Mathematics Education as a response to societal needs.

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ABSTRACT

In this paper I will discuss alternative proposals for Mathematics Education, taking into account its impact in the social processes. I start with discussing some of the recognized values for Mathematics Education. Essentially, the following set of six basic values are accepted: 1. *Utilitarian;* 2. *Academic;* 3. *Formative* (of reasoning); 4. *Cultural;* 5. *Sociological;* 6. *Aesthetic.* Affecting all these values, the question of assessment is of fundamental importance.

Keywords: values for Mathematics Education, Assessment system, Mathematics for all

RESUMO

Neste artigo discutem-se propostas alternativas para a Educação Matemática, tendo em conta o seu impacto nos processos sociais. O texto inicia pela análise de alguns dos valores reconhecidamente importantes para a Educação Matemática. Essencialmente, consideram-se seis: 1. Utilidade, 2. Acadêmico, 3. Formativo (do raciocínio), 4. Cultural, 5. Sociológico, 6. Estético. A avaliação é considerada de importância fundamental e afeta todos esses valores.

Palavras-chave: valores da Educação Matemática, Sistemas de avaliação, Matemática para todos

Utilitarian and academic values.

No one will deny that among these six values, all are equally important, although there has been disequilibrium among them.

Particularly between utilitarianism and academic, there has been much disagreement. Is academic mathematics useful? The utilitarian facet of mathematics has been, historically, in opposition to the academic. This affects particularly the aesthetic value. There are two visions of this value: first, the aesthetic and beauty intrinsic to mathematics, as seen in the formulae, proofs and reasoning, affecting mainly academic mathematics. It is a recurrent quest among

mathematics “what is the most beautiful mathematical result?” A recent enquire elected the formulae $e^{i\pi} + 1 = 0$, relating six basic elements of mathematics and attributed to Leonhard Euler, and the basic equation of regular polyhedra, due to Plato: $V - A + F = 2$. But also the enormous achievements on art and architecture, as well in technology, recognized by their beauty and harmony, are representative of the utilitarian value of a mathematics generally acquired out of school, through family, community and professions.

It is clear that Plato (429-347 B.C.), a disciple of Socrates (479-399 B.C.), recognizes two kinds of mathematics. A practical mathematics, especially learned of the Egyptians, and a new form of mathematics, due to the Greeks. If, for Egyptians, a trying is the result of a string connecting three pegs, for the Greeks a triangle is the integration of sis abstract notions, three points and three segments uniting these three points. There is an essential difference between the two ways of looking at things and, according to A. Sohn-Rethel, this distinction is the dividing line between manual work and intellectual work[1]. This distinction, which is recognized in the Mathematics of Plato, determines the epistemological basis that prevails in modern science. The distinction between manual and intellectual work, on which rest our systems of production and property, deserves a more detailed consideration of its place in mathematics education. Especially when one sees that, after Plato, prevailed a kind of mathematics aiming at the “best minds”, identified as the ruling elite, the owners of the means of production.

Two types of mathematics?

There is a big difference between what we call School Mathematics, that is taught in school and aims at its utilitarian value, and what we call Academic Mathematics or simply Mathematics. In fact, they are different in their objectives, methods and contents, although there may be some overlap in the basic contents which are present in both. Objectives and methods are very different. This distinction has been well highlighted by G. H. Hardy in his classic book *A Mathematician's Apology*, when he distinguishes between "the real mathematics” and “the trivial mathematics”[2]. His discourse leads to distinguishing the first as the mathematics of truth, which is practiced in the academy, and the second as the mathematics which serves. According to him, trivial mathematics is useful, while real mathematics is not, serves for nothing. This radical position reflects his position on the values of knowledge.

This idea of two mathematics was discussed, much earlier, by Plato. Examining the position of mathematics in the history of education in the Western world, there is an explicit reference in Plato to the importance of mathematics as a focal point of the educational system. Plato wrote in the form of dialogues, which are considered as the foundation of Western Philosophy. In the dialogue *Republic*, written around 400 B.C., Plato describes a mathematical curriculum for the Academy: arithmetic, plane geometry, solid geometry, astronomy, and music. He complains

1 A.Sohn-Rethel: *Intellectual and Manual Labor*, Macmillan Press, London, 1978.
2 G.H. Hardy: *A Mathematician’a Apology*, with a Foreword by C.P. Snow, Cambridge University Press, Cambridge, 1967.

that solid geometry is not well enough developed. Probably, this motivated his interest in the study of regular polyhedra. For him, geometry was regarded as propaedeutic, since it prepares for more advanced philosophical investigations, as we can infer from the fact that, according to tradition, in the doorway of his academy was written "*Let no one ignorant of geometry enter here*." The role of Mathematics in Plato's educational proposal is primarily to serve as an instrument for the selection of elites. As Plato put it, mathematics is the way to intellectual elitism. But he refers also to a form of mathematics, learned from the Egyptians, which is important for everyday life. But this is of interest to workers, merchants and soldiers, and shall not be the concern of intellectuals. These ideas prevailed also among the Romans and extended to the Middle Ages, formally organized as the *quadrivium*: arithmetic, geometry, music and astronomy, the basis of the higher level of studies, necessary for those pursuing higher education and the academy. The lower level of studies consisted of the *trivium*, a basic curriculum organized as grammar, rhetoric and dialectic.

The theme of two types of mathematics is of major actual importance. In the Symposium commemorative of 100 years of ICMI, held in Rome in March 2008, Anne Watson presented a very provocative paper entitled "School Mathematics as a Special Kind of Mathematics", later published in *The Learning of Mathematics*. In her talk, she says:

> "I shall claim that school mathematics is not, and perhaps never can be, a subset of the recognized discipline of mathematics, because it has different warrants, authorities, forms of reasoning, core activities, purposes and unifying concepts, and necessarily truncates mathematical activity in ways that are different from those of the discipline. By 'discipline of mathematics' I mean the activities that advance mathematical knowledge: the forms of engagement, kinds of questions, and standards of argument that are accepted as contributing to the conventional canon of pure or applied mathematics."[3]

School Mathematics, much more than to introducing a bunch of numbers and names and to imposing rules, must be a cultural practice that allows the recovery of the dignity of the student. As Paulo Freire said, literacy includes the individual in society and Mathematics allows the individual to recognize the world.

Back to the values of mathematics education.

Let us return to the discussion of the values of mathematics education. We note inadequacies in favoring the first of them, that is, the utilitarian. This erroneous character of a mathematics education oriented strongly towards pure utilitarianism is contradictory with the arrival of calculators and computers. Traditional mathematics education still emphasizes the utilitarian value, without recognizing the enormous changes in the modern world. Hence, the traditional

3 Anne Watson: School mathematics as a special kind of mathematics, *For ther Learning of Mathematics* **28**, 3 (November 2008), p.3.

approach is obsolete and inefficient.

Traditional utilitarianism does not take into consideration the new emphasis on applications to the real problems affecting world civilization. A legitimate approach to utilitarianism should focuses on these problems. There was little authenticity in the so-called "problem solving" of the beginning of the decade of the 80s. Just the same can be said of a more widely encompassing concept such as, for example, that given in the *Agenda for Action*, (NCTM 1980) where emphasis is given to traditional word problems. What they call real situations are, in fact, simulated, artificial situations and, although there is a desire to work with "really real" situations, such is not possible in a classroom, unless attitudes to mathematics are changed.

If we consider the educational system as a whole, mathematics has a very important place in school. "Readin', Ritin', and 'Rithmetic", the 3 Rs, are the backbone of a system that aims to provide equal opportunities for all students, and at the same time to prepare cadres that advance and improve the socioeconomic and political aspects of society. The 3 Rs have dominated the school scene for many decades. Should that continue?

The arrival of computers will certainly change the scenario of education, and the 21st century will bring a predominant role for information processing equipment. The use of computers will affect the dominance of readin', 'ritin', and 'rithmetic in education. As radical as it may be, the reading and writing may disappear, as it is suggested and explained by William Crossman in the project *Voice In-Voice Out*.[4] Also arithmetic is in jeopardy.[5] The very nature of mathematics education is challenged. In fact, computers bring a new vision within mathematics. Even the concept of proof, a basic support of Mathematics, goes into question.[6] This will surely affect pedagogical action. Curriculum, seen as a strategy for pedagogical action, will demand new components. I have proposed elsewhere a basic curriculum leading to the acquisition of communicative, symbolic and technological instruments.[7] Although this issue is very relevant to our discussion, let's pass directly to our purpose, that is, to identify some of the indicators of how mathematics is contributing to social objectives.

The sociological value of Mathematics Education.

Mathematics is a strategy for achieving long range social goals. It is impregnated of concepts of progress and development, but with some enduring values that are permanent in any global political model. This is the model that has been adopted by (and now dominates) the democratic spirit in its quest for social well-being. A growing force, also dominating, is the

4. William Crossman. *VIVO [Voice-In-Voice-Out]: The Coming Age of Talking Computers* , 2004.
5. Anthony Ralston: Let's Abolish Pencil-and-Paper Arithmetic, Journal of Computers in Mathematics and Science Teaching, Volume 18, Number 2, 1999, pp. 173-194.
6. A Special Issue on Formal Proof, *Notices of the American Mathematical Society*, vol.58, nº 11, December 2008, pp.1370-1414.
7. Ubiratan D'Ambrosio: Literacy, Matheracy, and Technoracy: A Trivium for Today, *Mathematical Thinking and Learning*,1(2),1999; pp.131-153.

ecological spirit which is intimately related with considerations of the superiority and absolute significance of our species, "the internationalization of holistic thought in science and in culture", as suggested by Richard A. Falk.[8] This requires new models of social, political, and economic organization. The challenge of our time - the risk of destruction of our species, the impact of data processing, the implications of genetics, etc. - throws new light on what is said with respect to the social responsibilities of the scientific community, as much in the initiation of research topics, as well as in their applications.

Our responsibilities as educators in a democracy span beyond reproducing the past through the transmission and adoption of current models. We are concerned with creating a future. We hope the future will bring new social structures, leading to a society without inequity, arrogance and bigotry. We hope and struggle for a future that is better than the present, although it takes different forms. This is our goal as educators. Readers may ask: *What does mathematics have to do with this?* And I reply: *Everything*.

Mathematics has profound roots in our cultural systems and as such has many values, as we discussed above. Although it has not been sufficiently studied, an analysis of the ideological components of mathematical thought reveals a strong link with the current socioeconomic model. This is also recognized in discussing the ideological components of education in general, as has been emphasized by M. Apple,[9] H. Giroux[10], and the proponents of critical theory. Along with some eminently conservative practices, affecting daily life, such as medicine -– which aims at standards of health convenient to keep consumption and production -- and law –- based on norms to support the current power hierarchy -- mathematics positions itself as a designer and promoter of variants of the current world order which, in the name of progress, violates social justice, environmental equilibrium, mental health and creativity and cultural roots. We can easily paraphrase Duncan Kennedy[11] saying that mathematics educators teach their students to believe that individuals and institutions are organized into hierarchies according to their mathematical abilities. The superiority of those who reach a higher level of mathematics is recognized by all and mathematical ability is sometimes considered a mark of genius. This is recognized by Paulo Freire, when he says that it should be a concern of mathematical educators to show the naturalness of mathematics. In his words:

> "Regrettably, this is not what is done and I am a Brazilian who pays, who pays very highly, for this. I have no doubt that inside me there is a mathematician that did not have the opportunity of awakening, and that I will die without having awoken this mathematician, who might have been a good one. … But this did not happen and I pay very dearly for this. In my generation of Brazilians from the Northeast, when we referred to mathematics, we were referring to something suited for gods and geniuses. There was a concession for the genius individual who might do mathematics without being a god. As a consequence, how many critical intelligences, how much curiosity,

8. Richard A. Falk: Solving the puzzles of global reform. *Alternatives*, 11(1):45-81, Jan. 1986. See especially p. 68.
9. Michael Apple. *Ideology and curriculum*. London, Routledge & Kegan Paul, 1979.
10. Henry Giroux. *Ideology, culture and the process of schooling*. Philadelphia, Temple University Press, 1981.
11. Duncan Kennedy. *Legal education and the reproduction of hierarchy*. Cambridge, AFAR, 1983.

how many enquirers, how many capacities that were abstract in order in order to become concrete, have we lost?"[12]

A critical perspective on cognition, the social structure, and the independence of the State, that is to say on the general organization of the world, places us in a position of an urgent need to examine the role of mathematics in our educational system, proceeding from a new perspective. Problems such as the deterioration of the environment, the invasion of privacy, lack of security, famine, and starvation, the threat of nuclear war, all demand sound new ideas to guide thought about the future.

Without any doubt, the future will be permeated with science and technology -- for good or for evil. Mathematics is the root of science and technology. A few years ago, the magazine *The Economist* published a long article entitled "We cannot be citizens of the 20th Century without mathematics." The responsibility of mathematics educators with respect to the future is of central importance and it is necessary to understand our role in the complex network of divided responsibilities. Thus we see the necessary structure to discuss the system used to provide a form of mathematics in our schools that will be more beneficial and progressive.

Naturally, this implies systems to assess the results of teaching or, even better, systems to monitor and assess the success of schools as institutions.

Assessment.

It is fundamental, therefore, to create an assessment system for school achievement. We must also consider how policy makers or legislators will use the information collected by the assessment system. There is an evident necessity for an educational effort on the part of the authorities.

Israel Scheffer has said that we need to provide policy makers with a model rather than just data. The metaphor implicit here is that there is a need for policy makers to understand the process of learning and possess a knowledge of the place of mathematics in daily life, with all its complexity, activities, experiences, purposes, necessities, tensions, and creativity. This requires a holistic understanding of the nature of our subject and its place in the complete scope of human knowledge. As Scheffer said

12. Paulo Freire, Ubiratan D'Ambrosio, Maria do Carmo Mendonça: A Conversation with Paulo Freire, *For the Learning of Mathematics* **17**, 3 (november 1997) pp.7-10; p.8.

> "The policymaker, (...) should be multilingual not only with respect to the several disciplines of inquiry, but also with respect to the ordinary languages of those people whose problems such inquiry addresses."[13]

How does this apply to policy makers in mathematics education?

Many aspects have to be considered in an assessment system. It is fundamentally a public concern. We speak of the responsibility of the school system. Naturally, an assessment system will be used by the state and by those responsible for administering the school system. On the other hand, the fact that those in charge respond to public demands is the principal characteristic of representative democracy. This is evidenced by the budgetary regime that prevails in the financing of education. High achievers are better rewarded. Metaphorically, let us feed better the healthier. To give more support to low achievers should be a priority for public systems. Not to penalize the high achievers, but to give equal importance to upgrade the low achievers. This is, for me, the greatest challenge facing politicians and decision makers of public resources.

Therefore, school assessment is primarily designed for the executive and legislative branches of the government. But it should also be accessible to all those more directly involved with daily running education, mainly teachers and school administrators, students and parents. As Myron Atkin has said in his discussion of the "Improvement of Science Teaching", in a special edition of *Daedalus* concerned with scientific matters, "the majority of persons want something that is practical or, at least, recognizable."[14] Thus, we should be concerned with the accessibility of data.

For decision makers, that is, to the administrators of the system, these data should be accompanied by pedagogical reflections. The information and data provided by the assessment system must be accompanied by explanations, interpretation, and critiques of the results. This should certainly be a predisposition of the assessment system and there is no way around it. The explanations, interpretation, and critiques of the results and data obtained through assessment must take into account the set of values of the discipline, particularly in mathematics education, as well socio-cultural parameters.

We cannot forget to consider the internal composition of the school, that is, the relationships between teachers, teachers and principals, principals and superintendents, etc. In summary, we have to work with all the important forces in the school system. There is a complex network of influences that permeate mathematics education. This was discussed by Paulus Gerdes in ICME-5 in the case of a predominantly traditional society, like Mozambique. It is also to be found in literature, basically illustrated by Gustave Flaubert's *Madame Bovary* and, more recently, by the book *Padre Padrone*, Giovanni Lada and the movie with same title. An interesting account of the expectations for education can be found in the work of Teresa

13. Israel Scheffer. On the education of policy makers. *Harvard Education Review*, 54(2): 152-65, May 1984; p. 154.
14. Myron Atkin: The improvement of science teaching. *Daedalus*, 112(2): 167-87, Spring 1983.

Amabile.[15] These expectations and interrelationships of the various actors on the educational stage are the fundamental components for assessing the entire work.

In summary, an efficient assessment system has to take into account the expectations of all those involved. These expectations range from one extreme of affect and the growth of creativity to the other extreme of pure utilitarianism. Naturally, this is not a dichotomy. Both respond to a particular point of view of society as a whole. This type of dichotomy is discussed clearly by Plato in Book VII of the *Republic*. The comments made by Henry Marrou[16] are very interesting. Undoubtedly, the greater or lesser emphasis on a specific aspect is a political decision intimately linked the global social objectives. For example, we may ask if, in the United States, the "Paideia Proposal" is more in line with traditional North American ideals than the "Back to Basics Movement". Do both have as their goal the same model of society?

The movement "Back to Basics" is explained by its name. The "Paideia Proposal" is synthesized in the citation by one of its founders, Mortimer J. Adler:

> "All students study mathematics for all the twelve years of basic schooling. ... Mathematics is central to the manipulation, and dissemination of information. Mathematical illiterates will be left behind. In addition, mathematical reasoning is one of the most human things that human beings can do."[17]

How can we assess this approach in comparison with what has been called, by Garth Boomer[18], the "catechistic teaching of mathematics", favored by the Back to Basics movement?

The basic problem is still unanswered. Should mathematics education aim at creativity and, thus, carried out in an open manner? Or should it be tied to a traditionally oriented model? Throughout history many examples of open programs oriented to creativity have been proposed. But it has been almost impossible to evaluate them. Impact evaluations, often called for, are unsatisfactory. In this case, the affective components are probably the only indicators in which we can have faith.[19] Assessment, in this case, focus the behavior of a small group, using both qualitative and quantitative instruments.

Assessment systems are directed by authorities and legislators who make decisions of an administrative and institutional nature. Its repercussions on curriculum and what happens in classrooms is inevitable. Every teacher who knows the evaluation scheme will be profoundly affected, in his work, by that scheme, since there is neither a reason nor a way to maintain a

15. *The social psychology of creativity*, cit.
16. Henry Marrou: *A history of education in antiquity*. Madison, University of Wisconsin Press, 1982. p. 73. (Original in French, 1948; translated by George Lamb, 1956.)
17. Mortimer J. Adler: *The Paideia Program: an educational syllabus*. New York, MacMillan Pub. Co., 1984, p. 84.
18. Garth Boomer: From catechism to communication; language, learning and mathematics. *Plenary address to the Australian Association of Teachers of Mathematics Conference*. Brisbane, Jan. 1986.
19. A very interesting alternmative for assessment, aiming at impact evaluation, can be found in *Letramento no Brasil. Habilidades Matemáticas*, org. Maria da Conceição Ferreira Reis Fonseca, Global Editora e Distribuidora Ltda., São Paulo, 2004

confidential assessment system. Every school system is affected by the theoretical structure upon which its assessment system is based, and, as a consequence, the assessment system profoundly affects teacher behavior. Therefore, assessment systems influence negatively educational systems.

Let us concentrate on the main questions we wish to address concerning the assessment system. These questions should provide indicators of the soundness of the educational system. They should, naturally, be grouped according to the reasons that exist for teaching mathematics in the school system.

A brief incursion into the philosophy of mathematics education.

More than anything else, this is a result of an epistemological barrier. Curricular dynamics are present in the classroom, but the mathematics curriculum is dominated by a very conservative process. It, therefore, includes topics that are set in a final form. This has been described quite well by Phillip Kitcher:

> "We can imagine that the experts demonstrate their expertise by producing verifiable solutions to problems that baffle us, that they produce plausible arguments against our contentions (arguments whose flaws are too well hidden for us to detect), and that they offer convincing psychological explanations of our mistake".[20]

This reflects Kitcher's position against mathematical apriorism. And we know that apriorism is present in mathematics education.

Another epistemological approach would be the Bachelardian approach, which has been ignored in education, and particularly in mathematics education. When Bachelard says "The logical state is a simple state and even simplistic,"[21] and recognizes that this state cannot be used for proof in the case of psychological reality, he opens a new path for an approach based on the psycho-emotional complexity of the student, which is just the opposite of the techniques transmitted by teachers. In fact, Bachelard recalls, and refers to William James. Unfortunately, the same as Bachelard, he has been marginalized, even ignored, by mathematics educators.

The dominant tendencies in the philosophy of mathematics education tend to conceal the fact that mathematics is intimately linked to reality and to individual perception of that reality. Reality informs an individual through a mechanism which I call sensual instead of sensorial,[22] precisely to reinforce the importance of our psycho-emotional nature. When we ask "How much do students learn with respect to the solution of complex problems?", we have to take into account a new state of consciousness, to which William James refers when he maintains

20. Phillip Kitcher: *The nature of mathematical knowledge*. New York, Oxford University Press, 1984, p. 54.
21. Gaston Bachelard: *Essai sur la connaissance approchée*. Paris, J. Vrin, 1081, p. 27.
22. Ubiratan D'Ambrosio: Uniting reality and action: a holistic approach to mathematics education. in Lynn Steen & D. J. Albers, eds., *Teaching teachers, teaching students*, Birkhäuser, Boston, 1981.

that the state of consciousness in which we recognize an object is new compared to the state of consciousness in which we distinguish the object. This is reflected in what can be called the sensual impact of reality on the individual. The essential transition from distinguishing an object and recognizing an object is the essence of my amplification of the sensorial to the sensual. It is, essentially, the same as what Paulo Freire means when he says that

> "I have no doubt that our presence in the world implied the invention of the world. I have been thinking a lot that the decisive step that made us capable of being human, women and men, was exactly the step by which the support in which we found ourselves became 'the world' and life became 'existence', or rather began to become existence."[23]

Indeed, this is the moment when the emotional plays a fundamental role in human existence. Unfortunately, mathematics educators have a tendency to suppress what is emotional in the individual's perception of reality. It is difficult to deny that spirituality also plays a role in the approach to complex problems. Both an inner voice and the motivation from the context and from the environment, in its broad cultural, social, natural senses, are partners in defining individual approaches to a problem. The influence of spirituality was the object of a careful research by Klaus G. Witz.[24]

An alternative approach to problem solving requires children to make a real dedication to holistic strategies. Problem solving is in fact viewed from a broader perspective that combines, creatively, modeled processes and training programs. Thus, evaluation becomes more qualitative than quantitative, more an affective oriented search than an result oriented search.

Assessment system must consider some new indicators to move from a result oriented to an affective oriented assessment When we change from a traditional problem solving approach to one based on modeling, it is necessary to know what are the indicators to be used in an assessment system which has an affective and qualitative orientation. An imaginative proposal is implicit in the analysis of the behavior of a detective, as for example was done by Umberto Eco and Thomas A. Sebeok[25] when they added *abductive* reasoning to general considerations of the reasoning processes. In as much as the debates over problem solving have focused on "inductive-deductive" modes of thought, abduction, which can be conceived as a conjecture about reality that needs to be tested, appears to be the basic component for work in a real situation. According to Charles S. Pierce, abduction appears, along with induction and deduction, as a special mode of thought in the cognitive process. Despite the many new ideas that have been involved in the comprehension of the mind since William James and Charles S. Pierce, their approaches to reasoning appear to conform to our understanding of mental and

23. Paulo Freire, Ubiratan D'Ambrosio, Maria do Carmo Mendonça: A Conversation with Paulo Freire, *For the Learning of Mathematics* **17**, 3 (november 1997) pp.7-10; p.7.

24. Klaus G. Witz: Spiritual aspirations connected with mathematics: the experience of American University students, The Edwin Mellen Press, Lewiston NY, 2007.
25. Umberto Eco and Thomas A. Sebeok: *The sign of three*. Bloomington University Press, 1983.

corporal processes, expressed in our move from sensorial to sensual mechanisms.

This transition from sensorial to sensual, which is intrinsic to the new socio-biological vision of cultural phenomena proposed by Charles J. Lumsden and Edward O. Wilson, is very important for mathematics education. It leads to understanding cultural phenomena through a sequence of components that they called learning, imitation, and *reification*. All, except *reification*, are also present in other species. *Reification* is presented as

> "mental activity in which concrete, simplified, and labeled form is given to words or other symbols, to vaguely perceived and relatively intangible phenomena such as the complex arrangement of objects or activities"[26]

The authors claim that these three components are found only in human beings. This idea I later developed into identifying the pulsions of survival and transcendence as characteristic of the human species.

In summary, we see that Lumsden and Wilson and, much before them, Pierce, envision codification processes as acquired by psychological mechanisms that go against the linear structure that characterizes and serves as the basis of the practice of mathematics education. These codes are acquired by a reification process, and stored for later use in different situations. Among these codes are the mathematical codes. This position suggests that the best way to teach mathematics is to plunge children into an environment where mathematical challenges are presented quite naturally. The work of Teresa Amabile, cited above, is along these same lines, much like the psycho-pedagogical structure implicit in the Logo proposal.[27] This is not different from what is related in *The Education of Henry Adams*[28] and is also seen in the root of the pedagogical thought of John Dewey.

A proposal of mathematics for all.

Trying to bring these considerations to the practice of mathematics in schools, we should turn to situations that are "really real". Projects of a global nature, such as the construction of a cabin or the mapping of a city or the evaluation of water consumption, provide information that demands the handling of problems and models. Problem solving occurs as a consequence, therefore it makes sense going beyond the results and searching for meaning of the solution. The methodology on which the *Foxfire Project* [29] is based considers the environment of the children and begins with what could be called "fact discovery", in the sense of collecting information about a situation. From there it moves to modeling and finally arrives at what can

26. Charles J. Lumsden and Edward O. Wilson: *Genes, mind and culture*; the evolutionary process. Cambridge, Mass., Harvard University Press, 1981, p. 381.
27. Seymour Papert. *Mindstorms*. New York, Basic Books, 1980.
28. Henry Adams. *The education of Henry Adams*. New York, The Library of America, 1983, p. 715-1192. (Original publication, 1905).
29. Eliot Wiggington, ed. *Foxfire 6*. Garden City, NY, Anchor Press/Doubleday, 1980.

be considered the realization of the model, that is, the transformation of results in action on objects. This is based on the cycle ... reality - individual - action - reality ..., as discussed in my plenary talk delivered in ICME 5, in Adelaide, Australia, 1984.[30]

This is, without a doubt, an open approach to mathematics education, with activities oriented, motivated, and induced from the environment and, consequently, reflecting previous knowledge. This leads to the Program *Ethnomathematics*, which reestablishes mathematics as a natural and spontaneous practice. Although there has been much research on the influence of ideas already encountered before school in an experimental approach to scientific education, principally by the Piaget and his followers,[31] the effort to identify mathematical practices and to recognize them as a base of great value in education is relatively recent. There has been not much research on the potential of a pedagogical model for mathematics based on the transition from preschool practices to practices of an academic nature. The Program Ethnomathematics aims at that.

The predominant position in all school systems is to give the same mathematics for all. First, it is assumed that all are prepared to acquire equally well the form of knowledge that appears to be appropriate for the given level and teaching/learning dynamics. Second, this mathematical knowledge is assumed to be compatible with existing mental structures. This is the kind of Kantian apriorism that predominates the philosophy of mathematics and is reflected in its totality in education.

30. Ubiratan D'Ambrosio: *Socio-cultural bases for mathematics education*, Unicamp. Campinas, 1985.
31. E.Causinille-Marmeche, M. Mehuet, M. G. Sére, A. Weil-Barais: The influence of a priori ideas on the experimental approach. *Science Education*, 69 (2), 1985, pp. 201-11.

Therefore, new approaches to the nature of mathematical knowledge, such as for example that of Kitcher[32] and the increasing attention given to ethnomathematics, open a new and broad area of research on what can be called the anthropological approach to mathematics with rethinking of constructions of cultural and psycho-emotional nature. From there, a new way of considering mathematics is conceived. It is based on cultural and psycho-emotional motivations that produce differences in the receptivity of mathematics, although the level and nature of the lectures are the same. Thus we have groups of individuals with higher or lower achievement in certain types of mathematics. Some claim dislike of algebra, other dislike geometry. What must be avoided in the school system is valuing one type of mathematics to the detriment of others. Explicitly, by bringing into the classroom a kind of mathematics more intimately related to activities that pleases one group, their performance should be better than when they deal with topics that are related to activities of another group. The same can be said for cultural activities.

More research is necessary in order to understand the different ways that children react. Because of the misleading tendency during the last decades to think of one mathematics for all, such research has been scarce. The differences in achievement are, very probably, the result of socio-cultural background. I have to say, as controversial as this may be, that we cannot exclude the possibility of genetic influences.
We should consider the research in this area, done by C.P. Benbow and J.C. Stanley,[33] even though it has been challenged and even condemned. At any rate, the key issue is to provide multiple and diversified curricular guidelines that are better adapted to the cultural and psycho-emotional patterns of children. This is much in the line of Howard Gardner. The multiplicity and diversity of mathematical experience lead to curricula based on situations, following an approach implicit in the work of Teresa Amabile in her projects on the social psychology of creativity and assessment.

Let's return to the modelling of real situations as the best method for working with such diversity. Naturally, this implies a recognition that values in mathematics education other than the utilitarian must not be given an inferior position. Surely, in many cases, the cultural, aesthetic, sociological, and formative values are even more important. The utilitarian values, that have prevailed in the last one hundred years, were felt throughout history in other domains of education. As I discussed in my address to ICME 3, in Karlsruhe, Germany,[34] the utilitarian vision of mathematics exists parallel to academic mathematics. It is evident that the disequilibrium between utilitarianism and other values that has occurred during the last one hundred years has caused a dehumanization of science, technology, and society as a whole. It is time to reestablish a humanistic focus in general education, particularly in mathematics education, in the proposal of education for all.

Consequently, the quality of mathematics curriculum, which, normally refers to international

32. Phillip Itcher: *The nature of mathematical knowledge*, op. cit.
33. C.P. Benbow and J.C. Stanley: Sex differences in mathematical reasoning ability: more facts, *Science*, **222**, 1983, pp.1029-1031.
34. Ubiratan D'Ambrosio: Objectives and goals of mathematics education, *New Trends in Mathematics Education IV*, UNESCO, Paris, 1979.

comparisons and are in detriment of less developed countries, has to be considered from the point of view of culture. Comparative assessment, like PISA, TIMSS and others, may cause more damage than good.

Quality is assessed not only by attitudes of achievement and performance, and even less by an analysis of the three levels of curriculum content that are generally considered (the intended, the implemented, and the achieved curriculum). The analysis of curriculum, even in this multi-layered way, may lead to false assessments of the systems, by masking the components of social injustice, prejudices, and various forms of discrimination based on social class, language, race, gender, religion and other cultural differences.

By proposing large attention given to the cultural environment as an alternative to the traditional curriculum, we are implicitly questioning mathematics as a biased system of codes that permits us to describe, shape, understand, and control reality. This is linked to a broad concept of what does it mean to understand reality.
An authentic system of codes would pass, from an initial process, derived from the family and cohort groups, to more sophisticated systems of codes proposed in the schools. The initial stage is very difficult, sometimes impossible, to institutionalize. It is generally imprecise and sometimes they are not even clearly understood. Studies in this area belong, traditionally, to the domain of anthropologists and only recently they have entered into the consideration by educators. Up until now, there has been a certain resistance to analyze such matters in the school system. They are linked to what Basil Bernstein[35] calls, in his treatment of language, "restricted code" as contrasted to "elaborated code". This are identified by Ivan Illich[36] with "vernacular language" or "vernacular universe". The Program Ethnomathematics is a possible response to the issues raised in this paragraph.

Some critical remarks, even objections, to the Program Ethnomathematics may be summarized in the following three points.

1. It is limited in techniques, because it is based on limited sources. On the other hand, its creative component is high, because it is free of formal rules and follows open criteria.

2. It is particularistic, in the sense of being limited to the context, although it is more effective than a deceitful, misleading, discourse of universal utilitarianism.

3. It operates using metaphors which allow for codes and symbols that are psycho-emotionally related, while mathematics operates with codes and symbols that result from pure rationality.

Naturally, this leads to a hierarchy for the transmission of knowledge and to the fundamental problem of the legitimization of knowledge. While ethnomathematics limits its value by basing its validity on the way in which something functions in a specific situation, or is pleasing and

35. Basil Bernstein: *Class, codes and control*. London, Routledge and Kegan Paul, 1971. v.1.
36. Ivan Illich: *Gender*. New York, Pantheon Books, 1982.

adapted to a particular vision of the world, mathematics bases its authority on a sequential hierarchy that begins with the teacher, the authority of printed matter, and reaches an authority of the academy. To see mathematics education in a way that personifies the value and the culture of children, that is, their ethnomathematics, seems to be the desired path to an humanistic rationalism.

The passage from ethnomathematics to mathematics can be seen as the passage from oral language to writing. Written language (reading and writing) rests on the knowledge of oral expression that children already have, and upon introducing written language we do not suppress oral language. To understand and respect the practice of ethnomathematics opens a great potential for the sense of questioning, recognition of specific parameters and the recognition of a global balance with nature. Ethnomathematical practices are recognized in all levels of social and professional life, but they are still regarded as irrelevant for mathematical knowledge.

When I say all levels of social and professional life, it must be understood also in sophisticated levels of knowledge such as physics, engineering, biology, etc. The delta function of Paul Dirac can be identified as ethnomathematics, just as the majority of the calculations made by the majority of engineers, physicists, and biologists can be called "ethnocalculations". Sylvanus Thompson, when he wrote *Calculus made easy*, in 1910, he was, in reality, publishing ethnomathematics.[37] And the procedures of open heart surgeons reveal important ethnomathematics.[38] Examples are abundant at this level. In the school system, that focus, essentially, the traditional curriculum and to ways of assessing performance, little attention is given to ethnomathematics.

Concluding remarks.

To finish let me refer to the difficulties of building a productive assessment system for the cultural, aesthetic, and formative values of mathematics education in which utilitarian values are privileged.

The cultural, aesthetic, and formative aspects of a student's evolution cannot be evaluated. A respectful and dignified vision of a person cannot be measured directly and it is very difficult to measure aesthetic values. The same thing happens with formative values; probably the only parameters that can possibly be introduced into the assessment system are those related to the emotional attitudes of a class.

As a main conclusion, I say that ethnomathematics and art go together as strategies to cope

37. Gustavo Alexandre de Miranda: *Silvanus Phillips Thompson e a Desmistificação do Cálculo: Resgatando uma História Esquecida*. Dissertação de Mestrado, PUC-SP, 2004.

38 Tod L. Shockey: Etnomatemática de uma Classe Profissional: Cirurgiões Cardiovasculares. *BOLEMA (Boletim de Educação Matemática)*, nº 17, ano 15, 2002. This paper summarizes the author's PhD dissertation, University of Virginia, 1999.

with the reality and representations of reality.

The recuperation of knowledge of the people, particularly art, is an important strategy for political awareness, which is a major goal of ethnomathematics. It is important to transmit mathematical knowledge, necessary for life in modern societies, based in multicultural reading of knowledge produced by the people.[39] This production is strongly represented by the arts.

39. This approach is adopted, with respect to blood and HIV, in the Project REPOhistory. Circulation, *Art Journal*, vol.59, nº4, 2000, pp.38-53.