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Efficiency of hyperelastic models for rubber-like materials

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ABSTRACT: This paper focuses on the modeling of rubber-like material behaviour under several modes of deformation using hyperelastic constitutive equations. A procedure based on genetic algorithms coupled to classical optimisation methods is proposed to identify the parameters of the models upon experimental data given in the literature. This leads to the classification of nineteen models with respect to criteria related to their capability to predict material behaviour.

1 INTRODUCTION

Hyperelastic models are used to simulate the non-linear elasticity of rubber materials under static loading conditions or to develop more sophisticated models. Many models have been proposed to describe the behaviour of elastomers, but few studies which evaluate the ability of hyperelastic models to reproduce rubber behaviour for all the modes of deformation can be found in literature. Recently, Seibert and Schöche (2000) compared six different models with their own experimental data. Danger of the series formulations is highlighted by showing bad predictions of biaxial response after uniaxial identification. Boyce and Arruda (2000) confronted five models with data in three different modes of deformation. More recently, Attard and Hunt (2004) used experimental data of seven different authors to demonstrate the efficiency of their model.

The aim of the present paper is to systematically compare nineteen models proposed in the literature in order to classify them with respect to their ability to fit experimental data.

2 COMPARED MODELS

Hyperelastic models are classified into three types of formulation, depending on the approaches adopted by their authors for their development.

2.1 *Phenomenological and empirical models*

The first type concerns general mathematical forms such as the Rivlin series. Their parameters are generally difficult to identify and their generalised form can lead to error when these models are used out of their identification range. Such models considered here are:

- the Mooney model (1940).
- the Mooney-Rivlin model (1948).
- the Biderman model (1958).
- the Haines-Wilson model (James et al. 1975) justified by Davet (1985) with experimental considerations.
- (he Ogden (1972) model.

These models are mathematical representations of the strain energy function W with no physical or experimental considerations.

2.2 *Approaches in derivatives $\partial W/\partial I_1$ and $\partial W/\partial I_2$*

Other authors preferred to extract directly the form of the fonction $\partial W/\partial I_1$ and $\partial W/\partial I_2$ from experimental data:

- Rivlin et Saunders (1951) observed that $\partial W/\partial I_1$ is independent on I_1 et I_2 and that $\partial W/\partial I_2$ does not depend on I_1,
- Gent and Thomas (1958) proposed an empirical form with only two material parameters,
- Hart-Smith (1966) noted that $\partial W/\partial I_1$ is constant for values of I_1 smaller than 12, but increases for higher values. He explained this phenomenon by the limit of extensibility of the polymeric chains,
- Valanis et Landel (1967) proposed a form of W in terms of the principal stretches λ_i with the assumption of separability on λ_i,

- Gent (1996) used the idea of a limit of chain extensibility and assumed that I_1 admits a maximum value,

- Yeoh and Fleming (1997) noted that the reduced stresses tends to a constant value independent of $(I_1 - 3)$ for $(I_1 - 3) > 5$.

2.3 *Physical-based models*

Over the last decades, development of phenomenological models tends to introduce physical considerations. The third type of models are those which are derived from physics of chains networks. Such models are based on statistical methods leading to different macroscopic models depending on the microscopic phenomena accounted for:

- Treloar (1943) used a gaussian statistical distribution to develop the neo-hookean model,

- Kuhn and Grün (1942) used a non-gaussian theory to take into account the limit chain extensibility. They introduced the inverse Langevin function,

- James and Guth (1943) derived the previous model and proposed a model where chains are re-distributed upon the three principal axes of deformation,

- Ishihara (1951) linearized equations of the non-gaussian theory and obtained a Rivlin series where parameters are linked. Thus, he introduced I_2 into a physical based model (confirmed by Wang and Guth (1952)).

The deviation in experimental data of the ideal chain models is classically imputed to the so-called phantom assumption which does not account for chains entanglement and chains can pass through each other. Authors introduced the idea of entanglement constraints or topology conservation constraints and adopt the following form of the strain energy functions:

$$W = W_{ph} + W_c \tag{1}$$

where W_{ph} is the phantom network part and W_c is the constraints or cross-linking part:

- Ball *et al.* (1981) developed the slip-link model where a first term corresponds to the phantom Gaussian model,

- Kilian *et al.* (1986) revived an idea of Wang and Guth by taking into account the *van der Waals* forces. Few years later the model is presented in a potential form (Ambacher et al. 1989),

- Flory *et al.* (1994) developed a model where junction points of the chains are constrained to move in a restricted neighbourhood due to the presence of other chains. The phantom part of the model is described by the neo-hookean model,

- Arruda and Boyce (1993) proposed a chain model with a distribution of chains in eight directions,

- Heinrich & Kaliske (1997) built a model where chains are constrained by a tube formed by the surrounding chains. This assumption is attributed to the high degree of entanglement of network chains. The model takes the form of the phenomenological model of Ogden with only two terms,

- Kaliske & Heinrich (1999) replaced the gaussian distribution of the above tube model by the non-gaussian approach and introduced an inextensibility parameter,

- Miehe *et al.* (2004) developed the non-affine micro-sphere model by associating Langevin chain models with the tube-model. The chains are distributed upon discrete directions and the micro-stretches are allowed to fluctuate around the macro-stretches with only one additional parameter.

3 IDENTIFICATION METHODS

It is now established that a unique experimental test is unable to characterize a rubber-like material. Moreover, it is difficult to identify model parameters by fitting only one curve corresponding to one type of deformation, especially when the number of these parameters is large and it is not sure that other types of deformation will be reproduced with good agreement. A good example is given in the paper of Seibert and Schöche (2000).

The incompressible assumption constrains the admissible kinematical field in rubber. In the principal axes, this equation allows the possible deformations to be governed by only two independent variables. Therefore a series of biaxial tests is sufficient to fully identify the constitutive models.

3.1 *Experimental data*

In order to investigate the identification of the material parameters, we choose complementary data from classical papers. The first set of data is due to Treloar (1944) and is widely used by other authors. We only focus on the 8% S vulcanized rubber which is known to exhibit highly reversible elastic behaviour and no crystallization on stretching up to 400%. The specimen was pre-stretched with a initial extension of

400% to eliminate the Mullins effect (Mullins 1948). Experimental measures were performed for equibiaxial extension (EQB), traction (T), pure shear (PS) and combined biaxial extension (BE).

The second set of data is due to Kawabata *et al.* (1981). With an apparatus built for general biaxial extension testing, they obtained data for a square sheet of polyisopren. The specimen were stretched from 1.04 to 3.7 in one direction (λ_1) and from 0.52 to 3.1 in the perpendicular direction (λ_2).

The two materials used respectively by Treloar and Kawabata *et al.* are similar. A unique set of material parameters should be able to reproduce these data with good agreement.

3.2 *Identification algorithms*

The problematic of identification makes analytical solutions to coincide with experimental measurements. The measure of the difference ϕ is classically defined by the mean square error. A minimization of ϕ is generally employed. In most cases the coincidence of data with theoretical responses can only be established on a restrictive set of data points (validity domain).

Among all possible minimization methods, we focus on classical gradient methods and genetic algorithms. The latest have been used for identification problem for few years (Furukawa & Yagama 1997; Liu et al. 2002; Yoshimoto et al. 2003).

3.2.1 *Gradient methods*

The solution of the minimization problem is often non-unique. Local solutions are generally obtained with classical methods such as conjugate gradient, Newton-like or Levenberg-Marquardt methods. Such iterative methods consiter the derivatives of ϕ and the solution depends on an initial point introduced by users. A series of points is build by looking for a descent direction which allows to find a new solution where the value of ϕ is lower than the present one.

3.2.2 *Genetic algorithms*

Genetic algorithms (GA) were introduced by Holland (1975). Later Michalewicz resumed the state of the art of such methods (1996). A genetic algorithm emulates biological evolutionary theories to solve optimization problems. According to the evolutionary theories, only the most fitting elements in a population are likely to survive and transmit their biological heredity to the next generations. This leads to the evolutive convergence of the species through operator such as competition among individuals, natural selection and mutation of the DNA.

The introduction of randomness in the GA makes exploration of the research space independent of the starting point. Thus the GA are likely to obtain a global optimum of the fitness function instead of a local one.

The original GA was based on a similitude between chromosome and binary code. Crossover of a sequence of bits and mutation bits were tempting to preserve the similitude with biology. Binary coding has long been considered as the best one but other codings are possible and some authors recommend a code as close to the space of parameters as possible. Here, we choose the integer coding.

There is no guaranty for convergence of the solution with the use of GA and no conclusion must be settle from a lonely run. Nevertheless, improvement can be observed by increasing the number of individual while convergence is less sensitive to the number of generation if it is not too small.

3.3 *Identification algorithms*

The choice of the identification algorithm is added to our strategy. Models are first identified with genetic algorithms and material parameters are used as initial parameters in the Levenberg-Marquardt method. In case of divergence of the latest method, the mean square method is used. In such a way, the results always take advantages of the genetic algorithms.

4 CLASSIFICATION

4.1 *Identification steps*

Both materials considered by Treloar and Kawabata *et.al* are similar in terms of composition and behaviour. We will try to determine an unique set of parameters can be identified to reproduce the two sets of experimental data. Two identifications steps are proposed here to achieve this aim :

1. parameters are identified on Treloar's data in traction, pure shear, equibiaxial extension and biaxial extension:
   (a) if the accuracy is good, paramaters are retained,
   (b) if the accuracy is poor, the validity domain is modified:
      i. if the model is not able to reproduce strain hardening at large strain, the domain of validity is reduced for uniaxial extension mode (λ_{max}),
      ii. elsewhere, other modes of deformation are progressively eliminated from the identification procedure. Then, the domain of validity (λ_{max}) for the other modes of deformation is observed on the response curves.
2. parameters identified by the above step are retained to simulate Kawabata biaxial experiments:

(a) if the accuracy is good enough, the parameters are considered as model parameters for both materials,

(b) elsewhere, new parameters are identified for the Kawabata data:

   i. if the accuracy is not good enough, the validity domain of the model for biaxial extension is reduced.

   ii. elsewhere, parameters are retained for biaxial mode and the domain of validity (λ_1 and λ_2) is then observed on the response curves.

The strategy described above leads to the determination of parameters for all models and of the validity domain for each mode of deformation.

4.2 *Classification*

The classification presented in Tables 1 and 2 is established with the following criteria. The larger is the domain of validity (λ_{max}, λ_1 and λ_2 for the different modes of deformation), the upper is the model in the table. Then, the greater is the number of parameters (nop) of the model, lower is the model. For equivalent models, more consideration is given to the one which can represent both sets of data with the same set of parameters. Finally, a subjective criterion is taken into account to decide between different equivalent models and preferences are awarded to physical-based models (column *Phys* in Table 2).

Tables 1 gives the limit λ_{max} of the validity domain for (T) traction, (PS) pure shear, (EQB) equibiaxial extension and (BE) biaxial extension for identification on Treloar data. Notations (under) and (over) indicates if stresses are predicted with underestimation or overestimation.

Tables 2 gives limit of the validity domain λ_{max} in both directions (λ_1 and λ_2) for Kawabata *et al.* data. Notations (under) and (over) indicates if stresses are predicted with underestimation or overestimation. Symbols ($=$) or ($\neq$) indicate if only one set of parameters is able to reproduce both sets of data.

4.3 *Example*

The following graphs illustrate the performance of the extended tube model. They are obtained with the same set of parameters for both sets of experimental data. The value of these parameters are given with the notations of Kaliske and Heinrich (1999).

5 CONCLUSIONS

This paper focuses on hyperelastic models found in the literature and investigates their capability to reproduce the mechanical behaviour under all kinematically admissible modes of deformation.

Table 1. Classification of hyperelastic models: validity domain for Treloar data

		Treloar data λ_{max}			
	Model name	T	PS	EQB	BE
1	extended tube	-	-	-	-
2	micro-sphere	-	-	-	-
3	Ogden	-	-	-	-
4	Haines-Wilson	-	-	-	-
5	Biderman	-	-	-	-
6	Hart-Smith	-	-	-	-
7	8-chains	-	-	under	under
8	Gent	-	-	-	-
9	Yeoh and Fleming	-	-	-	-
10	van der Waals	-	-	2.5	over
11	3-chains	-	-	under	under
12	tube model	4	3.5	3	-
13	Mooney	5	-	4	2
14	Ishihara	5	-	4	2.25
15	Gent and Thomas	5	-	3	-
16	Slip-link	5	4	2.5	over
17	constr. junctions	5	4	2.5	over
18	neo-hookean	5	2	3	2.5
19	Valanis and Landel	3.5	2.5	1.2	under

Table 2. Classification of hyperelastic models: validity domain for Kawabata *et al.* data; (nop) number of parameters; (Phys) physical-based model;

			Kawabata data λ_{max}			
	Model name		λ_1	λ_2	nop	Phys
1	extended tube	$=$	-	-	4	$\times$
2	micro-sphere	$=$	-	-	5	$\times$
3	Ogden	$\neq$	-	-	6	
4	Haines-Wilson	$\neq$	3.4	3	6	
5	Biderman	$\neq$	2.5	3	4	
6	Hart-Smith	$=$	1.9	1.5	3	
7	8-chains	$\neq$	1.9	1.9	2	$\times$
8	Gent	$\neq$	1.6	1.6	2	
9	Yeoh and Fleming	$\neq$	1.6	1.6	4	
10	van der Waals	$=$	2.2	2.2	4	$\times$
11	3-chains	$\neq$	1.3	1.3	2	$\times$
12	tube model	$=$	-	-	3	$\times$
13	Mooney	$\neq$	2.2	2	2	
14	Ishihara	$\neq$	1.9	1.9	3	$\times$
15	Gent and Thomas	$=$	1.6	1.6	2	
16	Slip-link	$\neq$	2.5	2.5	3	$\times$
17	constr. junctions	$\neq$	2.2	2.2	3	$\times$
18	neo-hookean	$=$	1.6	1.6	1	$\times$
19	Valanis and Landel	$\neq$	1.3	1.3	1	

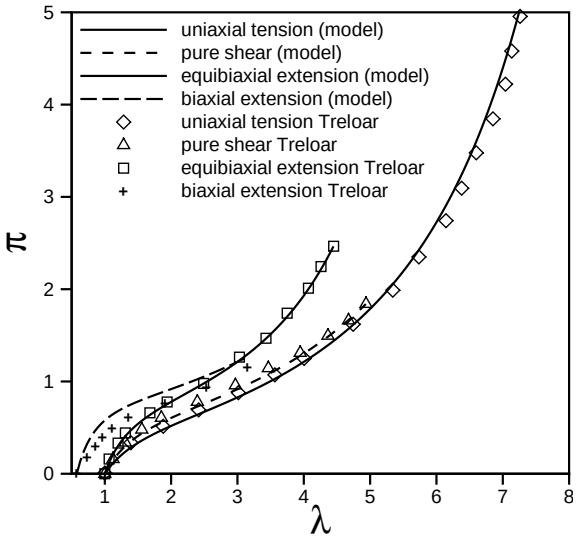

Figure 1. comparison between the extended tube model and experimental data of Treloar: $G_c = 0.202$; $G_e = 0.153$; $\beta = 0.178$; $\delta = 0.0856$

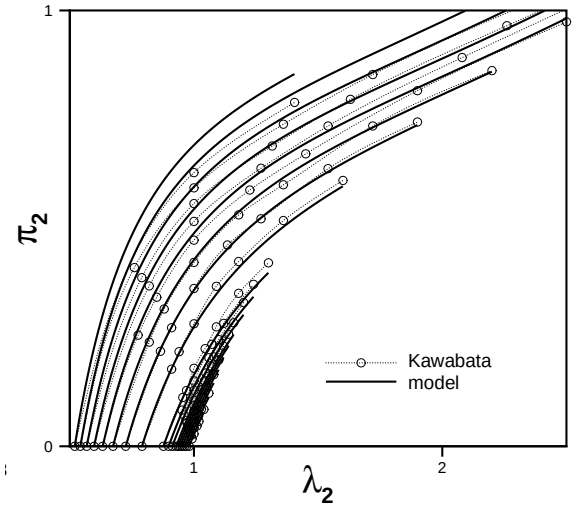

Figure 2. comparison between the extended tube model and experimental data of Kawabata *et al.*: $G_c = 0.202$; $G_e = 0.153$; $\beta = 0.178$; $\delta = 0.0856$

A methodology is proposed to identify the models with previously published experimental data. An identification procedure has been developed. An original point of this method is the use of genetic algorithms coupled to classical optimisation approaches. The proposed method leads to the identification of both material parameters and of the validity domain of the models.

Finally, a classification of the models is proposed considering the domain of validity for all modes of deformation, the number of parameters and the type of formulation used to derive the models. Depending on the considered domain of deformation, the neo-hookean model, the Mooney model and the Ogden model can be used respectively for small, moderate or large strain. Nevertheless, the study highlights non-classically used physical-based models which leads to better agreement with experiments and involves a smaller number of parameters: the extended-tube model and the micro-sphere model.

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