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NUMERICAL INVESTIGATIONS ON ULTIMATE STRENGTH OF A DOUBLE HULL VLCC UNDER COMBINED LOADS AND INITIAL IMPERFECTIONS

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ABSTRACT

Nonlinear finite element analyses are performed to determine the ultimate strength of a double hull VLCC under pure vertical, horizontal and biaxial bending. A parametric finite element model is developed and the influence of nonlinear material behavior, mesh size and model length on the hull girder ultimate strength is demonstrated exemplarily for hogging and sagging conditions. An appropriate parameter configuration with respect to numerical efforts and accuracy is used to perform static implicit analyses for horizontal bending and biaxial load cases. Convergence is reached by using the full Newton-Raphson scheme - an incremental iterative solution approach. The results are validated against the well-established Smith method. Due to welding, initial deflections and residual stresses are produced. For the proposed finite element model initial deflections of plating and stiffeners have been considered. Furthermore, the influence of welding residual stresses on the ultimate hull girder strength is demonstrated for the different load cases. Nonlinear finite element analyses are also performed to determine the residual strength of the damaged double hull VLCC under combined loads. Different symmetric grounding damages are implemented by removing structural components of the model. Expectedly, the results show that the ultimate strength of the structure decreases as the damage extent increases.

Keywords: Ultimate strength, nonlinear finite element analysis, Smith's method, damage case, welding residual stress, bending

1. INTRODUCTION

Ship structures are subjected to various types of loads during their lifetime. The estimation of the ultimate hull girder strength under combined loads is of essential importance to ensure their safety. Yao and Fujikubo [1] described different methods to determine the buckling and ultimate strength of ship and ship-like floating structures. Paik [2] proposed approaches to perform ultimate limit state analysis and design of plated structures. In both textbooks the progressive collapse behavior and strength are demonstrated for different ship hull girders.

Smith's method [3] is also a feasible and well established incremental iterative approach to perform progressive collapse analyses of ship hull girders in vertical bending. Smith [4] improved his method to analyze also unsymmetrical sections and hull girders subjected to biaxial bending. The applicability of Smith's method is demonstrated for different ships within the ISSC reports [5-6]. La Ferlita et al. [7] applied Smith's method in framework of an advanced salvage method for damaged ships. Tatsumi et al. [8] proposed a further study on progressive collapse analysis of a hull girder using Smith's method and uncertainties in ultimate strength prediction due to its basic assumptions are also discussed.

The Idealized Structural Unit Method (ISUM) originally developed by Ueda and Rashed [9] is a simplified approach with reduced numerical efforts to perform progressive collapse analyses of hull girders under combined loads. Oksina et al. [10] proposed a review of the current ISUM formulation and obtained results of plates under dynamic loadings.

The applicability of ISUM for ultimate strength analyses of stiffened plate structures is demonstrated exemplarily by Lindemann and Kaeding [11]. The proposed ISUM plate element formulation simulates the collapse behavior of plate structures under inplane and lateral loads.

The Finite Element Method (FEM) is a widely used tool to perform progressive collapse analyses of large structural systems under consideration of material and geometrical nonlinearities. Initial imperfections due to welding can be considered easily for ultimate strength predictions. Details about nonlinear FEM to perform static or dynamic analyses are given exemplarily by Bathe [12]. Nonlinear finite element analyses (FEA) are performed by Lindemann and Kaeding [13] to investigate the influence of shear and lateral loads on the collapse behavior of plate structures under longitudinal and transverse thrust. Despite enormous developments in finite element formulations, the numerical results should be validated against experimental data. In general, this is not possible for as-built ships but for scaled models. Lindemann et al. [14-15] performed ultimate strength tests of box girders specimens in vertical bending experimentally to validate nonlinear FEA results. The influence of welding residual stresses on the ultimate strength of the box girder specimens is investigated separately.

The applicability of nonlinear FEA is demonstrated for different ships in vertical bending within the ISSC report [6]. Darie et al. [16] performed ultimate strength analyses of a cape size bulk carrier under combined global vertical bending moment and local loads. Further nonlinear FEA are performed successfully by Darie and Rörup [17] to determine the hull girder ultimate strength of container ships in oblique sea. In both cases an implicit solver implemented in LS-DYNA was used. Tatsumi and Fujikubo [18] determined the hull girder ultimate strength of container ships for combined hogging moment and local bottom loads by using a static implicit approach in framework of the finite element method. Toh et al. [19] used a nonlinear explicit FEA code (LS-DYNA) to determine the ultimate strength of a bulk carrier in intact and damaged conditions. Kuznecovs et al. [20] also used an explicit solver (Abaqus) to analyze the ultimate limit state of a double-hull tanker subjected to biaxial bending in intact and collision-damaged conditions.

In this paper, nonlinear finite element analyses are performed using a static implicit solver (ANSYS) to determine the ultimate strength of a double hull VLCC under pure vertical, horizontal and biaxial bending. The influence of nonlinear material behavior, mesh size and model length on the hull girder ultimate strength is demonstrated exemplarily for hogging and sagging conditions. Convergence is reached by using the full Newton-Raphson scheme - an incremental iterative solution approach. The results are validated against the well-established Smith's method. Due to welding, initial imperfections are produced and the influence of residual stresses on the ultimate hull girder strength is demonstrated. Nonlinear finite element analyses are also performed to determine the residual strength of the damaged double hull VLCC under combined loads. Different symmetric damage cases due to grounding are implemented by removing structural parts of the model.

2. VLCC VESSEL

2.1 Dimensions and Properties

Progressive collapse analyses are performed for a double hull tanker (VLCC) proposed in the ISSC report (2000) [5]. The principal dimensions of the VLCC vessel are given in Table 1 respectively shown for the cross section in Fig. 1.

TABLE 1: PRINCIPAL DIMENSIONS OF VLCC [5]

Description	Symbol	Value	Unit
Length	L	315.0	m
Breadth	B	58.0	m
Depth	D	30.4	m

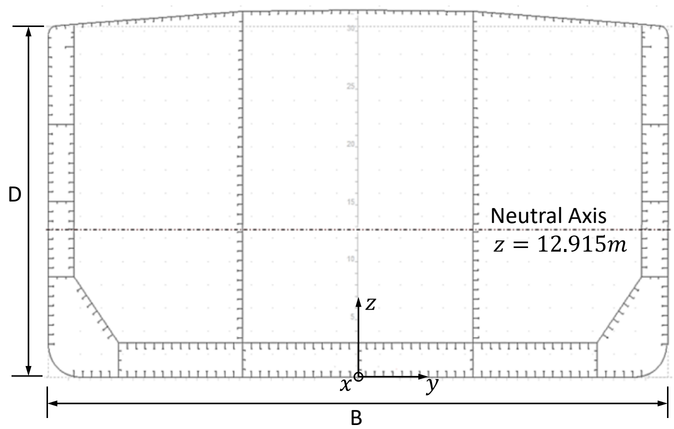

FIGURE 1: DIMENSIONS OF VLCC CROSS SECTION

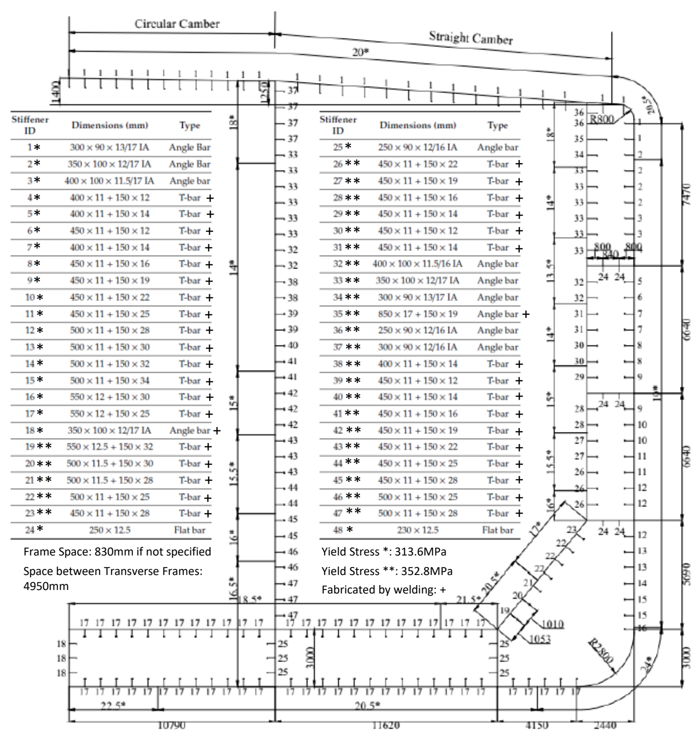

FIGURE 2: CROSS SECTION OF VLCC [5], [21]

In Figure 2 further details of the cross section are presented. Angle bars, T-bars and flat bars of different dimensions and material properties are used as longitudinal stiffener elements. The material properties of the plating and the stiffeners are given in Table 2. The frame space is given with 830 mm and the space between the transverse frames is equal to 4950 mm.

TABLE 2: MATERIAL PROPERTIES OF VLCC [5]

Description	Symbol	Value	Unit
Young's Modulus	E	210000	MPa
Poisson's Ratio	ν	0.3	-
Yield Stress *	σ_Y*	313.6	MPa
Yield Stress **	σ_Y**	352.8	MPa

Progressive collapse analyses are performed to determine the collapse behavior of the VLCC in intact condition as well as the residual strength for different damage cases. Exemplarily, three symmetrical damage cases due to grounding are assumed. The corresponding damage height and damage breadth are given in Table 3 respectively sketched out in Fig. 3. All damage cases cover the entire model length, which is given by half of the transverse frame space. For the numerical models the structural components located in the damaged region are removed because it is assumed that they don't contribute to the remaining load carrying capacity of the hull girder. Therefore, the initial position of the neutral axis shifts upwards.

TABLE 3: SYMMETRIC DAMAGE CASES OF VLCC

Damage Case	Damage Height	Damage Breadth
	[m]	[m]
1	1.065	17.43
2	1.895	40.67
3	3.0	40.67

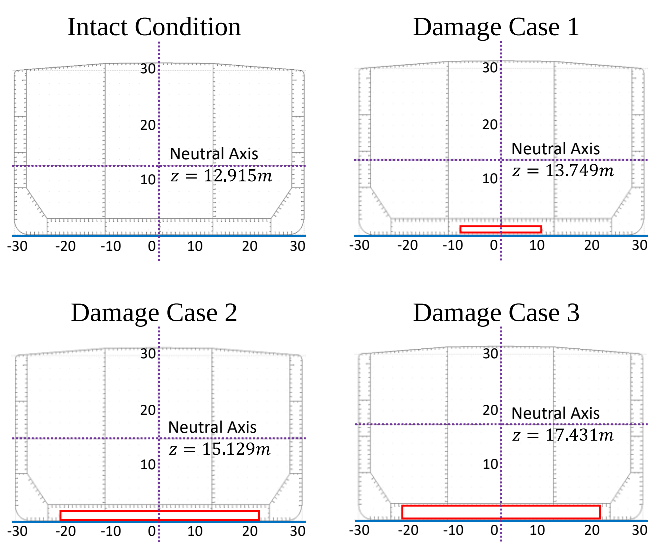

FIGURE 3: DAMAGED CROSS SECTION OF VLCC

2.2 Smith's Method based Model

In framework of Smith's method [3-4], the cross section is subdivided into structural elements composed of stiffeners and plates. In Figure 4 the plate thickness and the stiffener scantlings are proposed. Here, the commercial code MARS2000 provided by the classification society Bureau Veritas (BV) has been used. Different structural elements are applicable to determine the ultimate strength for load cases of pure vertical, horizontal or biaxial bending. In Figure 4 the structural elements of different plate thickness values and stiffener scantlings are shown for the intact VLCC vessel and used to model the midship cross section. Due to symmetry, only half of the cross section is shown. For the proposed damage cases (Table 3) the corresponding structural elements are removed. The cross section remains plane during the progressive collapse analysis and there exists no interaction between adjacent elements in the cross section [1].

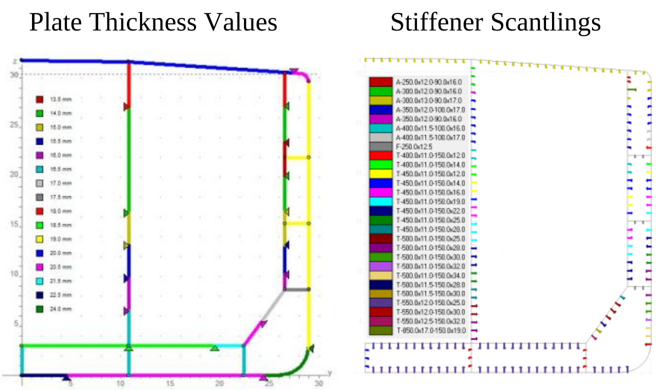

FIGURE 4: CROSS SECTIONAL ELEMENTS OF VLCC

2.3 Finite Element Model

The static implicit ANSYS solver has been used to perform nonlinear finite element analyses. A half transverse frame space and parametric finite element model of the VLCC vessel is developed by subdividing the cross section in sub-sections as shown in Fig. 5. All structural components are modelled with four-node bilinear shell elements (SHELL181) with six degrees of freedom per node and used with reduced integration.

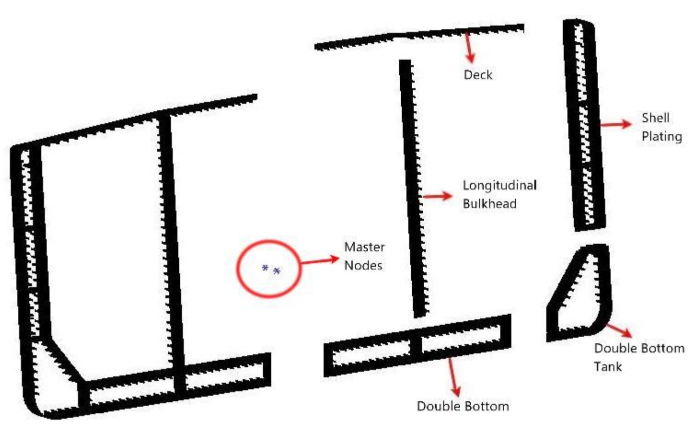

FIGURE 5: FE-MODEL OF VLCC

For all structural components, an elastic-perfectly plastic material behavior is assumed using the material properties given in Table 2. The influence of the tangent modulus for an elastic-linear plastic material behavior on the ultimate hull girder strength is investigated exemplarily for the pure vertical bending load case. In Figure 6 the discretization scheme used for all stiffened plate panels is proposed. The provided mesh is based on a convergence study proposed also for the vertical bending load case. Symmetry conditions have been applied. The loads are imposed by constraint equations of master nodes (Fig. 5) and shell element nodes located at the model edges. Transverse members are not modelled and replaced by boundary conditions.

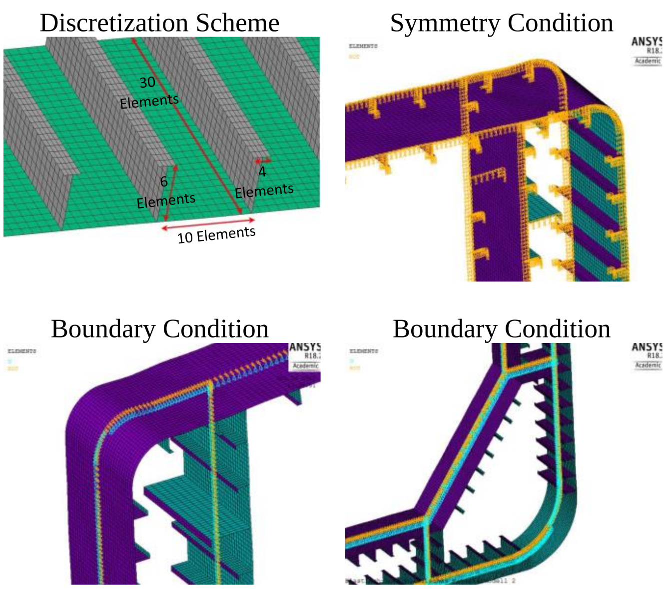

FIGURE 6: CROSS SECTION DETAILS OF VLCC

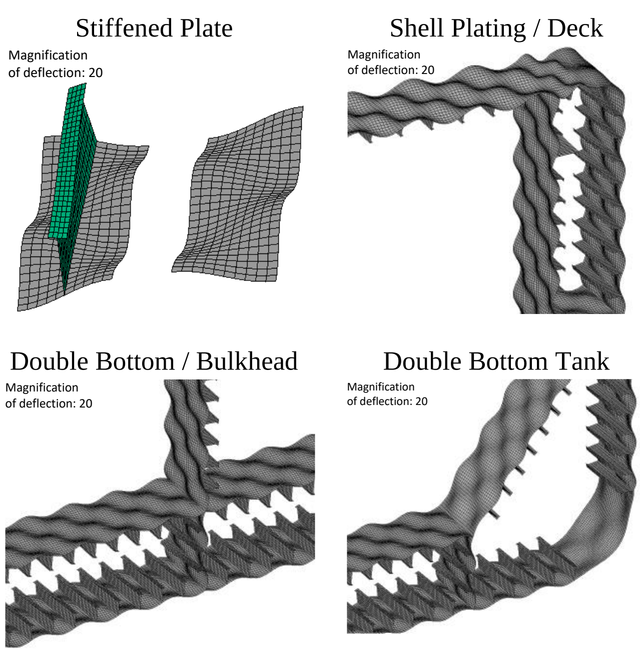

FIGURE 7: INITIAL DEFLECTIONS OF VLCC

In Figure 7 the principal distribution of initial deflections due to welding are shown for different structural components. For a stiffened plate the initial deflections are composed of the vertical panel deflection as well as the horizontal and vertical deflection of the stiffener. The applied concept is described within the ISSC report [5] respectively by Yao and Fujikubo [1].

The influence of welding residual stresses on the hull girder ultimate strength is investigated for the VLCC vessel based on the concept proposed by Fujikubo and Yao [22]. The magnitude of welding residual stresses depends on the heat input

$$\Delta Q = 78.8 \cdot f^2 \tag{1}$$

where f is the leg length of the weld [22]. The breadth of tensile yield stress regions can be determined for fillet and butt welds as

$$b_{tp} = \frac{t_w}{2} + 0.26\frac{\Delta Q}{(t_w + 2t_p)} \tag{2}$$

$$b_{ts} = \frac{t_w}{t_p} + 0.26\frac{\Delta Q}{(t_w + 2t_p)} \tag{3}$$

$$b_{te} = \frac{t_p}{2} + 0.13\frac{\Delta Q}{t_p} \tag{4}$$

depending on the heat input ΔQ, web and plate thickness (t_w; t_p). The resulting compressive stress σ_c is in equilibrium with the tensile yield stresses σ_t. In Figure 8 the initial stress distribution is shown for a stiffened plate panel. The concept is applied to the entire VLCC vessel as shown exemplarily in Figure 9.

$$\sigma_c = \frac{\sum b_{tei}t_{pi}\sigma_{tei} + \sum 2b_{tpi}t_{pi}\sigma_{tpi} + \sum b_{tsi}t_{wi}\sigma_{tsi}}{\sum b_i t_{pi} - (\sum b_{tei}t_{pi} + \sum 2b_{tpi}t_{pi}) + \sum b_{csi}t_{wi}} \tag{5}$$

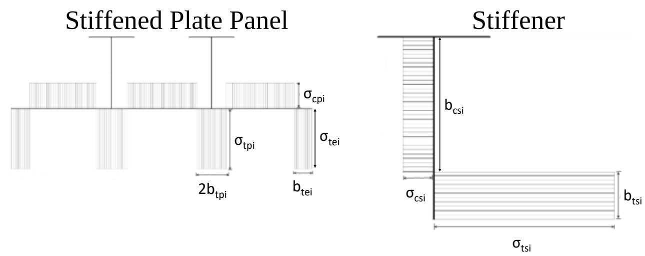

FIGURE 8: MODEL OF WELDING RESIDUAL STRESSES

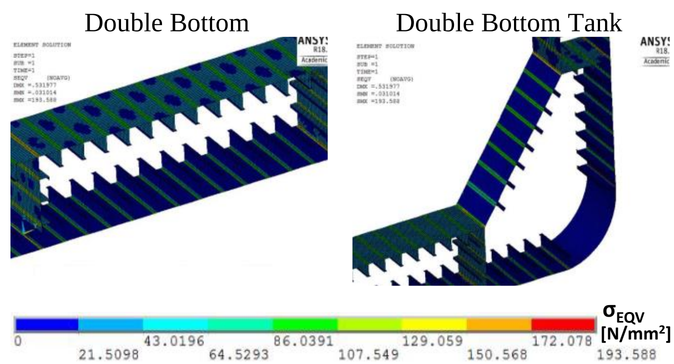

FIGURE 9: WELDING RESIDUAL STRESSES OF VLCC

3. VERTICAL BENDING

The longitudinal strength is the most fundamental aspect of ship's strength, which is the ability to withstand longitudinal bending under operational and extreme loads without suffering failure [5]. In Figure 10 the collapse behavior of the double hull VLCC and the equivalent stress (σ_{EQV}) distribution at ultimate strength are shown for hogging and sagging conditions. Static implicit nonlinear FEA are performed using the discretization scheme proposed in Fig. 6. Under Sagging condition, buckling appears and spreads over the entire deck into the upper part of the side shell structure. In the post-ultimate strength range the load carrying capacity slightly decreases as shown in Fig. 11 by the moment-curvature curve. The bottom is under tension but yielding is hardly observed for the structural members. Under hogging condition, yielding spreads over the deck, which is under tension and buckling takes place in the bottom structure.

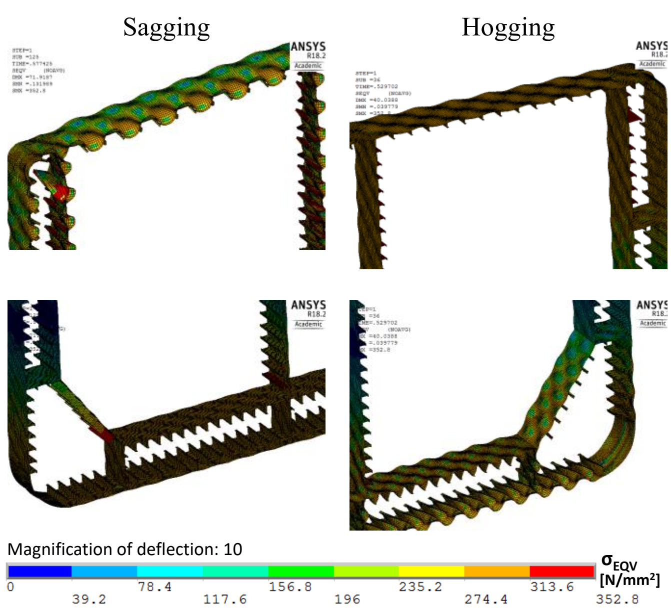

FIGURE 10: INTACT MODEL IN VERTICAL BENDING

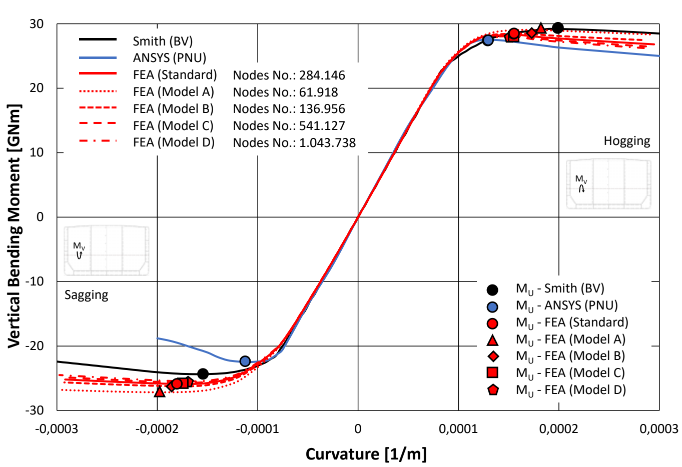

FIGURE 11: CONVERGENCY STUDY

In the post-ultimate strength range, the load-carrying capacity decreases slightly. Different discretized models are investigated, where "Model A" represents the coarse and "Model D" the finest mesh. The "Standard" mesh is used for further investigations because it delivers good results with reduced computational efforts compared to Model D. Only the PNU (ANSYS) FE model [6] delivers lower ultimate strength values for both loading conditions. In Table 4 the ultimate bending moments M_U and the percent difference compared to Smith's method results are given. Changing the material from linear elastic-perfectly plastic [23] to bilinear, then M_U increases, Fig. 12, for relatively large tangent modulus T values. For ultimate strength assessment of structures made of ductile materials an elastic-perfectly plastic material model without strain hardening or necking is often used because strains are usually not significant. This material model leads to a more pessimistic estimation of ultimate strength. [2]

TABLE 4: RESULTS - MATERIAL & MODEL SIZE

Model	M_U Hogging	Percent Diff.	M_U Sagging	Percent Diff.
	[GNm]	[%]	[GNm]	[%]
Smith	29.21	0.00	-24.37	0.00
PNU	27.50	-5.85	-22.50	-7.66
FEA	28.27	-3.23	-25.90	6.30
FEA (A)	29.05	-0.55	-27.17	11.52
FEA (B)	28.47	-2.53	-26.25	7.73
FEA (C)	28.09	-3.83	-25.69	5.46
FEA (D)	27.95	-4.30	-25.50	4.64
FEA (DSDS)	27.88	-4.55	-23.57	-3.26
T = E/100	28.77	-1.50	-26.81	10.03
T = E/50	29.22	0.05	-27.58	13.19
FEA (wrs)	28.36	-2.90	-25.94	6.48
FEA (awrs)	28.01	-4.11	-25.90	6.31

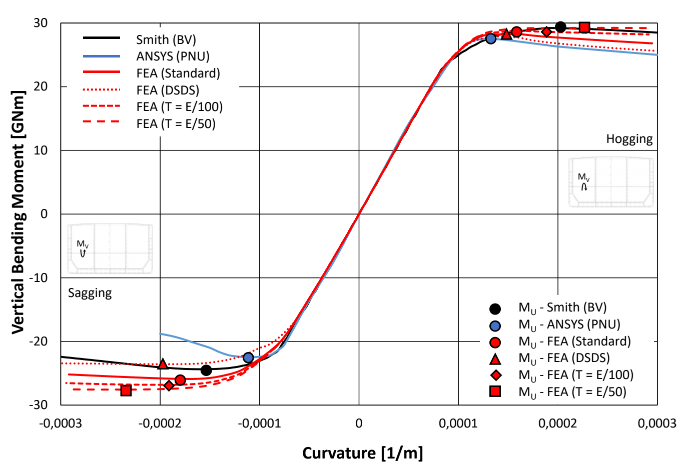

FIGURE 12: MATERIAL & MODEL SIZE

A further reduction of the ultimate strength can be determined by extending the FE model length from half transverse frame space "FEA (Standard)" to full length model between two transverse frames "FEA (DSDS)". The extended FE model is related to much higher numerical efforts due to higher number of elements of similar size. Therefore, the FEA (Standard) model is used to determine the influence of welding residual stresses on the maximum load carrying capacity of the double hull VLCC vessel. In Figure 14 the moment-curvature curve is shown for the FEA (wrs) model including welding residual stresses. The influence of welding residual stresses of plates and stiffened plate panels on the ultimate hull girder strength (Table 4) is very small. The results of the proposed FE model are confirming the conclusions drawn for the VLCC vessel within the ISSC report 2012 [6]. The inclination of the moment-curvature curve slightly changes for artificial welding residual stresses, FEA (awrs), where yielding is assumed over the entire element breadth.

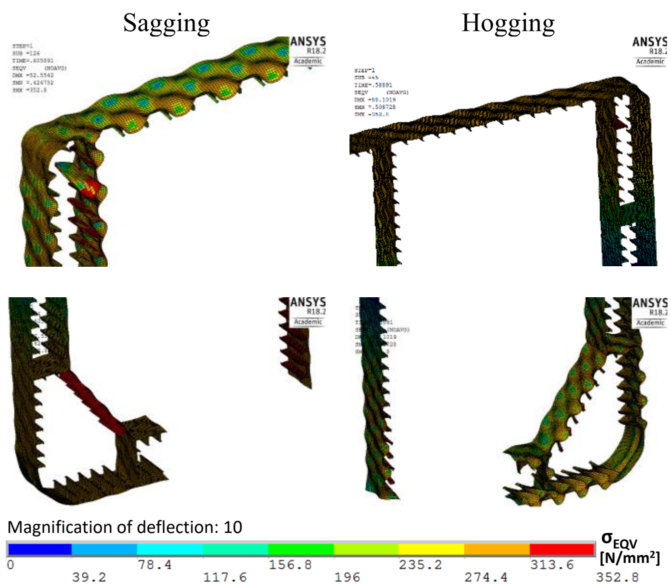

FIGURE 13: DAMAGED MODEL IN VERTICAL BENDING

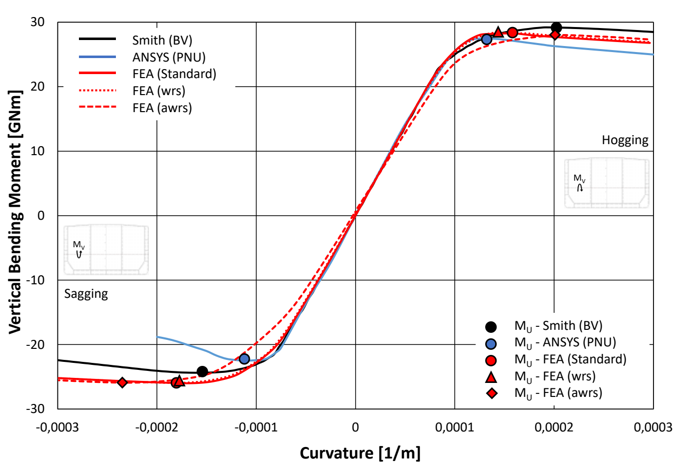

FIGURE 14: WELDING RESIDUAL STRESSES

Therefore, the compressive stress value for the elements between welding lines increases to ensure equilibrium. Even for the artificial welding residual stress model the ultimate strength is hardly reduced as given in Table 4.

In Figure 13 the collapse behavior of the VLCC vessel at ultimate strength is shown exemplarily for damage case 3. For the FE model not only the stiffeners but also the inner bottom plating is removed. The neutral axis shifts closer to the deck than to the double bottom structure. In sagging condition, buckling and yielding can be observed in the deck due to compressive loads. The remaining double bottom structure is under tension and yielding spreads into the side shell. In hogging condition, buckling is observed in the double bottom, lower side shell and longitudinal bulkhead. The upper deck structure, side shell and longitudinal bulkhead are under tension. In Table 5 the ultimate strength values are given for the three damage cases (dam1, dam2, dam3) and compared with the intact VLCC vessel. Furthermore, the percent difference between the FEA (Standard) and Smith's method (MARS2000) are proposed. In Figure 15 the corresponding moment-curvature curves are given. Due to the reduced stiffness for increasing damages, the residual strength and the inclination of the moment-curvature curves decrease.

TABLE 5: RESULTS – DAMAGE CASES

Model	M_U Hogging	Percent Diff.	M_U Sagging	Percent Diff.
	[GNm]	[%]	[GNm]	[%]
Smith	29.21	0.00	-24.37	0.00
Smith (dam1)	27.01	0.00	-23.54	0.00
Smith (dam2)	23.55	0.00	-21.85	0.00
Smith (dam3)	18.33	0.00	-18.34	0.00
FEA	28.27	-3.23	-25.90	6.30
FEA (dam1)	26.16	-3.17	-24.11	2.40
FEA (dam2)	22.87	-2.89	-21.51	-1.54
FEA (dam3)	17.85	-2.59	-17.30	-5.65

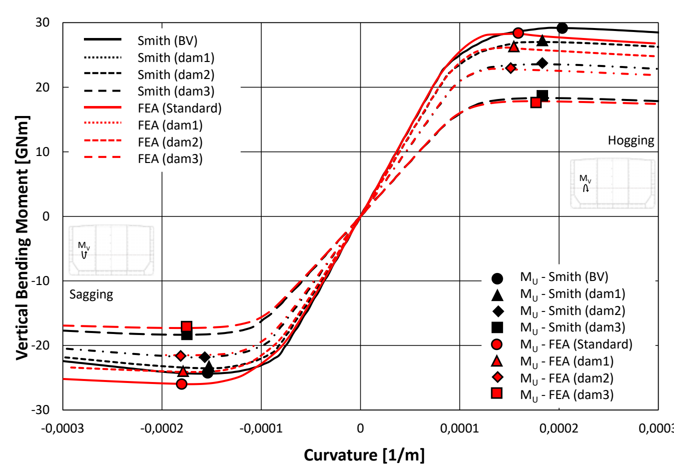

FIGURE 15: DAMAGE CASES

4. HORIZONTAL BENDING

To simulate the collapse behavior of the VLCC vessel in horizontal bending, progressive collapse analyses are performed with the FEA (Standard) model giving incremental rotations about the vertical axis to both master nodes (Figure 5). Inplane displacements are introduced to the outer shell elements nodes due to coupling conditions. In Figure 16 the collapse behavior at ultimate strength is shown for the intact VLCC vessel. At starboard side, the deck, side shell, bottom and the longitudinal bulkhead are under compression. Buckling and yielding already are observed. At portside, the structural components are under tension and yielding starts to spread over structural elements. In Figure 17 the moment curvature curves are shown for the intact vessel. Except the arrangement of stiffeners at the center girder, the cross section is symmetric. Therefore, the absolute ultimate strength values are identical for a positive respectively a negative horizontal bending moment. Due to a higher bending stiffness the ultimate strength is higher than in vertical bending.

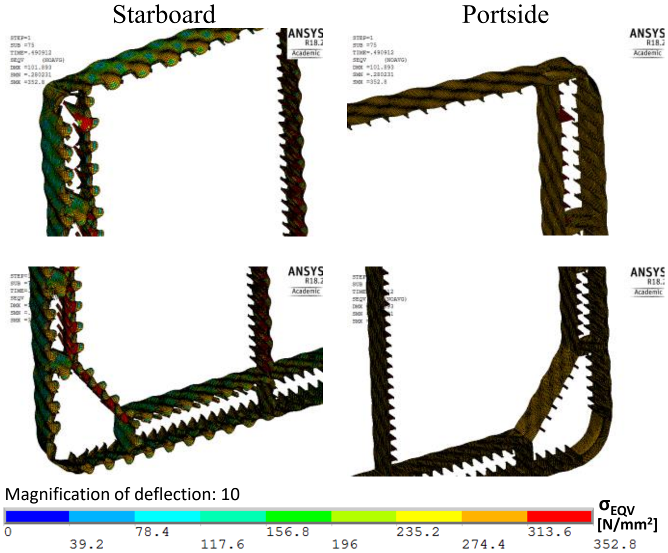

FIGURE 16: INTACT MODEL IN HORIZONTAL BENDING

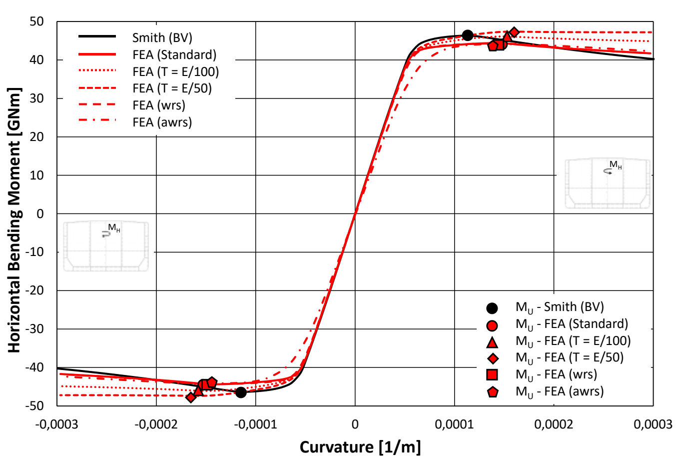

FIGURE 17: MATERIAL & WELDING RESIDUAL STRESSES

In Table 6 the nonlinear FEA results are given and compared to Smith's method. The proposed FE model delivers lower ultimate strength values for positive and negative horizontal bending moments but with slightly higher curvature values. The influence of the tangent modulus in horizontal bending is small. The ultimate strength und the corresponding curvature values increase slightly when the tangent modulus increases. The influence of welding residual stresses, FEA (wrs), on the collapse behavior of the VLCC vessel in horizontal bending is negligible. The artificial welding residual stresses, FEA (awrs), hardly reduce the ultimate strength and the moment-curvature curve is nearly identical compared to the FEA (Standard) curve. In Figure 18 the moment-curvature curves for the three different damage cases are proposed. The ultimate strength as well as the corresponding curvature value decreases when the symmetrical double bottom damage size increases.

TABLE 6: RESULTS – HORIZONATAL BENDING

Model	M_U Positive	Percent Diff.	M_U Negative	Percent Diff.
	[GNm]	[%]	[GNm]	[%]
Smith	46.37	0.00	-46.37	0.00
Smith (dam1)	45.78	0.00	-45.78	0.00
Smith (dam2)	42.57	0.00	-42.57	0.00
Smith (dam3)	39.25	0.00	-39.25	0.00
FEA	44.38	-4.30	-44.38	-4.29
FEA (dam1)	43.91	-4.07	-43.92	-4.06
FEA (dam2)	40.57	-4.71	-40.57	-4.71
FEA (dam3)	37.62	-4.15	-37.62	-4.15
T = E/100	46.12	-0.55	-46.11	-0.55
T = E/50	47.39	2.20	-47.40	2.21
FEA (wrs)	44.30	-4.47	-44.30	-4.48
FEA (awrs)	44.12	-4.84	-44.12	-4.45

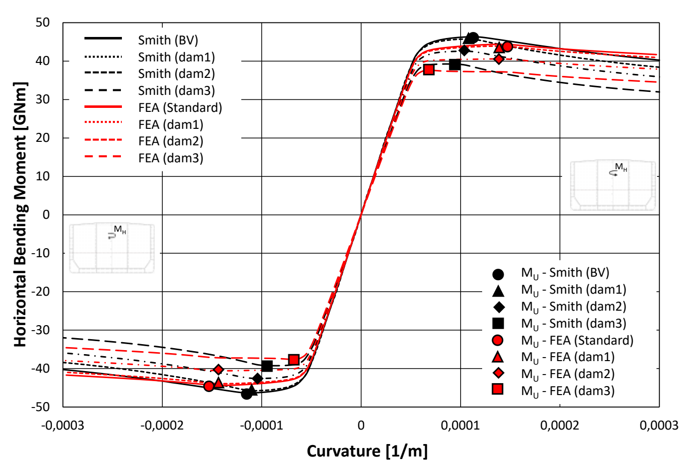

FIGURE 18: DAMAGE CASES

5. BIAXIAL BENDING

Hull girders are in general exposed to combined vertical and horizontal bending moments when the vessel is rolling in an oblique sea [1]. Paik [2] performed progressive collapse analyses for different ships using ALPS/HULL intelligent supersized finite element method and he proposed Eq. (6) to approximate the relation between the vertical (M_V) bending moment (i.e., hogging or sagging) and the horizontal (M_H) bending moment with respect to the ultimate strength values (M_{VU}; M_{HU}).

$$\left(\frac{M_V}{M_{VU}}\right)^{1.85} + \left(\frac{M_H}{M_{HU}}\right) = 1 \tag{6}$$

$$\left(\frac{M_V}{M_{VU}}\right)^{2} + \left(\frac{M_H}{M_{HU}}\right)^{2} = 1 \tag{7}$$

To compare results with a less conservative approach, the ellipse Eq. (7) is introduced in this paper. In Figure 19 and Fig. 21 the collapse behavior at ultimate strength is shown for the VLCC vessel in biaxial bending determined by the FEA (Standard) model, where the absolute rotation increments of both master nodes due to vertical bending (θ_V) and horizontal bending (θ_H) are identical. In Figure 20 the loading paths are shown for different ratios (θ_V/θ_H) and compared to Smith's method based results. Due to symmetry only the results for a positive horizontal bending moment are shown. For the different ratios (θ_V/θ_H) the maximum load carrying capacity of the VLCC determined by nonlinear FEA is smaller compared to Smith's method except for the sagging dominated load cases (θ_V/θ_H <= -5.0). Paik's (2018) estimation formula, Eq. (6), delivers more conservative results. The ultimate strength results determined by progressive collapse analyses are in between the range of Paik's estimation formula and the ellipse, Eq. (7), for sagging and hogging conditions.

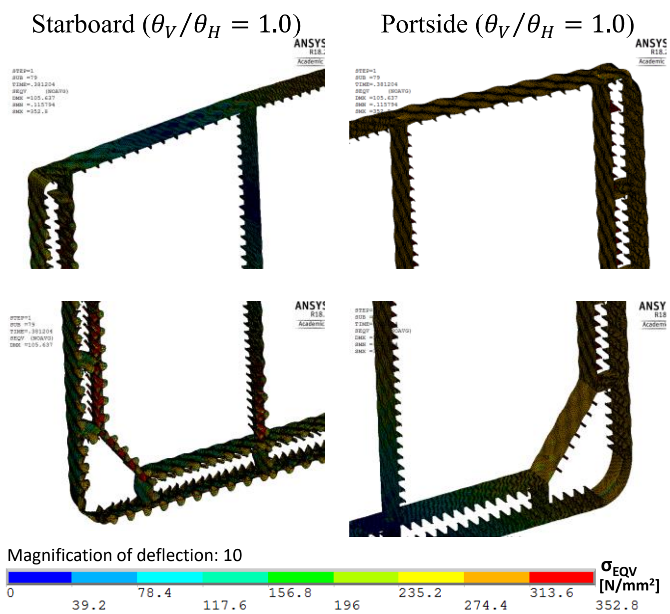

FIGURE 19: INTACT MODEL IN BIAXIAL BENDING

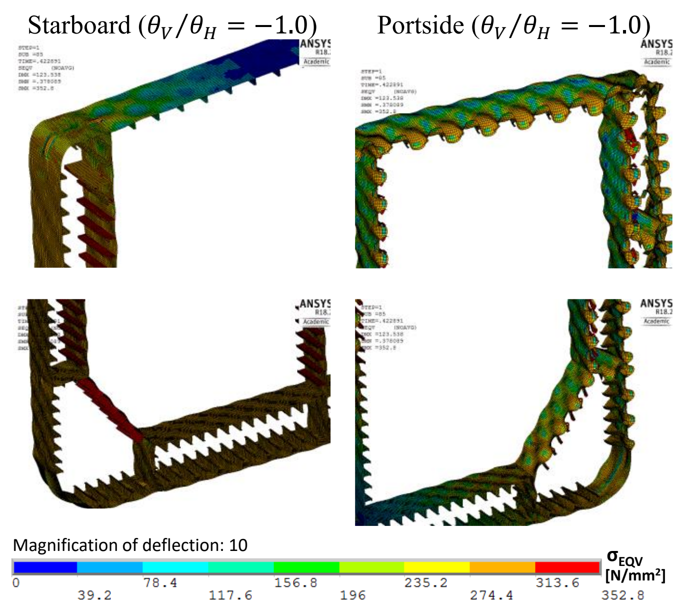

FIGURE 21: INTACT MODEL IN BIAXIAL BENDING

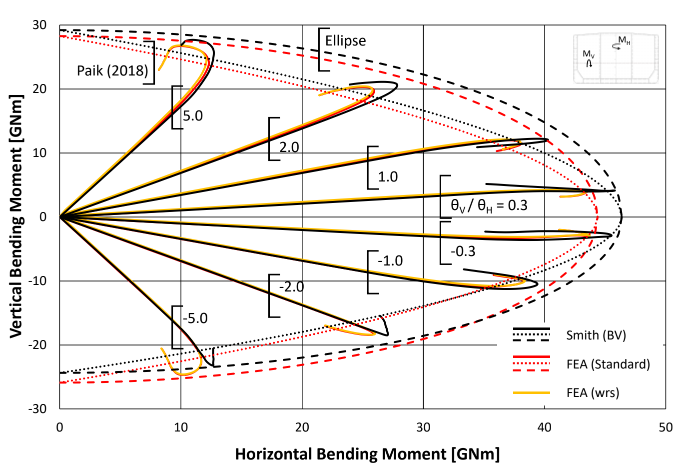

FIGURE 20: INTACT MODEL

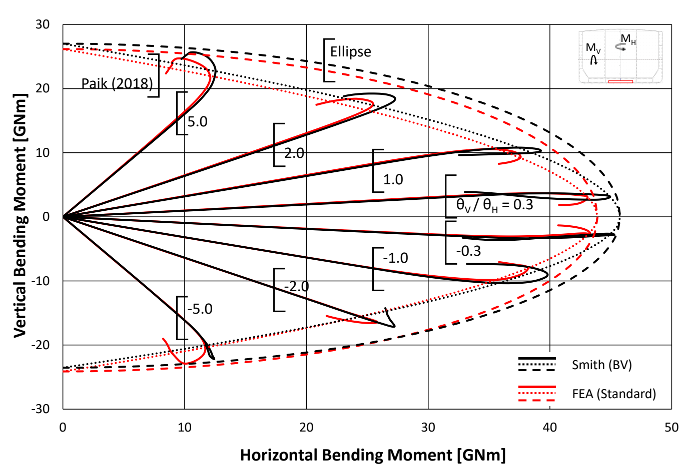

FIGURE 22: DAMAGE CASE 1

The influence of welding residual stresses, FEA (wrs), on the collapse behavior of the intact VLCC vessel in biaxial bending is negligible. Therefore, ultimate strength investigations are performed for the damaged VLCC without initial stresses. In Figure 22 the loading paths are shown for the first damage case (dam1). The maximum load carrying capacity is reduced for the different ratios (θ_V/θ_H) due to the symmetric double bottom damage. In hogging conditions, the FE model delivers lower ultimate strength results compared to Smith's method. In sagging conditions, the differences become smaller for an increasing influence of the vertical bending moment. For Paik's estimation formula, Eq. (6), and the ellipse, Eq. (7), the ultimate strength values (M_{VU}; M_{HU}) belonging to the damage case are used. The nonlinear FEA as well as Smith's method based ultimate strength results are again in between those of both simplified estimation approaches. The same observations are made for the remaining two damage cases, as shown in Fig. 24 and Fig. 26.

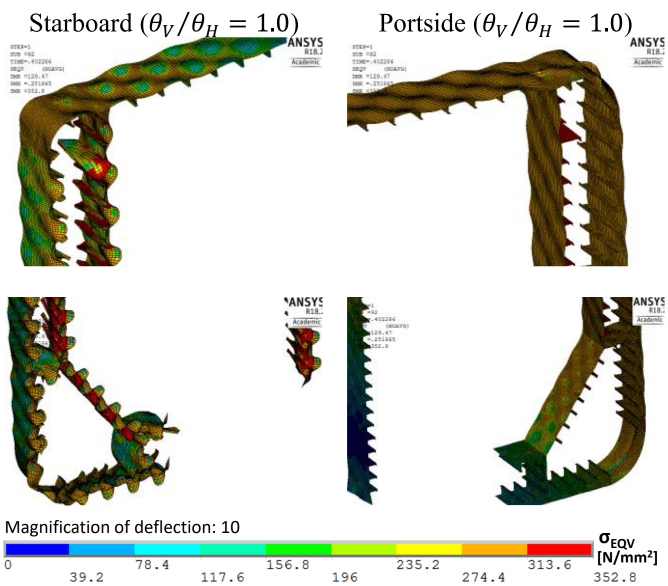

FIGURE 23: DAMAGED MODEL IN BIAXIAL BENDING

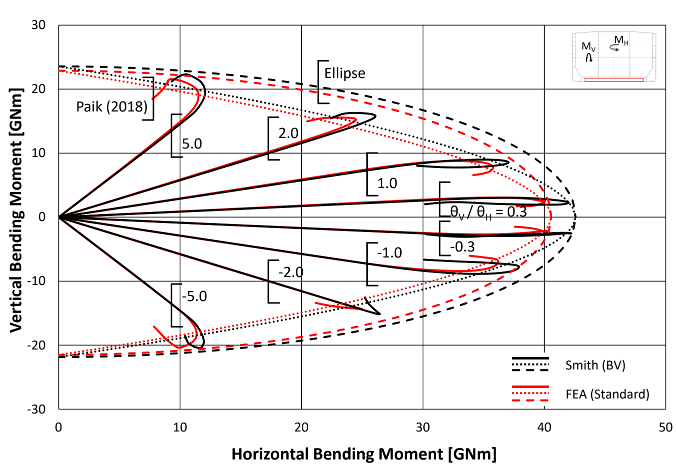

FIGURE 24: DAMAGE CASE 2

6. CONCLUSION

In this paper, nonlinear finite element analyses are successfully performed using a static implicit solver to determine the ultimate strength of a double hull VLCC under pure vertical, horizontal and biaxial bending. The main findings are:

- A relatively fine meshed FE model including welding induced initial deflections and elastic-perfectly plastic material model is used to simulate the collapse behavior with good accuracy.
- The results can be improved by increasing the FE model length but related to increased computational efforts.
- Welding residual stresses have a very small influence on the VLCC progressive collapse behavior.
- Nonlinear FEA deliver more conservative ultimate strength results compared to Smith's method for the intact VLCC and the grounding damage scenarios.

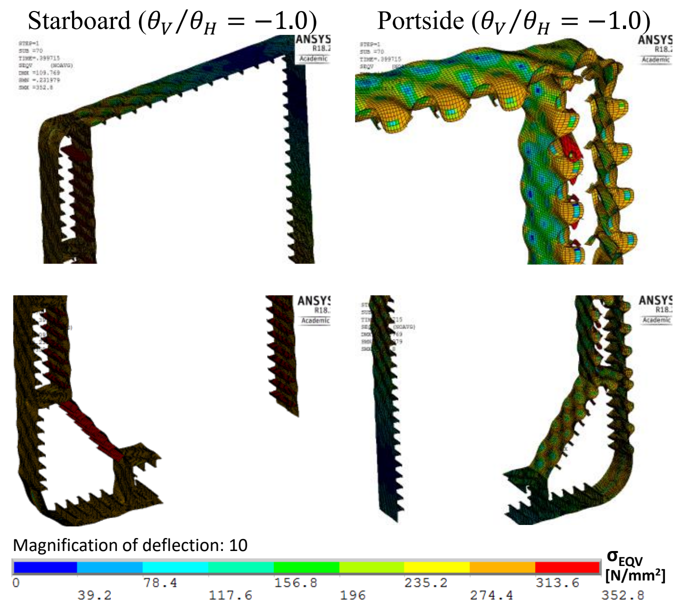

FIGURE 25: DAMAGED MODEL IN BIAXIAL BENDING

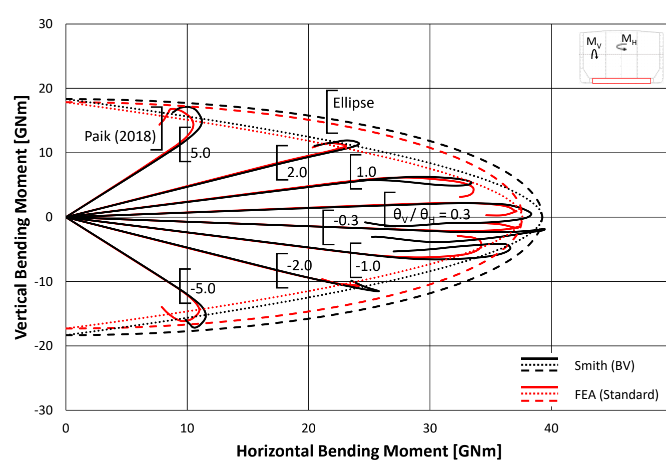

FIGURE 26: DAMAGE CASE 3

REFERENCES

[1] Yao, Tetsuya and Fujikubo, Masahiko. *Buckling and Ultimate Strength of Ship and Ship-like Floating Structures*. Butterworth-Heinemann, Elsevier Inc. Amsterdam (2016).

[2] Paik, Jeom K. *Ultimate Limit State Analysis and Design of Plated Structures*. 2nd ed. Wiley. Chichester (2018).

[3] Smith, CS. "Influence of Local Compressive Failure on Ultimate Longitudinal Strength of a Ship's Hull." *Proceedings of the International Symposium on PRADS*, pp. 73-79. Tokyo, Japan, 1977.

[4] Smith CS. "Structural Redundancy and Damage Tolerance in Relation to Ship Hull Strength." *Proceedings of International Symposium on the Role of Design, Inspection and Redundancy in Marine Structural Reliability*, pp. 91-117. Williamsburg, USA, 1983.

[5] ISSC Committee VI.2: Ultimate Hull Girder Strength. *Proceedings of the 14th International Ship and Offshore Structure Congress*, Vol 2: pp. 321-391, Nagasaki, Japan, 2000.

[6] ISSC Committee III.1: Ultimate Strength. *Proceedings of the 18th International Ship and Offshore Structure Congress*, Vol 1: 285-363, Rostock, Germany, 2012.

[7] La Ferlita, Alessandro, Rathje, Helge, Lindemann, Thomas, Kaeding, Patrick and Bronsart, Robert. "An Advanced Salvage Method for Damaged Ship Structures." *Proceedings of the SNAME Maritime Convention 2021*, Providence RI, USA, October 27-29, 2021.

[8] Tatsumi, Akira, Iijima, Kazuhiro and Fujikubo, Masahiko. "A Study on Progressive Collapse Analysis of a Hull Girder Using Smith's Method – Uncertainty in the Ultimate Strength Prediction." Lecture Notes in Civil Engineering Vol. 64 LNCE (2021) *Proceedings of the 14th International Symposium on Practical Design of Ships and Other Floating Structures*, pp. 128-144. Yokohama, Japan, September 22-29, 2019.

[9] Ueda, Yukio and Rashed, Sherif M.H. "The Idealized Structural Unit Method and its Application to Deep Girder Structures." *Computers & Structures*, Vol. 18 No. 2 (1984): pp. 277-293.

[10] Oksina, Anna, Lindemann, Thomas, Kaeding, Patrick and Fujikubo, Masahiko. "Idealized Structural Unit Method - A Review of the Current Formulation." *Proceedings of the OMAE 2016 International Conference on Ocean, Offshore & Arctic Engineering*, OMAE2016-54186, Busan, South Korea, June 19-24, 2016.

[11] Lindemann, Thomas and Kaeding, Patrick. "Application of the Idealized Structural Unit Method for Ultimate Strength Analyses of Stiffened Plate Structures." *Ship Technology Research (Schiffstechnik)*, Taylor & Francis, Vol. 64 No. 1 (2017): pp. 15-29.

[12] Bathe, Klaus-Jürgen. *Finite Element Procedures*, Prentice–Hall, New Jersey (1996).

[13] Lindemann, Thomas and Kaeding, Patrick. "Investigations on the Influence of Shear and Lateral Loads on the Collapse Behaviour of Plate Structures under Inplane Thrust." *Ship Technology Research (Schiffstechnik)*, Taylor & Francis, Vol. 66 No. 1 (2019): pp. 38-56.

[14] Lindemann, Thomas, Backhaus, Eldor and Kaeding, Patrick. "Experimental Determination of the Ultimate Strength of Box Girder Specimens." *Proceedings of the ASME 2016 35th International Conference on Ocean, Offshore and Arctic Engineering*, OMAE2016-54140, Busan, South Korea, June 19-24, 2016.

[15] Lindemann, Thomas, Backhaus, Eldor, Bi, Zhihan and Kaeding, Patrick. "Ultimate Strength of Box Girders considering Welding Residual Stresses." Sustainable Development and Innovations in Marine Technologies, *Proceedings of the 18th International Congress of the International Maritime Association of the Mediterranean (IMAM2019)*, pp. 11-18, Varna, Bulgaria, September 9-11, 2019.

[16] Darie Ionel, Rörup Jörg and Wolf Viktor. „Ultimate Strength of a Cape Size Bulk Carrier under Combined Global and Local Loads." *Proceedings of the 12th International Symposium on Practical Design of Ships and Other Floating Structures*, Vol 2: pp. 1173-1180. Changwon, Korea, October 20-25, 2013.

[17] Darie, Ionel and Rörup, Jörg. "Hull girder ultimate strength of container ships in oblique sea." Progress in the Analysis and Design of Marine Structures. *Proceedings of the 6th International Conference on Marine Structures*, pp. 225-234, Lisbon, Portugal, May 8-10, 2017.

[18] Tatsumi, Akira and Fujikubo, Masahiko. "Ultimate strength of container ships subjected to combined hogging moment and bottom local loads part 1: Nonlinear finite element analysis." Marine Structures, 69: (2020): https://doi.org/10.1016/j.marstruc.2019.102683

[19] Toh, Kimihiro, Maeda, Shunsuke and Yoshikawa, Takao. "A Study on the Simplified Calculation Method for the Residual Ultimate Strength of Damaged Hull Structures." *Proceedings of the OMAE 2016 International Conference on Ocean, Offshore & Arctic Engineering*, OMAE2016-54673, Busan, South Korea, June 19-24, 2016.

[20] Kuznecovs, Artjoms, Ringsberg, Jonas W, Johnson, Erland and Yamada, Yasuhira. "Ultimate limit state analysis of a double-hull tanker subjected to biaxial bending in intact and collision-damaged conditions." Ocean Engineering, Elsevier, 209 (2020). DOI10.1016/j.oceaneng.2020.107519

[21] Zhu, Zhiyao, Ren, Huilong, Li, Chenfeng and Zhou, Xueqian. "Ultimate Limit State Function and Its Fitting Method of Damaged Ship under Combined Loads." *Journal of Marine Science and Engineering*, 8, 117 (2020). DOI10.3390/jmse8020117

[22] Fujikubo Masahiko and Yao Tetsuya. "Elastic Local Buckling Strength of Stiffened Plate considering Plate/Stiffener Interaction and Welding Residual Stress." *Marine Structures*, 12: (1999): pp. 543-564.

[23] Yao, Tetsuya, Fujikubo, Masahiko, Kondo, Kimio and Nagahama, Shin-ichi. "Progressive Collapse Behaviour of Double Hull Tanker Under Longitudinal Bending." *Proceedings of the Fourth International Offshore and Polar Engineering Conference*, pp. 570-577, Osaka, Japan, April 10-15, 1994.