring some extra reading material to enrich the debate.

a) Analyze the graph that shows the average wages for men and women over 14 years of age in 2017?  
 b) Read the statement below:

*The average wage earned by women represents approximately 77.5% of that earned by men*

Do you agree with this statement? Justify your answer

c) Was the average wage earned by men was more or less than the monthly average income of all Brazilians over 14 years old in 2017? What about the average wage earned by woman?  
 d) Using an electronic spreadsheet, elaborate a horizontal bar chart based on the data presented in the graph above.

2) Analyze the table below:

Ethnic group	Average wage
White	R\$ 2814
Black	R\$ 1570
Brown	R\$ 1606

Data from: <https://educabge.gov.br/ciencias/bras11/posso-povo/19626-trabalho-e-rendimento.html>.

a) Which type of graph is more appropriate to illustrate the data presented in this table: a pie chart, a bar chart or a line chart? Elaborate the graph you have chosen in an electronic spreadsheet.  
 b) Considering that the monthly average wage of the Brazilians over 14 years of age was R\$ 2178 in 2017, represent this average with a line in the graph you have elaborated.  
 c) Considering that the monthly average wage of the Brazilians over 14 years of age was R\$ 2178 in 2017, represent this average with a line in the graph you have elaborated.

To reflect:  
 What is your opinion about gender/ethnic wage inequality? Discuss with your classmates.

Unless these data are discussed under a critical point or view (Skovsmose, 1994) and these situations are problematized, the sexist and racist discourse that is structurally present in our society will be naturalized. The school plays a fundamental role in reporting, in a Freirean sense any offense of this kind. Although the questions that proposed reflection are at the bottom of the page, it is necessary to

recognize the authors' choice in selecting rich thematic, one that is relevant to be discussed in the classroom. With this kind of task, the teacher can problematize the country's power relations and inequality (Gutstein 2003).

Overall, the Brazilian textbooks, despite having points to be improved, can be considered adequate tools to change the students' view in many aspects, fostering critical thinking and citizenship (mathemacy). However, the teacher plays the central role in this process. In this section, examples and theoretical justifications on this assumption are given.

## CONCLUSION

The objective of the present study was to demonstrate that the Brazilian textbooks can be seen as a strategy to promote changes, contributing with student formation, not only regarding mathematical knowledge, but also regarding their role in society, as responsible and critical citizens (Mass et al., 2019). These problematizations were exemplified through social affairs, such as road safety awareness, the environment, taxes, alcohol consumption and gender/ethnic inequality.

Even addressing relevant topics that need to be discussed and problematized, the textbooks frequently approach them in a way that critical thinking and mathematics seem to be disconnected. The teacher plays a fundamental role in proposing discussions and reflections on the topics initially proposed by the textbook, enabling changes and, possibly, actions.

The Brazilian mathematics education is known to follow very traditional education models. According to Skovsmose (2007), in traditional teaching, the steps and procedures contained in the textbooks are commonly followed in details, page by page, the teacher's orality predominates and the exercise paradigm is manifested the whole time, i.e., there is no room for debate. A critical view of this scenario can lead to deeper reflections. Perhaps the textbook superficially presents social, political, and economic discussions, because they approach an optimized reading that mathematics teachers do in schools today: they produce contexts without deepening. In other words, perhaps the textbook does not bring a deepening of discussions, precisely because, if it did, it would not be so adopted by teachers, since many do not want discussions on topics that they are unfamiliar with, and that may reflect the lack of

political discussion of society itself, including teachers, teacher trainers, educational training courses, among other sectors and people.

It is also possible to look at activities and think that the authors of the textbooks naively did not explore the potential of the questions for extra mathematical discussions. However, it is known that the amount of money involved in these productions is large, and it can be conjectured that publishers and authors know very well the profile of our mathematics teachers. Therefore, perhaps they do not present these discussions simply because they would be ignored by teachers or even could cause teachers not to choose the collection because it is too "difficult". There are several examples in the history of the production of Brazilian mathematics textbooks that show collections that were critical success and failure to sell.

To change, educators must be willing to become vulnerable, leave the comfort zone, put themselves in a learning position, side by side with the students, in a relationship where the individuals, seen as teacher-students and students-teacher, learn together (Freire, 2013). This is a gradual transformation, the willingness of the educator to change will illuminate obscurities imposed by the dominant class, allowing and encouraging a critical reading of society (Freire & Shor, 1987).

The textbook should bring more open and complex situations, related to *actual* problems of everyday life. Unfortunately, "mathematics is commonly seen as consisting essentially of computation and formulas, yielding exact and infallible answers, without relevance to everyday life, accessible only by experts, and not open to criticism" (Mukhopadhyay and Greer, 2001, p. 297). The restrictive historical view that mathematics is object to absolute and irrefutable truths must be broken down (Boaler, 1993). It is necessary to criticize what has been established. According to Freire,

But, for me, what is important, what is indispensable, is to be critical. Criticism creates the necessary intellectual discipline, asking questions to the reading, to the writing, to the book, to the text. We should not submit to the text or be submissive in front of the text. The thing is to fight with the text, even though loving it, no? To engage in a conflict with the text (Freire, 1987, p. 22)

From this *conflict* with the text – in this case, the ones included in mathematics textbooks – the students can understand that there is "a

major inertial force holding back radical reform of mathematics education” (Mukhopadhyay & Greer, 2001, p. 297), and are able to unveil the interests behind the words.

It should be noted that it is not assumed that Critical Mathematical Education is the "reference" or "what is best in curriculum terms". But it is expected that the discussions presented here will lead to the reflection that there may be an impact on society when students have the opportunity to discuss social, political, and economic issues. Perhaps the optimistic perspective could be that textbooks can contribute to bring these themes to the classroom, with textbooks that are not produced "to be approved in PNLD evaluations" and that teachers, educational trainers, and other sectors of society accept the challenge of putting these contemporary discussions on the scene.

In conclusion, the present study can lead authors, publishing companies and PNLD members to reflect on possible improvements to Mathematics textbooks and provide teachers with relevant information on how to apply the tasks proposed by the circulating textbooks through a critical mathematics view.

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## **AUTHORSHIP CONTRIBUTION STATEMENT**

Authors RAS and LM discussed the methodology and theoretical foundation. The first author, RAS, collected and analyzed the data of the 20 Middle School textbooks and the second author, LM, collected and analyzed the data of the 21 High School textbooks. Authors RAS and LM discussed the results and contributed to the writing and correction of the article in an equivalent manner.

## DATA AVAILABILITY STATEMENT

The data that supports the results of this study are available in the textbooks approved by the National Textbook Program (PNLD). The textbooks can be accessed by contacting different Brazilian public schools.

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